A new construction of pooling designs based on bilinear forms

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Abstract

In [H. Ngo, D. Du, New constructions of non-adaptive and error-tolerance pooling designs, Discrete Math. 243 (2002) 167–170], by using subspaces in a vector space Ngo and Du constructed a family of well-known pooling designs. In this paper, we construct a family of pooling designs by using bilinear forms on subspaces in a vector space, and show that our design and Ngo-Du's design have the same error-tolerance capability but our design is more economical than Ngo-Du's design under some conditions.

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Key words: Pooling design, disjunct matrix, error-tolerance, bilinear forms

1 Introduction

The basic problem of group testing is to identify the set of defective items in a large population of items. A group test is applicable to an arbitrary subset of items with two possible outcomes: a negative outcome indicates that all items in the subset are negative, and a positive outcome indicates otherwise. A pooling design is a specification of all tests such that they can be performed simultaneously with the goal being to identify all positive items with a small number of tests [1]. A pooling design is usually represented by a binary matrix with columns indexed with items and rows indexed with pools. A cell (i, j) contains a 1-entry if and only if the ith pool contains the jth item. By treating a column as a set of row indices intersecting the column with a 1-entry, we can talk about the union of several columns. A binary matrix is s^e-*disjunct* if every column has at least $e+1$ 1-entries not contained in the union of any other s columns [8]. An s^0-disjunct matrix is also called *s-disjunct*. An s^e-disjunct matrix is called *fully s^e-disjunct* if it is neither $(s+1)^e$-disjunct nor s^{e+1}-disjunct. An s^e-disjunct matrix is $\lfloor e/2 \rfloor$-error-correcting [3, 6, 7].

Let $\mathbb{F}_q^n$ be the n-dimensional vector space over the finite field $\mathbb{F}_q$ and $\begin{bmatrix} [n] \\ k \end{bmatrix}_q$ denote the set of all k-dimensional subspaces of $\mathbb{F}_q^n$. Then the size of the set $\begin{bmatrix} [n] \\ k \end{bmatrix}_q$

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is $\begin{bmatrix} n \\ k \end{bmatrix}_q = \prod_{i=n-k+1}^{n}(q^i - 1)/\prod_{i=1}^{k}(q^i - 1)$. Ngo and Du [9] constructed a family of pooling designs by means of the containment relation of subspaces in $\mathbb{F}_q^n$.

Definition 1.1 ([9]) For positive integers d, k, n with $d < k < n$, let $M_q(d, k, n)$ be the binary matrix with rows indexed with $\begin{bmatrix} [n] \\ d \end{bmatrix}_q$ and columns indexed with $\begin{bmatrix} [n] \\ k \end{bmatrix}_q$ such that $M_q(A, B) = 1$ if and only if $A \subseteq B$.

D'yachkov et al. [2] studied the error-tolerance capability of $M_q(d, k, n)$ and obtained the following result.

Theorem 1.1 ([2]) *If $k-d \geq 2$ and $1 \leq s \leq q(q^{k-1}-1)/(q^{k-d}-1)$, then $M_q(d,k,n)$ is s^e-disjunct, where*

$$e = q^{k-d}\begin{bmatrix} k-1 \\ d-1 \end{bmatrix}_q - (s-1)q^{k-d-1}\begin{bmatrix} k-2 \\ d-1 \end{bmatrix}_q - 1. \tag{1}$$

In particular, if $s \leq \min\{q+1, q(q^{k-1}-1)/(q^{k-d}-1)\}$, then $M_q(d,k,n)$ is fully s^e-disjunct.

In this paper, we construct a family of pooling designs by using bilinear forms on subspaces in a vector space, and show that our design and Ngo-Du's design have the same error-tolerance capability but our design is more economical than Ngo-Du's design under some conditions.

2 Construction

Now we construct a family of pooling designs by means of the containment relation of bilinear forms on subspaces in a vector space.

Let $\mathcal{B}_q(k, m)$ denote the set of all k-pairs (C, f) where C is a k-dimensional subspace of $\mathbb{F}_q^m$ and f is a bilinear form on C. That is, f is a bilinear map from $C \times C$ to $\mathbb{F}_q$. For $(C, f) \in \mathcal{B}_q(k, m)$ and $(D, g) \in \mathcal{B}_q(d, m)$, the pair (D, g) is called a *d-pair* of (C, f) if $D \subseteq C$ and $f|_D = g$, where $f|_D$ is the restriction of f on D. If (D, g) is a d-pair of (C, f), we also say that (C, f) contains (D, g).

Definition 2.1 For positive integers d, k, m with $d < k < m$, let $B_q(d, k, m)$ be the binary matrix with rows indexed with $\mathcal{B}_q(d, m)$ and columns indexed with $\mathcal{B}_q(k, m)$ such that $B_q((D, g), (C, f)) = 1$ if and only if (D, g) is a d-pair of (C, f).

In order to study the matrix $B_q(d, k, m)$, we first introduce an useful lemma.

Lemma 2.1 *Let d, k, m be positive integers with $d \leq k \leq m$. Then*

(i) *The size of the set $\mathcal{B}_q(k, m)$ is $q^{k^2}\begin{bmatrix} m \\ k \end{bmatrix}_q$.*

(ii) *For a given k-pair $(C, f) \in \mathcal{B}_q(k, m)$, let $(C, f)^{(d)}$ be the set of all d-pairs contained in (C, f). Then the size of the set $(C, f)^{(d)}$ is $\begin{bmatrix} k \\ d \end{bmatrix}_q$. Moreover, the function $\begin{bmatrix} x \\ d \end{bmatrix}_q$ about x is strictly increasing for $x \geq d$.*

(iii) *For a given d-pair* $(D, g) \in \mathcal{B}_q(d, m)$*, the number of k-pairs of* $\mathcal{B}_q(k, m)$ *containing* (D, g) *is* $q^{k^2-d^2}\begin{bmatrix} m-d \\ k-d \end{bmatrix}_q$.

Proof. (i) There are $\begin{bmatrix} m \\ k \end{bmatrix}_q$ many k-dimensional subspaces of $\mathbb{F}_q^m$ and there are q^{k^2} bilinear forms on each k-dimensional subspace, so (i) holds.

(ii) There are $\begin{bmatrix} k \\ d \end{bmatrix}_q$ many d-dimensional subspaces of C and the restriction of f on each d-dimensional subspace is unique, so (ii) holds.

(iii) There are $\begin{bmatrix} m-d \\ k-d \end{bmatrix}_q$ many k-dimensional subspaces of $\mathbb{F}_q^m$ containing D and there are $q^{k^2-d^2}$ bilinear forms on each k-dimensional subspace such that their restrictions on D are all g, so (iii) holds. □

Now we introduce main results of this paper.

Theorem 2.2 *Let* $k - d \geq 2$ *and* $1 \leq s \leq q(q^{k-1} - 1)/(q^{k-d} - 1)$*. Then* $B_q(d, k, m)$ *is* s^e*-disjunct, where e as in* (1)*. In particular, if* $s \leq \min\{q+1, q(q^{k-1}-1)/(q^{k-d}-1)\}$*, then* $B_q(d, k, m)$ *is fully* s^e*-disjunct.*

Proof. Let $(C_0, f_0), (C_1, f_1), \ldots, (C_s, f_s)$ be any $s+1$ distinct columns of $B_q(d, k, m)$. By Lemma 2.1 (i) (C_0, f_0) contains $\begin{bmatrix} k \\ d \end{bmatrix}_q$ many d-pairs. For each $1 \leq j \leq s$, let V_{0j} be the largest subspace of $C_0 \cap C_j$ such that $f_0|_{V_{0j}} = f_j|_{V_{0j}}$. Then $(C_0, f_0)^{(d)} \cap (C_j, f_j)^{(d)} = (V_{0j}, f_0|_{V_{0j}})^{(d)}$ for each $1 \leq j \leq s$. To obtain the maximum d-pairs in

$$(C_0, f_0)^{(d)} \cap \bigcup_{j=1}^{s}(C_j, f_j)^{(d)} = \bigcup_{j=1}^{s}(V_{0j}, f_0|_{V_{0j}})^{(d)},$$

by Lemma 2.1 (ii) we may assume that $\dim V_{0j} = k - 1$ for each $1 \leq j \leq s$. Then the size of the set $(V_{01}, f_0|_{V_{01}})^{(d)}$ is $\begin{bmatrix} k-1 \\ d \end{bmatrix}_q$. Since $k - 2 \leq \dim(V_{0j} \cap V_{0l}) \leq k - 1$ for all $1 \leq j, l \leq s$, by Lemma 2.1 (i) the size of each of the sets

$$(V_{02}, f_0|_{V_{02}})^{(d)} \backslash (V_{01}, f_0|_{V_{01}})^{(d)}, \ldots, (V_{0s}, f_0|_{V_{0s}})^{(d)} \backslash (V_{01}, f_0|_{V_{01}})^{(d)}$$

is at most $\begin{bmatrix} k-1 \\ d \end{bmatrix}_q - \begin{bmatrix} k-2 \\ d \end{bmatrix}_q$. Consequently, the number of d-pairs that belong to $(C_0, f_0)^{(d)}$ but not to $\bigcup_{j=1}^{s}(C_j, f_j)^{(d)}$ is at least

$$\begin{bmatrix} k \\ d \end{bmatrix}_q - \begin{bmatrix} k-1 \\ d \end{bmatrix}_q - (s-1)\left(\begin{bmatrix} k-1 \\ d \end{bmatrix}_q - \begin{bmatrix} k-2 \\ d \end{bmatrix}_q\right) = e + 1.$$

Let $s \leq \min\{q + 1, q(q^{k-1} - 1)/(q^{k-d} - 1)\}$. We show that $B_q(d, k, m)$ is fully s^e-disjunct. Let (U, h) be a $(k - 2)$-pair contained in (C_0, f_0). Then the number of $(k - 1)$-pairs between (U, h) and (C_0, f_0) is equal to the number of $(k - 1)$-dimensional subspaces between U and C_0. Since this number is $q + 1$, we can choose s distinct ones among them, say $(V_j, f_0|_{V_j})$ $(1 \leq j \leq s)$. For each $(V_j, f_0|_{V_j})$, by Lemma 2.1 (iii) we can choose a k-pair (C_j, f_j) such that $V_j = C_0 \cap C_j$ and $f_0|_{V_j} = f_j|_{V_j}$. Then both (C_0, f_0) and (C_j, f_j) contain the same $(k-2)$-pair (U, h). Therefore, the desired result follows. □

Next we show that our design and Ngo-Du's design have the same error-tolerance capability under some conditions.

By Theorem 1.1 and Theorem 2.2, we obtain the following result.

Corollary 2.3 *For positive integers d, k, m, n with $d < k < \min\{m, n\}$, if $k - d \geq 2$ and $1 \leq s \leq \min\{q+1, q(q^{k-1}-1)/(q^{k-d}-1)\}$, then both $B_q(d,k,m)$ and $M_q(d,k,n)$ are fully s^e-disjunct, where e as in* (1).

Theorem 2.4 *For positive integers d, k, n with $d < k < n$, if $M_q(d,k,n)$ is fully s^{e_1}-disjunct and $B_q(d,k,n)$ is fully s^{e_2}-disjunct, then $e_1 = e_2$.*

Proof. Since $M_q(d,k,n)$ is fully s^{e_1}-disjunct, there exist $s+1$ distinct columns $V_0, V_1, \ldots, V_s$ of $M_q(d,k,n)$ such that the number of d-dimensional subspaces of V_0 not contained in $V_1, \ldots, V_s$ is $e_1 + 1$. Pick a bilinear forms f_0 on V_0. Let $V_{0j} = V_0 \cap V_j$ and $f_{0j} = f_0|_{V_{0j}}$ for each $1 \leq j \leq s$. Then $k_j := \dim V_{0j} \leq k-1$ and (V_{0j}, f_{0j}) is a k_j-pair of (V_0, f_0). For each $1 \leq j \leq s$, by Lemma 2.1 (iii) we can choose a $(V_j, f_j) \in \mathcal{B}_q(k,n)$ such that $f_{0j} = f_j|_{V_{0j}}$. Then $(V_0, f_0), (V_1, f_1), \ldots, (V_s, f_s)$ are $s+1$ distinct columns of $B_q(d,k,n)$. By Lemma 2.1 the number of d-pairs that belong to $(V_0, f_0)^{(d)}$ but not to $\bigcup_{j=1}^{s}(V_j, f_j)^{(d)}$ is equal to the number of d-dimensional subspaces of V_0 not contained in $V_1, \ldots, V_s$, it is $e_1 + 1$. Since $B_q(d,k,n)$ is fully s^{e_2}-disjunct, $e_1 \geq e_2$.

Since $B_q(d,k,n)$ is fully s^{e_2}-disjunct, there exist $s+1$ distinct columns (C_0, f_0), (C_1, f_1), …, (C_s, f_s) of $B_q(d,k,n)$ such that the number of d-pairs that belong to $(C_0, f_0)^{(d)}$ but not to $\bigcup_{j=1}^{s}(C_j, f_j)^{(d)}$ is $e_2 + 1$. Let V_{0j} be the largest subspace in the set $\{V \subseteq C_0 \cap C_j \mid f_0|_V = f_j|_V\}$ for each $1 \leq j \leq s$. We assert that $V_{01}, \ldots, V_{0s}$ are s distinct subspaces of C_0. In fact, if $V_{0j} = V_{0l}$ for some $1 \leq j < l \leq s$, without loss of generality, let $V_{01} = V_{02}$. Then the number of d-pairs that belong to $(C_0, f_0)^{(d)}$ but not to $\bigcup_{j=2}^{s}(C_j, f_j)^{(d)}$ is $e_2 + 1$. Pick a d-pair (D, g) that belong to $(C_0, f_0)^{(d)}$ but not to $\bigcup_{j=2}^{s}(C_j, f_j)^{(d)}$. By Lemma 2.1 (iii) we can pick $(\tilde{C}_1, \tilde{f}_1) \in \mathcal{B}_q(k,n)$ such that $(\tilde{C}_1, \tilde{f}_1) \neq (C_0, f_0), D \subseteq \tilde{C}_1$ and $\tilde{f}_1|_D = g$. Then $(C_0, f_0), (\tilde{C}_1, \tilde{f}_1), (C_2, f_2), \ldots, (C_s, f_s)$ are $s+1$ distinct columns of $B_q(d,k,n)$, and the number of d-pairs that belong to $(C_0, f_0)^{(d)}$ but not to $(\tilde{C}_1, \tilde{f}_1)^{(d)} \cup \bigcup_{j=2}^{s}(C_j, f_j)^{(d)}$ is at most e_2, this is a contradiction by $B_q(d,k,n)$ being fully s^{e_2}-disjunct. For each $1 \leq j \leq s$, we choose a $V_j \in \left[{[n] \atop k}\right]_q$ such that $C_0 \cap V_j = V_{0j}$. Then $C_0, V_1, \ldots, V_s$ are $s+1$ distinct columns of $M_q(d,k,n)$ and the number of d-dimensional subspaces of C_0 not contained in $V_1, \ldots, V_s$ is equal to the number of d-pairs that belong to $(C_0, f_0)^{(d)}$ but not to $\bigcup_{j=1}^{s}(C_j, f_j)^{(d)}$, it is $e_2 + 1$. Since $M_q(d,k,n)$ is fully s^{e_1}-disjunct, $e_1 \leq e_2$. □

Next we show that our design is more economical than Ngo-Du's design under some conditions.

Note that $M_q(d,k,n)$ is an $\left[{n \atop d}\right]_q \times \left[{n \atop k}\right]_q$ matrix and $B_q(d,k,m)$ is a $q^{d^2}\left[{m \atop d}\right]_q \times q^{k^2}\left[{m \atop k}\right]_q$ matrix. Since $\left[{x \atop d}\right]_q$ is a strictly increasing continuous function about x, for given

q, k, m, there exists a positive real number x_0 such that $\begin{bmatrix} x_0 \\ d \end{bmatrix}_q = \begin{bmatrix} m \\ d \end{bmatrix}_q q^{d^2}$. Let $n_0 = \lceil x_0 \rceil$. Then $\begin{bmatrix} n_0 \\ d \end{bmatrix}_q \geq q^{d^2} \begin{bmatrix} m \\ d \end{bmatrix}_q$, the row-to-column ratio of $M_q(d, k, n_0)$ is

$$\frac{\begin{bmatrix} n_0 \\ d \end{bmatrix}_q}{\begin{bmatrix} n_0 \\ k \end{bmatrix}_q} = \frac{\prod_{i=d+1}^{k} (q^i - 1)}{\prod_{i=n_0-k+1}^{n_0-d+1} (q^i - 1)}$$

and the row-to-column ratio of $B_q(d, k, m)$ is

$$\frac{q^{d^2} \begin{bmatrix} m \\ d \end{bmatrix}_q}{q^{k^2} \begin{bmatrix} m \\ k \end{bmatrix}_q} = \frac{\prod_{i=d+1}^{k} (q^i - 1)}{q^{k^2-d^2} \prod_{i=m-k+1}^{m-d+1} (q^i - 1)}.$$

If $q^{k^2-d^2} \prod_{i=m-k+1}^{m-d+1} (q^i - 1) > \prod_{i=n_0-k+1}^{n_0-d+1} (q^i - 1)$, then the row-to-column ratio of $B_q(d, k, m)$ is smaller than that of $M_q(d, k, n_0)$. Therefore we obtain the following result.

Theorem 2.5 *Let* $\begin{bmatrix} x_0 \\ d \end{bmatrix}_q = q^{d^2} \begin{bmatrix} m \\ d \end{bmatrix}_q$ *and* $n_0 = \lceil x_0 \rceil$. *If* $q^{k^2-d^2} \prod_{i=m-k+1}^{m-d+1} (q^i - 1) > \prod_{i=n_0-k+1}^{n_0-d+1} (q^i - 1)$, *then* $B_q(d, k, m)$ *is more economical than* $M_q(d, k, n_0)$.

Combining Corollary 2.3 and Theorem 2.5, we obtain the following result.

Corollary 2.6 *Let* d, k, m *be integers with* $1 \leq d < k < m$ *and* $k - d \geq 2$. *Suppose that* $1 \leq s \leq \min\{q + 1, q(q^{k-1} - 1)/(q^{k-d} - 1)\}$ *and* $q^{k^2-d^2} \prod_{i=m-k+1}^{m-d+1} (q^i - 1) > \prod_{i=n_0-k+1}^{n_0-d+1} (q^i - 1)$. *Then both* $B_q(d, k, m)$ *and* $M_q(d, k, n_0)$ *are fully* s^e*-disjunct, where* e *as in* (1) *and* n_0 *as in Theorem 2.5. Moreover,* $B_q(d, k, m)$ *is more economical than* $M_q(d, k, n_0)$

Example 2.1 Let $d = 2, k = 3$ and $s = 2 = q$. By Corollary 2.3, both $B_2(2, 3, m)$ and $M_2(2, 3, n_0)$ are fully 2^6-disjunct. For $5 \leq m \leq 11$, the following table tells us that $B_2(2, 3, m)$ is more economical than $M_2(2, 3, n_0)$, where $n_0 = 7, 8, 9, 10, 11, 12, 13$ corresponding to $m = 5, 6, 7, 8, 9, 10, 11$, respectively:

name	rows	columns
$B_2(2,3,5)$	2480	79360
$M_2(2,3,7)$	2667	11811
$B_2(2,3,6)$	10416	714240
$M_2(2,3,8)$	10795	97155
$B_2(2,3,7)$	42672	6047232
$M_2(2,3,9)$	43435	788035
$B_2(2,3,8)$	172720	49743360
$M_2(2,3,10)$	174251	6347715
$B_2(2,3,9)$	694960	403473920
$M_2(2,3,11)$	698027	50955971
$B_2(2,3,10)$	2788016	3250030080
$M_2(2,3,12)$	2794155	408345795
$B_2(2,3,11)$	11168432	26089457152
$M_2(2,3,13)$	11180715	3269560515

In [4, 5, 6, 10], Nan and the authors of these papers proposed a new model for pooling designs, and generalized Macula's design and Ngo-Du's design. Now we generalize our construction under the new model.

For $(C,f) \in \mathcal{B}_q(k,m)$ and $(D,g) \in \mathcal{B}_q(d,m)$, the pair (W,h) is called the *intersection* of (C,f) and (D,g) if $h = g|_W = f|_W$ where W is the largest subspace of $D \cap C$ such that $g|_W = f|_W$.

Definition 2.2 For positive integers i,d,k,m with $i \le d < k < m$, let $B_q(i;d,k,m)$ be the binary matrix with rows indexed with $\mathcal{B}_q(d,m)$ and columns indexed with $\mathcal{B}_q(k,m)$ such that $B_q((D,g),(C,f)) = 1$ if and only if the intersection of (C,f) and (D,g) is an i-pair.

Lemma 2.7 ([5]) *Suppose* $\max\{0, r+t-m\} \le j \le r$ *and* $j \le t \le m$. *Let* P *be a* t*-dimensional subspace of* $\mathbb{F}_q^m$ *and let* W *be a* j*-dimensional subspace of* P. *Then the number of* r*-dimensional subspaces of* $\mathbb{F}_q^m$ *intersecting* P *at* W *is* $f(j,r,m;t) = q^{(r-j)(t-j)}\begin{bmatrix} m-t \\ r-j \end{bmatrix}_q$. *Moreover the function* $f(j,r,m;t+\alpha)$ *about* α *is decreasing for* $0 \le \alpha \le m+j-t-r$.

Theorem 2.8 *Let* $1 \le s \le q(q^{k-1}-1)/(q^{k-i}-1)$ *and* $m-(s+1)k \ge d-i \ge 1$. *Then* $B_q(i;d,k,m)$ *is an* s^e*-disjunct matrix, where*

$$e = q^{(d-i)((s+1)k+d)}\begin{bmatrix} m-(s+1)k \\ d-i \end{bmatrix}_q \left(q^{k-i}\begin{bmatrix} k-1 \\ i-1 \end{bmatrix}_q - (s-1)q^{k-i-1}\begin{bmatrix} k-2 \\ i-1 \end{bmatrix}_q\right) - 1.$$

Proof. Let $(C_0,f_0),(C_1,f_1),\ldots,(C_s,f_s)$ be any $s+1$ distinct columns of $B_q(i;d,k,m)$. By Theorem 2.2, the number of i-pairs that belong to $(C_0,f_0)^{(i)}$ but not to $\bigcup_{j=1}^{s}(C_j,f_j)^{(i)}$ is at least $q^{k-i}\begin{bmatrix} k-1 \\ i-1 \end{bmatrix}_q - (s-1)q^{k-i-1}\begin{bmatrix} k-2 \\ i-1 \end{bmatrix}_q$. Let $U := C_0 + C_1 + \cdots + C_s$. Then $k \le \dim U \le (s+1)k$. Given an i-pair (W,h) that belongs to $(C_0,f_0)^{(i)}$ but not to

$\bigcup_{j=1}^{s}(C_j, f_j)^{(i)}$. By Lemma 2.7, the number of d-dimensional subspaces D in $\mathbb{F}_q^m$ satisfying $D \cap U = W$ is at least $q^{(d-i)((s+1)k-i)}\begin{bmatrix} m-(s+1)k \\ d-i \end{bmatrix}_q$. For each D, the number of $(D, g) \in \mathcal{B}_q(d, m)$ satisfying $g|_W = h$ is $q^{(d-i)(d+i)}$. Note that the intersection of (D, g) and (C_0, f_0) is (W, h) but the intersection of (D, g) and (C_j, f_j) is not (W, h) for each j. Therefore, the desired result follows. □

Remarks. Similar to Theorems 2.2 (Theorems 2.8), we may obtain new pooling designs from alternating bilinear forms, symmetric bilinear forms or Hermitian forms. We suppress the details.

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