On the distance and distance Laplacian spectral radius of graphs with cut edges *

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Abstract. Suppose that the vertex set of a graph G is $V(G) = \{v_1, \cdots, v_n\}$. Then we denote by $Tr_G(v_i)$ the sum of distances between v_i and all other vertices of G. Let $Tr(G)$ be the $n \times n$ diagonal matrix with its (i, i)-entry equal to $Tr_G(v_i)$ and $D(G)$ be the distance matrix of G. Then $L_D(G) = Tr(G) - D(G)$ is the distance Laplacian matrix of G. The largest eigenvalues of $D(G)$ and $L_D(G)$ are called distance spectral and distance Laplacian spectral radius of G, respectively. In this paper we describe the unique graph with distance and distance Laplacian spectral radius among all connected graphs of order n with given cut edges.

Key words: Spectral radius; cut edges

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1. Introduction

Let G be a connected simple graph on n vertices with the vertex set $V(G) = \{v_1, \ldots, v_n\}$. The *distance* d_{uv} between the vertices u and v is the length of shortest path between u and v in G. For $u \in V(G)$, the transmission $Tr_G(u)$ of u is the sum of distances between u and all other vertices of G. Let $Tr(G)$ be the $n \times n$ diagnonal matrix with its (i,i)-entry equal to $Tr_G(v_i)$ and $D(G)$ be the distance matrix of G. Then the distance Laplacian matrix of G is $L_D = Tr(G) - D(G)$. The largest eigenvalues $\rho_D(G)$ and $\rho_L(G)$ of $D(G)$ and $L_D(G)$ are distance and distance Laplacian spectral radius of G, respectively.

The distance spectral radius of a connected graph has been studied extensively. S. Bose, M. Nath and S. Paul [3] determined the unique graph with maximal distance spectral radius in the class of graphs without a pendent vertex. A. Ilić [5] obtained the tree with given matching number which minimizes distance spectral radius. G. Yu et al [13, 14] determined respectively the extremal graph and unicyclic graph with the maximum and minimum distance spectral radius.

M. Aouchiche and P. Hansen introduced in [1] the distance Laplacian and distance signless Laplacian spectra of graphs, respectively, and proved in [2] that the star S_n of order n attains minimum distance Laplacian spectral radius among all trees. R. Xing and B. Zhou [11] gave the unique graph with minimum distance and distance signless Laplacian spectral radii among bicyclic graphs with fixed number of vertices. R. Xing, B. Zhou and J. Li [12] determined the graphs with minimum distance signless Laplacian spectral radius among the trees, unicyclic graphs, bipartite graphs, the connected graphs with fixed pendant vertices and fixed connectivity, respectively. M. Nath and S. Paul [7] characterized the graphs whose complement is a tree or unicyclic graph having $n+1$ as the second smallest distance Laplacian eigenvalue.

A cut edge in a connected graph G is an edge whose deletion breaks the graph into two components. Denote by $\mathfrak{g}_n^k$ the set of graphs on n vertices with k cut edges. H. Liu, M. Lu and F. Tian [6] obtained the graph with the maximum spectral radius in $\mathfrak{g}_n^k$. R. Wu and Y. Fan [9] gave the graph with the maximum signless Laplacian spectral radius in $\mathfrak{g}_n^k$.

In this paper we determine the graphs with the minimum distance and distance Laplacian spectral radius in $\mathfrak{g}_n^k$.

2. The graph with the minimum distance spectral radius in $\mathfrak{g}_n^k$

Suppose that u and v are two non-adjacent vertices of graph G. Then we denote by $G+uv$ the graph obtained from G by adding the edge uv.

Lemma 2.1 [8]. *Let G be a connected graph with two non-adjacent vertices $u, v \in V(G)$. Then $\rho_D(G+uv) < \rho_D(G)$.*

Lemma 2.2 [10]. *Suppose that G is a connected graph and that uv is a non-pendent cut edge of G. Let G' be the graph obtained from G by contradicting uv to the vertex u and attaching a pendent edge to u. Then $\rho_D(G) > \rho_D(G')$.*

Let $E(G)$ denote the edge set of a graph G. A clique of a graph G is a complete subgraph of G.

Lemma 2.3 [4]. *Let G be a graph with a clique K_s such that $G - E(K_s)$ has exactly s components. Let G_1 and G_2 be two nontrivial components of $G - E(K_s)$ such that $u \in V(K_s) \cap V(G_1)$ and $v \in V(K_s) \cap V(G_2)$. If $G' = G - \sum_{w \in N_{G_2}(v)} vw + \sum_{w \in N_{G_2}(v)} uw$, then $\rho_D(G) > \rho_D(G')$.*

If two graph G and H are isomorphic then we write $G \cong H$, and otherwise $G \ncong H$. Denote by K_n^k the graph obtained by attaching k pendant edges to some one vertex of the complete graph K_{n-k}.

Theorem 2.4. *If $G \in \mathfrak{g}_n^k$, then $\rho_D(G) \geq \rho_D(K_n^k)$ with equality if and only if $G \cong K_n^k$.*

Proof. Let G be a graph with the minimum distance spectral radius in $\mathfrak{g}_n^k$ and $E_1 = \{e_1, e_2, \ldots, e_k\}$ be the set of cut edges of G. Then by Lemma 2.1, we can determine that each component of $G - E_1$ is a clique since otherwise we add all possible edges in each component of $G - E_1$ to obtain a graph $\overline{G} \in \mathfrak{g}_n^k$ such that $\rho_D(G) > \rho_D(\overline{G})$, a contradiction with the minimality of G. Thus we can denote these components by $K_{n_0}, K_{n_1}, \ldots, K_{n_k}$, where $n_i = |V(K_{n_i})|$ for $0 \leq i \leq k$ and $\sum\limits_{0 \leq i \leq k} n_i = n$.

Let $V_{n_i} = \{v \in K_{n_i}\colon v$ is an end vertex of a cut edge of $G\}$. Then we have the following two claims.

Claim 1. $|V_{n_i}| = 1$ for each $0 \leq i \leq k$.

Suppose that there is some ℓ such that $|V_{n_\ell}| > 1$. Then there exist $u, v \in V_{n_\ell}$ such that $N(u) \setminus N(v) \neq \emptyset$, where $N(u)$ is the set of neighbors of u in G. Suppose that $N(u) \setminus N(v) = \{z_1, z_2, \ldots, z_s\}$. Then $s \geq 1$. Let $G^* = G - \{uz_1, uz_2, \ldots, uz_s\} + \{vz_1, vz_2, \ldots, vz_s\}$. Then $G^* \in \mathfrak{g}_n^k$. By Lemma 2.3, $\rho_D(G) > \rho_D(G^*)$, a contradiction. Therefore, $|V_{n_i}| = 1$ $(0 \leq i \leq k)$.

Claim 2. G does not contain non-pendent edges.

The claim 2 follows from Lemma 2.2.

Combining the two claims and Lemma 2.1, we know that $G \cong K_n^k$. □

3. The graph with minimum distance Laplacian spectral radius in $\mathfrak{g}_n^k$

If $x = (x_1, x_2, \cdots, x_n)^T$ then it can be considered as a function defined on $V(G) = \{v_1, \cdots, v_n\}$ which maps vertex v_i to x_i, i.e. $x(v_i) = x_i$, and so

$$x^T L_D(G) x = \sum_{\{u,v\} \subseteq V(G)} d_{uv}(x(u) - x(v))^2$$

which shows that $L_D(G)$ is positive semidefinite.

Assume that x is an eigenvector of $L_D(G)$ corresponding to eigenvalue λ. Then for $v_i \in V(G)$, $\lambda x_i = \sum_{v_j \in V(G)} d_{v_i v_j}(x_i - x_j)$.

Lemma 3.1 [1]. *If u and v are two non-adjacent vertices of graph G, then $\rho_L(G+uv) \leq \rho_L(G)$.*

Let $Tr_{max}(G)$ be the maximum transmission of vertices of G. Then we have the following

Lemma 3.2. *Let G be a connected graph. Then $\rho_L(G) \geq Tr_{max}(G)$.*

Proof. By Rayleigh's inequalities we know that for any unitary vector $x \in R^n$, $\rho_L(G) \geq x^T L_D(G) x$ with equality if and only if x is an eigenvector of $L_D(G)$ corresponding to $\rho_L(G)$. Let e_i be the i-th unitary vector of the standard basis $(1 \leq i \leq n)$. Since $e_i^T L_D(G) e_i = Tr_G(v_i)$, the inequality $\rho_L(G) \geq Tr_{max}(G)$ holds. □

Suppose that v is a vertex of graph G and that u is a vertex of a tree T. If we regard u and v as the same vertex, then we call v (or u) the *attached vertex* of T on G and we write T by T_v. Let $J_{p\times q}$ be $p \times q$ matrix whose entries are all 1.

Lemma 3.3. *If $2 \leq k \leq n-3$ then $\rho_L(K_n^k) = 2n-1$.*

Proof. Let $V(K_{n-k}) = \{v_1, v_2, \cdots, v_{n-k}\}$, where v_1 is an attached vertex of the star S_k. Let $x = (x_1, x_2, \cdots, x_n)^T$ be a unitary eigenvector of $L_D(K_n^k)$ corresponding to $\rho_L(K_n^k)$ where $x_i = x(v_i)$ $(1 \leq i \leq n)$. For any $2 \leq i \neq j \leq n-k$, we have

$$\begin{cases} \rho_L(K_n^k)x_i = x_i - x_1 + 2\sum\limits_{t=n-k+1}^{n}(x_i - x_t) + \sum\limits_{t=2}^{n-k}(x_i - x_t), \\ \rho_L(K_n^k)x_j = x_j - x_1 + 2\sum\limits_{t=n-k+1}^{n}(x_j - x_t) + \sum\limits_{t=2}^{n-k}(x_j - x_t), \end{cases}$$

from which we obtain that $(\rho_L(K_n^k) - (n+k))(x_i - x_j) = 0$. By Lemma 3.2 we know that $\rho_L(K_n^k) \geq Tr_{max}(K_n^k) = 2n-3 > n+k$, and so $x_i = x_j = x_2$. Thus we have

$$\begin{cases} \rho_L(K_n^k)x_{n-k+1} = 2(x_{n-k+1}-x_{n-k+2})+2(x_{n-k+1}-x_{n-k+3}) \\ \qquad +\cdots+2(x_{n-k+1}-x_n)+(x_{n-k+1}-x_1) \\ \qquad +2(n-k-1)(x_{n-k+1}-x_2), \\ \rho_L(K_n^k)x_{n-k+2} = 2(x_{n-k+2}-x_{n-k+1})+2(x_{n-k+2}-x_{n-k+3}) \\ \qquad +\cdots+2(x_{n-k+2}-x_n)+(x_{n-k+2}-x_1) \\ \qquad +2(n-k-1)(x_{n-k+2}-x_2), \\ \qquad \cdots \\ \qquad \cdots \\ \rho_L(K_n^k)x_n = 2(x_n-x_{n-k+1})+2(x_n-x_{n-k+2})+\cdots+ \\ \qquad 2(x_n-x_{n-1})+(x_n-x_1)+2(n-k-1)(x_n-x_2), \\ \rho_L(K_n^k)x_1 = (x_1-x_{n-k+1})+(x_1-x_{n-k+2})+\cdots+ \\ \qquad (x_1-x_n)+(n-k-1)(x_1-x_2), \\ \rho_L(K_n^k)x_2 = 2(x_2-x_{n-k+1})+2(x_2-x_{n-k+2})+\cdots+ \\ \qquad 2(x_2-x_n)+(x_2-x_1). \end{cases}$$

From the above equations we can obtain the following matrix

$$M = \begin{pmatrix} (2n-1)I_k - 2J_{k\times k} & -J_{k\times 1} & -2(n-k-1)J_{k\times 1} \\ -J_{1\times k} & n-1 & -(n-k-1) \\ -2J_{1\times k} & -1 & 2k+1 \end{pmatrix}.$$

By direct calculation, we know that $\det(\lambda I_{k+2} - M) = \lambda(\lambda - n)(\lambda - (2n-1))^k$. This shows that $\rho_L(K_n^k) = 2n-1$. □

Let $\mathfrak{B}_n^k$ be the set of the graphs obtained by attaching k pendent edges to some vertices of the complete graph K_{n-k}.

Lemma 3.4. *Suppose that $G \in \mathfrak{B}_n^k$. Then $\rho_L(G) \geq \rho_L(K_n^k)$ with equality if and only if $G \cong K_n^k$.*

Proof. When $k = 0, 1$ and $n-1$, G is isomorphic to K_n, K_n^1 and S_n, respectively.

If $k = 2$ and $G \ncong K_n^2$, then by a simple computation we can obtain that $\rho_L(G) = \frac{3n}{2} + 1 + \frac{1}{2}\sqrt{n^2+4} > 2n-1$.

Suppose that $k = 3$. If $G \ncong K_n^3$ then since $G \in \mathfrak{B}_n^3$, G is isomorphic to G_1 or G_2 shown in Fig. 1.

G1 G2

Fig. 1.

It is clear that $\vec{1} = (1, 1, \ldots, 1)^T$ is an eigenvector of $L_D(G_1)$ corresponding to eigenvalue 0, and so we can choose an eigenvector x of $L_D(G_1)$ corresponding to $\rho_L(G_1)$ which is orthogonal to $\vec{1}$. For any $1 \le i \ne j \le n-5$, we have

$$\begin{cases} \rho_L(G_1)x_i = \sum\limits_{1 \le t \le n-3} (x_i - x_t) + 2(x_i - x_{n-2}) + 2(x_i - x_{n-1}) \\ \qquad +2(x_i - x_n), \\ \rho_L(G_1)x_j = \sum\limits_{1 \le t \le n-3} (x_j - x_t) + 2(x_j - x_{n-2}) + 2(x_j - x_{n-1}) \\ \qquad +2(x_j - x_n). \end{cases}$$

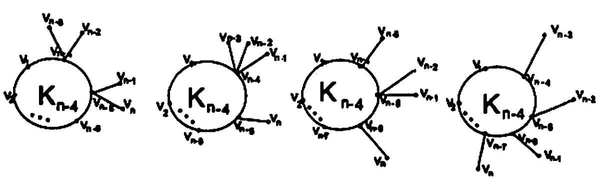
from which we obtain that $(\rho_L(G_1) - (n+3))(x_i - x_j) = 0$. By Lemma 3.2 we know that $\rho_L(G_1) \ge Tr_{max}(G_1) \ge Tr_{G_1}(v_n) = 2n-1 > n+3$, and so $x_i = x_j = x_1$. Thus we have

$$\begin{cases} \rho_L(G_1)x_1 = 8x_1 - x_{n-4} - x_{n-3} - 2x_{n-2} - 2x_{n-1} - 2x_n, \\ \rho_L(G_1)x_{n-4} = -(n-5)x_1 + nx_{n-4} - x_{n-3} - x_{n-2} - x_{n-1} \\ \qquad -2x_n, \\ \rho_L(G_1)x_{n-3} = -(n-5)x_1 - x_{n-4} + (n+1)x_{n-3} - 2x_{n-2} \\ \qquad -2x_{n-1} - x_n, \\ \rho_L(G_1)x_{n-2} = -2(n-5)x_1 - x_{n-4} - 2x_{n-3} + (2n-2)x_{n-2} \\ \qquad -2x_{n-1} - 3x_n, \\ \rho_L(G_1)x_{n-1} = -2(n-5)x_1 - x_{n-4} - 2x_{n-3} - 2x_{n-2} \\ \qquad +(2n-2)x_{n-1} - 3x_n, \\ \rho_L(G_1)x_n = -2(n-5)x_1 - 2x_{n-4} - x_{n-3} - 3x_{n-2} - 3x_{n-1} \\ \qquad +(2n-1)x_n. \end{cases}$$

Let $\varphi(\lambda) = \lambda(\lambda - (n+1))(\lambda - 2n)\psi(\lambda)$ where $\psi(\lambda) = \lambda^3 - 5\lambda^2 n + 8\lambda n^2 - 4n^3 - 3\lambda^2 + 11\lambda n - 10n^2 - 2n$. Note that $\psi(2n) = -2n < 0$. Thus, $\rho_L(G_1)$ is the largest root of the equation $\psi(\lambda) = 0$, which implies that $\rho_L(G_1) > 2n - 1$.

Similarly, we can prove that $\rho_L(G_2) > 2n - 1$.

Suppose that $k = 4$. If $G \ncong K_n^4$ then since $G \in \mathfrak{B}_n^4$, G is isomorphic to some one of the four graphs shown in Fig. 2.



G_1^* G_2^* G_3^* G_4^*

Fig. 2.

Let $x = (x_1, x_2, \cdots, x_n)^T$ be a unitary eigenvector of $L_D(G_1^*)$ corresponding to $\rho_L(G_1^*)$.

For any $1 \le i \ne j \le n-6$, we have

$$\begin{cases} \rho_L(G_1^*)x_i = \sum\limits_{1 \le t \le n-4} (x_i - x_t) + 2(x_i - x_{n-3}) + 2(x_i - x_{n-2}) \\ \qquad\qquad +2(x_i - x_{n-1}) + 2(x_i - x_n), \\ \rho_L(G_1^*)x_j = \sum\limits_{1 \le t \le n-4} (x_j - x_t) + 2(x_j - x_{n-3}) + 2(x_j - x_{n-2}) \\ \qquad\qquad +2(x_j - x_{n-1}) + 2(x_j - x_n), \end{cases}$$

from which we have $(\rho_L(G_1^*) - (n+4))(x_i - x_j) = 0$. By Lemma 3.2 we know that $\rho_L(G_1^*) \ge Tr_{max}(G_1^*) \ge Tr_{G_1^*}(v_n) = 2n - 1 > n + 4$, and so $x_i = x_j = x_1$.

Thus, we have

$$\begin{cases} \rho_L(G_1^*)x_1 = 10x_1 - x_{n-5} - x_{n-4} - 2x_{n-3} - 2x_{n-2} - 2x_{n-1} \\ \quad -2x_n, \\ \rho_L(G_1^*)x_{n-5} = -(n-6)x_1 + (n+1)x_{n-5} - x_{n-4} - 2x_{n-3} \\ \quad -2x_{n-2} - x_{n-1} - x_n, \\ \rho_L(G_1^*)x_{n-4} = -(n-6)x_1 - x_{n-5} + (n+1)x_{n-4} - x_{n-3} \\ \quad -x_{n-2} - 2x_{n-1} - 2x_n, \\ \rho_L(G_1^*)x_{n-3} = -2(n-6)x_1 - 2x_{n-5} - x_{n-4} + (2n-1)x_{n-3} \\ \quad -2x_{n-2} - 3x_{n-1} - 3x_n, \\ \rho_L(G_1^*)x_{n-2} = -2(n-6)x_1 - 2x_{n-5} - x_{n-4} - 2x_{n-3} \\ \quad +(2n-1)x_{n-2} - 3x_{n-1} - 3x_n, \\ \rho_L(G_1^*)x_{n-1} = -2(n-6)x_1 - x_{n-5} - 2x_{n-4} - 3x_{n-3} - 3x_{n-2} \\ \quad +(2n-1)x_{n-1} - 2x_n, \\ \rho_L(G_1^*)x_n = -2(n-6)x_1 - x_{n-5} - 2x_{n-4} - 3x_{n-3} - 3x_{n-2} \\ \quad -2x_{n-1} + (2n-1)x_n. \end{cases}$$

Let $\varphi(\lambda) = \lambda^7 - (10n+8)\lambda^6 - (-41n^2-68n-22)\lambda^5 - (88n^3 + 228n^2+156n+28)\lambda^4 - (-104n^4-376n^3-410n^2-160n-17)\lambda^3 - (64n^5+304n^4+472n^3+300n^2+74n+4)\lambda^2 - (-16n^6-96n^5 - 200n^4-184n^3-77n^2-12n)\lambda$. Then $\rho_L(G_1^*)$ is the largest root of the equation $\varphi(\lambda) = 0$.

By Lemma 3.3, we have $\rho_L(K_n^4) = 2n-1$. Set $\phi(\lambda) = (\lambda - (2n-1))\lambda^6$. Take $h(\lambda) = \varphi(\lambda) - \phi(\lambda)$. Then $h(\rho_L(K_n^4)) = -64n^2 + 192n - 80 < 0$, from which we know that $\varphi(\rho_L(K_n^4)) < \phi(\rho_L(K_n^4)) = 0$, and so $\rho_L(G_1^*) > \rho_L(K_n^4)$.

We easily compute that $Tr_{G_i^*}(v_n) = 2n$ ($i = 2,3,4$), and so by Lemma 3.2, $\rho_L(G_i^*) > 2n-1$.

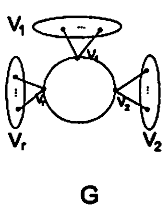


Fig. 3.

Finally, we assume that $5 \leq k \leq n-3$.

Let G be shown in Fig. 3, where $|V_i| = n_i$ $(1 \leq i \leq r \leq k)$ with $1 \leq n_1 \leq n_2 \leq \ldots \leq n_{r-1} \leq n_r$. Then we can observe that $Tr_{max}(G) \geq 2n - n_1 + k - 3$. Note that $n_1 \leq \lfloor \frac{k}{2} \rfloor$. By Lemma 3.2, we have $\rho_L(G) \geq Tr_{max}(G) > 2n-1$. □

Lemma 3.5. *Suppose that uv is a non-pendent cut edge of G such that $G - \{uv\}$ consists of two cliques K_s and K_t. Then $\rho_L(G) > \rho_L(K_n^k)$.*

Proof. Clearly, neither s nor t is less than 1. Without loss of generality we can assume that $s \leq \lfloor \frac{n}{2} \rfloor$. If $s < \lfloor \frac{n}{2} \rfloor$ then for $v \in V(K_s)$, we can obtain $Tr_G(v) = 3t + s - 2 > 2n - 1$, and so by Lemma 3.2, we have $\rho_L(G) > 2n-1$.

Now we assume $s = \lfloor \frac{n}{2} \rfloor$. If n is even then $s = t = \frac{n}{2}$. We consider $V(G)$ partitioned into $U_1 \cup U_2 \cup \{v_{2s-1}\} \cup \{v_{2s}\}$, where $U_1 = \{v_1, v_2, \cdots, v_{s-1}\}$ and $U_2 = \{v_s, v_{s+1}, \cdots, v_{2s-2}\}$. Let $x = (x_1, x_2, \cdots, x_{2s})^T$ be a unitary eigenvector of $L_D(G)$ corresponding to $\rho_L(G)$. Thus we have

$$M = \begin{pmatrix} A & B \\ C & D \end{pmatrix},$$

where

$$A = \begin{pmatrix} (4s-1)I_{s-1} - J_{(s-1)\times(s-1)} & -3J_{(s-1)\times(s-1)} \\ -3J_{(s-1)\times(s-1)} & (4s-1)I_{(s-1)} - J_{(s-1)\times(s-1)} \end{pmatrix}$$

$$B = \begin{pmatrix} -2J_{(s-1)\times 1} & -J_{(s-1)\times 1} \\ -J_{(s-1)\times 1} & -2J_{(s-1)\times 1} \end{pmatrix},$$

$$C = \begin{pmatrix} -2J_{1\times(s-1)} & -J_{1\times(s-1)} \\ -J_{1\times(s-1)} & -2J_{1\times(s-1)} \end{pmatrix},$$

and

$$D = \begin{pmatrix} 3s-2 & -1 \\ -1 & 3s-2 \end{pmatrix}.$$

By direct calculation, we know that $\det(\lambda I_{2s} - M) = \lambda(\lambda - (4s-1))^{2s-4}(\lambda - 3s)(\lambda - (\frac{9}{2}s - 2 \pm \frac{1}{2}\sqrt{9s^2 - 8s}))$, from which we have $\rho_L(G) = \frac{9}{2}s - 2 + \frac{1}{2}\sqrt{9s^2 - 8s} > 2n - 1$. By Lemma 3.3, we have $\rho_L(G) > \rho_L(K_n^k)$.

If $s = \frac{n-1}{2}$. Similarly, we can prove that $\rho_L(G) > \rho_L(K_n^k)$. □

Combining Lemma 3.4 with Lemma 3.5 we can easily obtain the following result

Theorem 3.6. *K_n^k attains the minimum distance Laplacian spectral radius in $\mathfrak{g}_n^k$.*

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