A NOTE ON DERIVATIONS IN SUBTRACTION ALGEBRAS

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ABSTRACT. The aim of this paper is to introduce the notions of f−derivation and symmetric bi-derivation in c−subtraction algebras and to study some properties of these derivations.

1. INTRODUCTION

B. M. Schein [19] considered systems of the form $(\Phi; \circ, \backslash)$, where Φ is a set of functions closed under the composition "$\circ$" of functions (and hence $(\Phi; \circ)$ is a function semigroup) and the set theoretic subtraction "$\backslash$" (and hence $(\Phi; \backslash)$ is a subtraction algebra in the sense of [1]). He proved that every subtraction semigroup is isomorphic to a difference semigroup of invertible functions. B. Zelinka [22] discussed a problem proposed by B. M. Schein concerning the structure of multiplication in a subtraction semigroup. He solved the problem for subtraction algebras of a special type, called the atomic subtraction algebras. Y. B. Jun, H. S. Kim and E. H. Roh [9] introduced the notion of ideals in subtraction algebras and discussed characterization of ideals. In [10], Y. B. Jun and H. S. Kim established the ideal generated by a set, and discussed related results. In [4], Y. Çeven and M. A. Öztürk introduced some additional concepts on subtraction algebras, so called subalgebra, bounded subtraction algebra and union of subtraction algebra.

It is very interesting and important that the similar properties of derivation which is the one of the basic theory in analysis and applied mathematics are also satisfied in the ring theory. The commutativity of prime rings with derivations was initiated by E. Posner in [18]. Over the last two decades, a lot of work has been done on this subject. In [16] M. A. Öztürk and Y. Çeven defined derivation on subtraction algebras and examined some properties of derivation. In [13] and [14], Gy. Maksa defined bi-derivation in ring theory mutually to partial derivations and examined some properties of derivation. After this studied, many observers studied symmetric bi-derivation as derivation in ring theory.

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The aim of this paper is to introduce the notions of $f-$derivation and symmetric bi-derivation in $c-$subtraction algebras and to study some properties of these derivations.

2. Preliminaries

An algebra $(X;-)$ with a single binary operation "$-$" is called subtraction algebra if for all $x,y,z \in X$ the following conditions hold:

$(S1)$ $x-(y-x)=x$,

$(S2)$ $x-(x-y)=y-(y-x)$,

$(S3)$ $(x-y)-z=(x-z)-y$.

The subtraction determines an order relation on X as the following:

$$a \leq b \Leftrightarrow a-b=0$$

where $0=a-a$ is an element of X and this property does not depend on the choice of $a \in X$. The ordered set $(X;\leq)$ is a semi-Boolean algebra in the sense of [1], that is, it is a meet semilattice with zero 0 in which every interval $[0,a]$ is a Boolean algebra with respect to induced order. Here $a \wedge b = a-(a-b)$ and the complement of an element $b \in [0,a]$ is $a-b$.

In a subtraction algebra, the following are true [9], [12]:

$(a1)$ $(x-y)-y=x-y$,

$(a2)$ $x-0=x$ and $0-x=0$,

$(a3)$ $(x-y)-x=0$,

$(a4)$ $x-(x-y)\leq y$,

$(a5)$ $(x-y)-(y-x)=x-y$,

$(a6)$ $x-(x-(x-y))=x-y$,

$(a7)$ $(x-y)-(z-y)\leq x-z$,

$(a8)$ $x\leq y$ if and only if $x=y-w$ for some $w\in X$,

$(a9)$ $x\leq y$ implies $x-z\leq y-z$ and $z-y\leq z-x$ for all $z\in X$,

$(a10)$ $x,y\leq z$ implies $x-y=x\wedge(z-y)$,

$(a11)$ $(x\wedge y)-(x\wedge z)\leq x\wedge(y-z)$,

$(a12)$ $(x-y)-z=(x-z)-(y-z)$.

Definition 1. [9]*A nonempty subset A of a subtraction algebra X is called an ideal of X if it satisfies*

(1) $0\in A$,

(2) $(\forall x\in X)(\forall y\in A)(x-y\in A \Rightarrow x\in A)$

for all $x,y,z\in X$.

Lemma 1. [9]*An ideal A of a subtraction algebra X has the following property*

$$(\forall x\in X)(\forall y\in A)(x\leq y \Rightarrow x\in A).$$

Definition 2. [11]*Let X be a subtraction algebra. For any $a,b\in X$, let $G(a,b)=\{x\in X: x-a\leq b\}$. X is said to be complicated subtraction algebra ($c-$subtraction algebra) if the set $G(a,b)$ has the greatest element for any $a,b\in X$.*

Note that $0, a, b \in G(a, b)$. The greatest element of $G(a, b)$ is denoted $a + b$.

Proposition 1. [11]*If X be a c−subtraction algebra, then for all $x, y \in X$*

(i) $x \leq x + y$, $y \leq x + y$,
(ii) $x + 0 = x = 0 + x$,
(iii) $x + y = y + x$,
(iv) $x \leq y \Rightarrow x + z \leq y + z$,
(v) $x \leq y \Rightarrow x + y = y$,
(vi) $x + y$ *is the least upper bound of x and y.*

Theorem 1. [11]*If X is a c−subtraction algebra, then $(X, +, 0)$ is a commutative monoid.*

Definition 3. [16]*Let X be a c−subtraction algebra and $d : X \to X$ be a function. We call d a derivation on X if it satisfies the following condition*

$$d(x \wedge y) = (d(x) \wedge y) + (x \wedge d(y))$$

for all $x, y \in X$.

3. f−Derivations in Subtraction Algebras

The following definition introduces the notion of f−derivation for a subtraction algebra.

Definition 4. *Let X be a c−subtraction algebra. A function $d_f : X \to X$ is called f−derivation on X if there exists a function $f : X \to X$ such that*

$$d_f(x \wedge y) = (d_f(x) \wedge f(y)) + (f(x) \wedge d_f(y))$$

for all $x, y \in X$.

It is obvious in the Definition 4 that if f is an identity function, then d_f is a derivation on X.

Example 1. *Let $X = \{0, a, b, c\}$ be a c−subtraction algebra with the following Cayley table* [11].

−	0	a	b	c
0	0	0	0	0
a	a	0	a	0
b	b	b	0	0
c	c	b	a	0

Define functions on X,

$$d_f : X \to X \quad , \quad d_f(x) = \begin{cases} 0 & , \quad x = 0, b \\ c & , \quad x = a, c \end{cases}$$

and

$$f : X \to X \quad , \quad f(x) = \begin{cases} 0 & , \quad x = 0, b \\ c & , \quad x = a, c \end{cases} .$$

Then we can see that d_f is a $f-$derivation on X. But d_f isn't a derivation on X.

Example 2. *Let X be the c–subtraction algebra in Example 1 and define functions*

$$d_f : X \to X \quad , \quad d_f(x) = \begin{cases} 0 & , \quad x = 0, b \\ c & , \quad x = a, c \end{cases}$$

and

$$f : X \to X \quad , \quad f(x) = \begin{cases} 0 & , \quad x = 0, b \\ c & , \quad x = a, c \end{cases} .$$

Then we can see that d_f is a $f-$derivation on X. But, d_f is a derivation on X.

Proposition 2. *Let X be a c–subtraction algebra and d_f be a $f-$derivation on X. Then the following properties hold:*

(i) $d_f(x) = d_f(x) \wedge f(x) \leq f(x)$, for all $x \in X$.

(ii) $d_f(x \wedge y) \leq f(x) + f(y)$, for all $x, y \in X$.

(iii) If $f(0) = 0$, then $d_f(0) = 0$.

(iv) If I is an ideal of a subtraction algebra X, then $f(I) \subseteq I$ implies $d_f(I) \subseteq I$.

Proof. (i) Since $x \wedge x = x$ and from $(a4)$, we get

$$\begin{aligned} d_f(x) &= d_f(x \wedge x) \\ &= (d_f(x) \wedge f(x)) + (f(x) \wedge d_f(x)) \\ &= d_f(x) \wedge f(x) \\ &= d_f(x) - (d_f(x) - f(x)) \leq f(x) \end{aligned}$$

for all $x \in X$.

(ii) From $(a4)$, $d_f(x) \wedge f(y) \leq f(y)$ and $d_f(y) \wedge f(x) \leq f(x)$ for all $x, y \in X$. Since $f(x), f(y) \in G(f(x), f(y))$, we have $d_f(x) \wedge f(y) \leq f(x) + f(y)$ and $d_f(y) \wedge f(x) \leq f(x) + f(y)$. Since $d_f(x \wedge y)$ is the least upper bound $d_f(x) \wedge f(y)$ and $d_f(y) \wedge f(x)$, we obtain $d_f(x \wedge y) \leq f(x) + f(y)$.

(iii) Suppose that $f(0) = 0$. From (i), we have $d_f(0) \leq f(0) = 0$. Since, from $(a2)$, $0 \leq d_f(0)$, we get $d_f(0) = 0$.

(iv) From (i), we know that $d_f(x) \leq f(x)$ for all $x \in I$. Then $0, d_f(x) - f(x) = 0 \in I$. From the definition of an ideal of X, we obtain $d_f(x) \in I$. □

Theorem 2. *Let X be a c–subtraction algebra and d_f be a $f-$derivation on X. Then*

$$d_f(x) \wedge d_f(y) \leq d_f(x \wedge y) \leq d_f(x) + d_f(y).$$

Proof. Since $d_f(y) \leq f(y)$, we have $d_f(x) - f(y) \leq d_f(x) - d_f(y)$ and $d_f(x) - (d_f(x) - d_f(y)) \leq d_f(x) - (d_f(x) - f(y))$, that is, $d_f(x) \wedge d_f(y) \leq d_f(x) \wedge f(y)$.

Similarly, since $d_f(x) \leq f(x)$, we obtain $d_f(x) \wedge d_f(y) \leq d_f(y) \wedge f(x)$. Then we get

$$d_f(x) \wedge d_f(y) \leq (d_f(x) \wedge f(y)) + (f(x) + d_f(y)) = d_f(x \wedge y).$$

Furthermore, since $d_f(x) \wedge f(y) \le d_f(x) \le d_f(x) + d_f(y)$ and $d_f(y) \wedge f(x) \le d_f(y) \le d_f(x) + d_f(y)$, we have $d_f(x \wedge y) \le d_f(x) + d_f(y)$. □

Proposition 3. *Let X be a c−subtraction algebra and d_f be a f−derivation on X. If f is an increasing function on X, then*

(i) $f(x) \le d_f(x+y) \Rightarrow f(x) = d_f(x)$ *for all* $x, y \in X$,
(ii) $f(x) \ge d_f(x+y) \Rightarrow d_f(x) \ge d_f(x+y)$ *for all* $x, y \in X$.

Proof. (i) Since $x = x \wedge (x+y)$, we obtain

$$\begin{aligned} d_f(x) &= d_f(x \wedge (x+y)) \\ &= (d_f(x) \wedge f(x+y)) + (f(x) \wedge d_f(x+y)). \end{aligned}$$

From Proposition 2(i) and since f is an increasing function, $d_f(x) \le f(x) \le f(x+y)$. Therefore

$$d_f(x) = d_f(x) + f(x) = f(x)$$

for all $x \in X$.

(ii) Since $x = x \wedge (x+y)$, we get

$$\begin{aligned} d_f(x) &= d_f(x \wedge (x+y)) \\ &= (d_f(x) \wedge f(x+y)) + (f(x) \wedge d_f(x+y)) \\ &= d_f(x) + d_f(x+y). \end{aligned}$$

Hence $d_f(x+y) \le d_f(x)$ for all $x, y \in X$. □

Let X be a c−subtraction algebra and d_f be a f−derivation on X. Define a set $F := \{x \in X : f(x) = d_f(x)\}$.

Proposition 4. *Let X be a c−subtraction algebra and d_f be a f−derivation on X. If f is an increasing function on X, then $y \le x$ and $x \in F$ imply $y \in F$.*

Proof. Let $y \le x$ and $x \in F$. Since f is an increasing function, we obtain $d_f(y) \le f(y) \le f(x) = d_f(x)$. Note that,

$$\begin{aligned} d_f(y) = d_f(x \wedge y) &= (d_f(x) \wedge f(y)) + (f(x) \wedge d_f(y)) \\ &= f(y) + d_f(y). \end{aligned}$$

Therefore $f(y) \le d_f(y)$, that is, $d_f(y) = f(y)$. □

Definition 5. *Let X be a c−subtraction algebra and d_f be a f−derivation on X. If $x \le y$ implies $d_f(x) \le d_f(y)$, then d_f is called an isotone mapping.*

Proposition 5. *Let X be a c−subtraction algebra and d_f be a f−derivation on X. If d_f is an isotone mapping and f is decreasing function on X, then $x, y \in F$ implies $x + y \in F$.*

Proof. Since $x \leq x+y$ and $y \leq x+y$, we have $f(x+y) \leq f(x)$, $f(x+y) \leq f(y)$ and $d_f(x) \leq d_f(x+y)$, $d_f(y) \leq d_f(x+y)$. From the definition of set F, we get

$$f(x+y) \leq f(x)+f(y) = d_f(x)+d_f(y) \leq d_f(x+y).$$

Also $d_f(x+y) \leq f(x+y)$. Thus $d_f(x+y) = f(x+y)$, that is, $x+y \in F$. □

4. Symmetric Bi-derivations in Subtraction Algebras

Definition 6. *Let X be a c−subtraction algebra. A mapping $D : X \times X \to X$ is called symmetric if it is satisfies following condition*

$$D(x,y) = D(y,x)$$

for all $x,y \in X$.

A mapping $d : X \to X$ defined by $d(x) = D(x,x)$ is called the trace of D, where D is a symmetric mapping.

Definition 7. *Let X be a c−subtraction algebra. A symmetric mapping $D : X \times X \to X$ is called symmetric bi-derivation if it is satisfies following condition*

$$D(x \wedge z, y) = (D(x,y) \wedge z) + (x \wedge D(z,y))$$

for all $x,y,z \in X$.

It is obvious that if D is a symmetric bi-derivation, then

$$D(x, y \wedge z) = (D(x,y) \wedge z) + (y \wedge D(x,z))$$

for all $x,y,z \in X$.

Example 3. *Let X be the c−subtraction algebra in Example 1 and define function on X,*

$$D : X \times X \to X \quad , \quad D(x,y) = \begin{cases} 0 \ , & (x,y) = (0,0) \\ 0 \ , & (x,y) = (0,a) \ or \ (x,y) = (a,0) \\ 0 \ , & (x,y) = (0,b) \ or \ (x,y) = (b,0) \\ 0 \ , & (x,y) = (0,c) \ or \ (x,y) = (c,0) \\ a \ , & (x,y) = (a,a) \\ b \ , & (x,y) = (b,b) \\ c \ , & (x,y) = (c,c) \\ 0 \ , & (x,y) = (a,b) \ or \ (x,y) = (b,a) \\ a \ , & (x,y) = (a,c) \ or \ (x,y) = (c,a) \\ b \ , & (x,y) = (b,c) \ or \ (x,y) = (c,b) \end{cases}.$$

Then we can see that D is a symmetric bi-derivation on X.

Remark 1. *Let X be a c−subtraction algebra and D be a symmetric bi-derivation on X. In this case, for any fixed $a \in X$ and for all $x \in X$, a mapping $d_a : X \to X$ defined by $d_a(x) = D(x,a)$ is a derivation on X.*

Proposition 6. *Let X be a c−subtraction algebra and D be a symmetric bi-derivation on X with the trace d. Then $d(x) \leq x$ for all $x \in X$.*

Proof. Since $x \wedge x = x$, we have

$$\begin{aligned} d(x) &= D(x,x) \\ &= D(x \wedge x, x) \\ &= (D(x,x) \wedge x) + (x \wedge D(x,x)) \\ &= D(x,x) \wedge x = d(x) \wedge x \end{aligned}$$

for all $x \in X$. Therefore $d(x) \leq x$ for all $x \in X$ by $(S2)$ and $(a4)$. □

Proposition 7. *Let X be a c−subtraction algebra and D be a symmetric bi-derivation on X with the trace d. Then $D(x,y) \leq x$, $D(x,y) \leq y$ for all $x, y \in X$.*

Proof. Since $x \wedge x = x$, we obtain

$$\begin{aligned} D(x,y) &= D(x \wedge x, y) \\ &= (D(x,y) \wedge x) + (x \wedge D(x,y)) \\ &= D(x,y) \wedge x \end{aligned}$$

for all $x \in X$. Therefore $D(x,y) \leq x$ for all $x, y \in X$ by $(S2)$ and $(a4)$. Similarly, we see that $D(x,y) \leq y$ for all $x, y \in X$. □

Proposition 8. *Let X be a c−subtraction algebra and D be a symmetric bi-derivation on X with the trace d. Then*

(i) $d(G(a,b)) \subseteq G(a,b)$,
(ii) $G(d(a), d(b)) \subseteq G(a,b)$,
(iii) $d(x+y) \leq x+y$ for all $x, y \in X$,
(iv) $d(x) + d(y) \leq x+y$ for all $x, y \in X$,
(v) If I is an ideal of a subtraction algebra X, then $d(I) \subseteq I$.

Proof. (i) For all $z \in G(a,b)$, we have $z - a \leq b$. From Proposition 6, we get $d(z) \leq z$ and $d(z) - a \leq z - a \leq b$ by $(a9)$. Hence we obtain $d(z) \in G(a,b)$.

(ii) For all $z \in G(d(a), d(b))$, $z - d(a) \leq d(b) \leq b$. Hence we have $z - b \leq d(a) \leq a$ or $z - a \leq b$. Then we get $z \in G(a,b)$.

(iii) and (iv) are straightforward from Proposition 6.

(v) For all $x \in I$, we know that $d(x) \leq x$, that is, $d(x) - x = 0 \in I$. From the definition of an ideal of X, we obtain $d(x) \in I$. □

Theorem 3. *Let X be a c−subtraction algebra and D be a symmetric bi-derivation on X with the trace d. Then*

$$d(x \wedge y) = (d(x) \wedge y) + D(x,y) + (x \wedge d(y))$$

for all $x, y \in X$.

Proof. Note that

$$\begin{aligned} d(x \wedge y) &= D(x \wedge y, x \wedge y) \\ &= (D(x, x \wedge y) \wedge y) + (x \wedge D(y, x \wedge y)) \\ &= \{[(D(x,x) \wedge y) + (x \wedge D(x,y))] \wedge y\} \\ &\qquad + \{x \wedge [(D(y,x) \wedge y) + (x \wedge D(y,y))]\} \\ &= \{[(d(x) \wedge y) + D(x,y)] \wedge y\} + \{x \wedge [D(x,y) + (x \wedge d(y))]\} \end{aligned}$$

for all $x, y \in X$. Since $d(x) \wedge y \leq y$ by $(a4)$ and $D(x,y) \leq y$ by Proposition 7, we have $(d(x) \wedge y) + D(x,y) \leq y$. Similarly, we get $D(x,y) + (x \wedge d(y)) \leq x$. Therefore we obtain

$$d(x \wedge y) = [(d(x) \wedge y) + D(x,y)] + [D(x,y) + (x \wedge d(y))].$$

From Theorem 1, we have

$$d(x \wedge y) = (d(x) \wedge y) + D(x,y) + (x \wedge d(y)).$$

□

Corollary 1. *Let X be a c−subtraction algebra and D be a symmetric bi-derivation on X with the trace d. Then*

(i) $D(x,y) \leq d(x \wedge y)$,
(ii) $x \wedge d(y) \leq d(x \wedge y)$,
(iii) $d(x) \wedge y \leq d(x \wedge y)$,
(iv) $d(x) \wedge d(y) \leq d(x \wedge y)$.

Proof. (i), (ii) and (iii) are straightforward from Theorem 3.

(iv) Since $d(y) \leq y$, we get $d(x) - y \leq d(x) - d(y)$ and $d(x) - (d(x) - d(y)) \leq d(x) - (d(x) - y)$, that is, $d(x) \wedge d(y) \leq d(x) \wedge y$. Therefore, from (iii), we obtain that $d(x) \wedge d(y) \leq d(x \wedge y)$. □

Corollary 2. *Let X be a c−subtraction algebra and D be a symmetric bi-derivation on X with the trace d. Then $D(x, x+y) \leq d(x \wedge (x+y)) = d(x)$ and $x \wedge d(x+y) \leq d(x \wedge (x+y)) = d(x)$ for all $x, y \in X$.*

Corollary 3. *Let X be a c−subtraction algebra and D be a symmetric bi-derivation on X with the trace d. Then*

(i) $x \geq d(x+y) \Rightarrow d(x) \geq d(x+y)$,
(ii) $x \leq d(x+y) \Rightarrow x = d(x+y)$,
(iii) $x \leq y$ and $d(y) = y \Rightarrow x = d(x)$.

Proof. (i) Let $x \geq d(x+y)$ for all $x, y \in X$. Since $d(x+y) = x \wedge d(x+y) \leq d(x)$, we have $d(x+y) \leq d(x)$ for all $x, y \in X$.

(ii) Let $x \leq d(x+y)$ for all $x, y \in X$. Since $x = x \wedge d(x+y) \leq d(x)$ and $d(x) \leq x$, we get $x = d(x)$.

(iii) Let $x \leq y$ and $d(y) = y$ for all $x, y \in X$. Since $x \leq y$, $x \wedge y = x$. Hence

$$d(x) = d(x \wedge y)$$

$$= (d(x) \wedge y) + D(x, y) + (x \wedge d(y)) = x$$

for all $x, y \in X$. □

Definition 8. *Let X be a c−subtraction algebra and D be a symmetric bi-derivation on X with the trace d. If $x \leq y$ implies $d(x) \leq d(y)$, then d is called an isotone mapping.*

Remark 2. *Let X be a c−subtraction algebra and D be a symmetric bi-derivation on X with the trace d. Denote $Fix_d(X) = \{x \in X : d(x) = x\}$. From Corollary 3, we can see that $Fix_d(X)$ is down-closed set, that is, $x \in Fix_d(X)$ and $y \leq x$ imply $y \in Fix_d(X)$. If d is isotone, then $d(x) \leq d(x+y)$ and $d(y) \leq d(x+y)$. Therefore $d(x) + d(y) \leq d(x+y)$. If d is isotone and $x, y \in Fix_d(X)$, then $x + y \leq d(x+y)$. Hence $d(x+y) = x + y$.*

Proposition 9. *Let X be a c−subtraction algebra and D be a symmetric bi-derivation on X with the trace d. Define $d^2(x) = d(d(x))$ for all $x \in X$. Then $d^2 = d$, that is, $d(x) \in Fix_d(X)$.*

Proof. From Proposition 6, we obtain that $d(x) \wedge x = d(x)$. From Proposition 7 and Theorem 3, we have

$$\begin{aligned} d^2(x) &= d(d(x)) \\ &= d(d(x) \wedge x) \\ &= \left(d^2(x) \wedge x\right) + D(d(x), x) + (d(x) \wedge d(x)) \\ &= d(x) \end{aligned}$$

since $d^2(x) \leq d(x) \leq x$ and $D(d(x), x) \leq d(x)$ for all $x \in X$. □

Proposition 10. *Let X be a c−subtraction algebra, D_1 and D_2 be symmetric bi-derivations on X and d_1 and d_2 be their traces, respectively. If d_1 and d_2 are isotone, then $d_1 = d_2$ if and only if $Fix_{d_1}(X) = Fix_{d_2}(X)$.*

Proof. $(\Rightarrow)$: If $d_1 = d_2$, then $Fix_{d_1}(X) = Fix_{d_2}(X)$.

$(\Leftarrow)$: Let $Fix_{d_1}(X) = Fix_{d_2}(X)$ and $x \in X$. Since $d_1(x) \in Fix_{d_1}(X) = Fix_{d_2}(X)$ by Proposition 9, we obtain $d_2(d_1(x)) = d_1(x)$. Similarly, we can get $d_1(d_2(x)) = d_2(x)$. Since d_1 and d_2 are isotone, we have $d_2(d_1(x)) \leq d_2(x) = d_1(d_2(x))$ and so $d_2(d_1(x)) \leq d_1(d_2(x))$. Similarly, we can get $d_1(d_2(x)) \leq d_2(d_1(x))$. This shows that $d_2(d_1(x)) = d_1(d_2(x))$. Hence $d_1(x) = d_2(d_1(x)) = d_1(d_2(x)) = d_2(x)$, that is, $d_1 = d_2$. □

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