# Hamiltonicity of $L_1$ – Graphs with Bounded Dilworth Numbers

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**Abstract**

Let $u$ and $v$ be two vertices in a graph $G$. We say vertex $u$ dominates vertex $v$ if $N(v) \subseteq N(u) \cup \{u\}$. If $u$ dominates $v$ or $v$ dominates $u$, then $u$ and $v$ are comparable. The Dilworth number of a graph $G$, denoted $Dil(G)$, is the largest number of pairwise incomparable vertices in the graph $G$. A graph $G$ is called $\{H_1, H_2, ..., H_k\}$ – free if $G$ contains no induced subgraph isomorphic to any $H_i$, $1 \leq i \leq k$. A graph $G$ is called an $L_1$ – graph if, for each triple of vertices $u$, $v$, and $w$ with $d(u,v) = 2$ and $w \in N(u) \cap N(v)$, $d(u) + d(v) \geq |N(u) \cup N(v) \cup N(w)| - 1$. Let $G$ be a $k$ ($k \geq 2$) – connected $L_1$ – graph. If $G$ is $\{K_{1,5}, K_{1,5}+e\}$ – free and $Dil(G) \leq 2k-1$, then $G$ is Hamiltonian or $G \in \mathcal{F}$, where $K_{1,5} + e$ is a graph obtained by joining a pair of nonadjacent vertices in $K_{1,5}$ and $\mathcal{F} = \{G : K_{p,p+1} \subseteq G \subseteq K_p \vee (p+1)K_1, 2 \leq p \leq 3\}$, where $\vee$ denotes the join operation of two graphs.

## 1. Introduction

We consider only finite undirected graphs without loops and multiple edges. Notation and terminology not defined here follow that in [7]. If $S \subseteq V(G)$, then $N(S)$ denotes the neighbors of $S$, that is, the set of all vertices in $G$ adjacent to at least one vertex in $S$. For a subgraph $H$ of $G$ and $S \subseteq V(G) - V(H)$, let $N_H(S) = N(S) \cap V(H)$ and $|N_H(S)| = d_H(S)$. If $S = \{s\}$, then $N_H(S)$ and $|N_H(S)|$ are written as $N_H(s)$ and $d_H(s)$ respectively. For disjoint subsets $A$, $B$ of the vertex set $V(G)$ of a graph $G$, let $e(A,B)$ be the number of the edges in $G$ that join a vertex in $A$ and a vertex in $B$. The distance between two vertices $u$ and $v$, $d(u,v)$, in a connected graph $G$ is the least number of edges in a path connecting $u$

and $v$. $\mathcal{K}$ is defined as $\{G : K_{p,p+1} \subseteq G \subseteq K_p \vee (p+1)K_1,\ p \geq 2\}$ and $\mathcal{F}$ is defined as $\{G : K_{p,p+1} \subseteq G \subseteq K_p \vee (p+1)K_1,\ 2 \leq p \leq 3\}$, where $\vee$ denotes the join operation of two graphs. $K_{1,5} + e$ is a graph obtained by joining a pair of nonadjacent vertices in $K_{1,5}$. A graph $G$ is 1 - tough if $\omega(G - S) \leq |S|$ for every subset $S$ of $V(G)$ with $\omega(G - S) > 1$, where $\omega(G - S)$ denotes the number of components in the graph $G - S$.

A graph $G$ is called $\{H_1, H_2, ..., H_k\}$ - free if $G$ contains no induced subgraph isomorphic to any $H_i$, $1 \leq i \leq k$. If $k = 1$ and $H_1 = K_{1,3}$, then the $G$ is called claw - free. For an integer $i$, a graph $G$ is called an $L_i$ - graph if $d(u) + d(v) \geq |N(u) \cup N(v) \cup N(w)| - i$ or equivalently $|N(u) \cap N(v)| \geq |N(w) - (N(u) \cup N(v))| - i$ for each triple of vertices $u$, $v$, and $w$ with $d(u, v) = 2$ and $w \in N(u) \cap N(v)$. It can easily be verified that every claw - free graph is an $L_1$ - graph (see [3]). Ainouche [1] introduced the concept of quasi - claw - free graphs, extending the concept of claw - free graphs. A graph $G$ is called quasi - claw - free if it satisfies the property: $d(x, y) = 2 \Rightarrow$ there exists $u \in N(x) \cap N(y)$ such that $N[u] \subseteq N[x] \cup N[y]$. Obviously, every claw - free graph is quasi - claw - free. Asratian and Khachatrian began to investigate the Hamiltonicity of $L_i$ - graphs and they in [5] proved that all connected $L_0$ - graphs of order at least three are Hamiltonian. Saito [13] shown that if a graph $G$ is a 2 - connected $L_1$ - graph of diameter two then either $G$ is Hamiltonian or $G \in \mathcal{K}$. More results related to the Hamiltonian properties of $L_i$ - graphs can be found in [2], [3], [4], [6], [12], and [10].

The definition of the Dilworth number of a graph can be found in [9] (also see [8]). Let $u$ and $v$ be two vertices in a graph $G$. We say vertex $u$ domoniates vertex $v$ if $N(v) \subseteq N(u) \cup \{u\}$. If $u$ dominates $v$ or $v$ dominates $u$, then $u$ and $v$ are comparable. The Dilworth number of a graph $G$, denoted $Dil(G)$, is the largest number of pairwise incomparable vertices in the graph $G$. Using the Dilworth numbers of graphs, Li in [11] obtained sufficient conditions for the Hamiltonicity of $\{quasi - claw, K_{1,5}, K_{1,5} + e\}$ - free graphs. The objective of this paper is to prove a similar theorem for the Hamiltonicity of $L_1$ - graphs which are $\{K_{1,5},\ K_{1,5} + e\}$ - free. The next theorem is the main result of this paper.

**Theorem 1** *Let $G$ be a $k$ ($k \geq 2$) - connected $L_1$ - graph. If $G$ is $\{K_{1,5}, K_{1,5} + e\}$ - free and $Dil(G) \leq 2k - 1$, then $G$ is Hamiltonian or $G \in \mathcal{F}$.*

Since every claw - free graph is an $L_1$ - graph and $\{K_{1,5}, K_{1,5} + e\}$ - free and every graph in $\mathcal{F}$ is not claw - free, Theorem 1 has the following corollary.

**Corollary 1** *Let $G$ be a $k$ ($k \geq 2$) - connected claw - free graph. If*

*Dil(G)* $\leq 2k-1$, *then G is Hamiltonian.*

We need the following additional notations in the reminder of this paper. If $C$ is a cycle of $G$, let $\overrightarrow{C}$ denote the cycle $C$ with a given orientation. For $u$, $v \in C$, let $\overrightarrow{C}[u,v]$ denote the consecutive vertices on $C$ from $u$ to $v$ in the direction specified by $\overrightarrow{C}$. The same vertices, in reverse order, are given by $\overleftarrow{C}[v,u]$. Both $\overrightarrow{C}[u,v]$ and $\overleftarrow{C}[v,u]$ are considered as paths and vertex sets. If $u$ is on $C$, then the predecessor, successor, next predecessor and next successor of $u$ along the orientation of $C$ are denoted by $u^-$, $u^+$, $u^{--}$, and $u^{++}$, respectively. If $A \subseteq V(C)$, then $A^-$ and $A^+$ are defined as $\{v^- : v \in A\}$ and $\{v^+ : v \in A\}$, respectively. If $H$ is a connected component of a graph $G$ and $u$ and $v$ are two vertices in $H$, let $uHv$ denote a path between $u$ and $v$ in $H$.

## 2. Lemmas

The following Lemma 1 is a result obtained in [3].

**Lemma 1** *If $G$ is a 2 - connected $L_1$ - graph, then either $G$ is 1 - tough or $G \in \mathcal{K}$.*

The following Lemma 2 is a result extracted from the proof of Theorem 3 in [10]. For the sake of completeness, we include the proof of Lemma 2 here.

**Lemma 2** *Let $G$ be a 2 - connected nonhamiltonian $L_1$ - graph. Suppose $C$ is a longest cycle with a given orientation in $G$, $H$ is a connected component of $G[V(G)-V(C)]$, $N(V(H)) \cap V(C) = \{a_1, a_2, ..., a_l\}$ such that $h_i a_i \in E$, where $h_i \in V(H)$ for each $i$, $1 \leq i \leq l$, and $a_1$, $a_2$, ..., $a_l$ are labeled in the order of the orientation of $C$. Then $G \in \mathcal{K}$ or $a_i^- a_i^+ \in E$ for each $i$, $1 \leq i \leq l$.*

**Proof of Lemma 2.** If $G \in \mathcal{K}$, then the proof is finished. Now we assume that $G \notin \mathcal{K}$. Then Lemma 1 implies that $G$ is 1 - tough. Since $G$ is 2 - connected, $l \geq 2$. Set $A := \{a_1, a_2, ..., a_l\}$ and for each $i$, $1 \leq i \leq l$, let $b_i$ and $d_i$ be the predecessor and successor respectively of $a_i$ along $C$. Set $B := \{b_1, b_2, ..., b_l\}$ and $D := \{d_1, d_2, ..., d_l\}$.

Next we will prove that for each $i$, $1 \leq i \leq l$, $b_i d_i \in E$. Suppose not, then there exists a $k$, $1 \leq k \leq l$, such that $b_k d_k \notin E$. Clearly, $d(h_k, d_k) = 2$ and $a_k \in N(h_k) \cap N(d_k)$. Since $G$ is an $L_1$ - graph,

$$|N(h_k) \cap N(d_k)| \geq |N(a_k) - (N(h_k) \cup N(d_k))| - 1 \geq |\{b_k, d_k, h_k\}| - 1 = 2.$$

By the choice of $C$, we have $N(h_k) \cap N(d_k) \cap (V(G) - V(C)) = \emptyset$. Then there exists a vertex $a_j \in N(V(H)) \cap V(C)$ such that $a_j \in N(h_k) \cap N(d_k)$.

Let $X$ be the set $N(h_k) \cap V(C) := \{x_1, x_2, ..., x_{l_1}\}$ with the $x_i$'s ordered with increasing index in the direction of orientation of $C$. Then $X \subseteq A$ and $l_1 \geq 2$. For each $i$, $1 \leq i \leq l_1$, let $s_i$ and $t_i$ be the predecessor and successor respectively of $x_i$ along $C$. Set $S := \{s_1, s_2, ..., s_{l_1}\}$ and $T := \{t_1, t_2, ..., t_{l_1}\}$. Clearly, $S \cup \{h_k\}$ is an independent set in $G$ and for each $i$, $1 \leq i \leq l_1$, $N(h_k) \cap N(s_i) \cap (V(G) - V(C)) = \emptyset$. Moreover for each $i$, $1 \leq i \leq l_1$, $d(h_k, s_i) = 2$ and $x_i \in N(h_k) \cap N(s_i)$. Since $G$ is an $L_1$ - graph, we have

$$|N(h_k) \cap N(s_i)| \geq |N(x_i) - (N(h_k) \cup N(s_i))| - 1.$$

Obviously, $N_S(x_i) \subseteq N(x_i) - (N(h_k) \cup N(s_i) \cup \{h_k\})$. Thus,

$|N_S(x_i)| \leq |N(x_i) - (N(h_k) \cup N(s_i))| - 1$. Therefore,

$|N_S(x_i)| \leq |N(h_k) \cap N(s_i)| = |N_X(s_i)|$. Hence,

$e(X, S) = \sum_{i=1}^{l_1} |N_S(x_i)| \leq \sum_{i=1}^{l_1} |N_X(s_i)| = e(X, S)$.

It follows, for each $i$, $1 \leq i \leq l_1$, that

$$N(x_i) - (N(h_k) \cup N(s_i) \cup \{h_k\}) = N_S(x_i) \subseteq S. \qquad (1)$$

Similarly, for each $i$, $1 \leq i \leq l_1$,

$$N(x_i) - (N(h_k) \cup N(t_i) \cup \{h_k\}) = N_T(x_i) \subseteq T. \qquad (2)$$

We claim that there exists an $i$ such that $s_{i+1} \neq t_i$, where $1 \leq i \leq l_1$ and the index $(l_1 + 1)$ is regarded as 1. Suppose not, then for each $i$, $1 \leq i \leq l_1$, $s_{i+1} = t_i$. Clearly, for each $i$, $1 \leq i \leq l_1$, $N(t_i) \cap V(H) = \emptyset$, otherwise $C$ is not of maximum length, also for any pair of $i$, $j$, $1 \leq i, j \leq l_1$ and $i \neq j$, $t_i$, $t_j$ do not have neighbors in the same component of the graph $G[V(G) - V(C) - V(H)]$, otherwise $C$ is again not of maximum length. Therefore, $G - \{x_1, x_2, ..., x_{l_1}\}$ has at least $l_1 + 1$ components, contradicting the fact that $G$ is 1 - tough.

Without loss of generality, assume that $s_1 \neq t_{l_1}$. Observe that $s_1 \in N(t_1)$, otherwise from (2), we have $s_1 \in T$, which is impossible. Since $s_1 t_1 \in E$, $s_2 \neq t_1$. Observe again that $s_2 \in N(t_2)$, otherwise from (2), we have $s_2 \in T$, which is also impossible. Repeating this process, we have $s_j t_j \in E$, for each $j$, $1 \leq j \leq l_1$. This implies that $b_k d_k \in E$, a contradiction. Hence $b_id_i \in E$ for each $i$, $1 \le i \le l$. Namely, $a_i^- a_i^+ \in E$ for each $i$, $1 \le i \le l$. QED

## 3. The Proof of the Main Theorem

**Proof of Theorem 1.** Let $G$ be a graph satisfying the conditions in Theorem 1. Suppose that $G$ is nonhamiltonian and $G \notin \mathcal{F}$. Choose a longest cycle $C$ in $G$ and specify an orientation of $C$. Assume that $H$ is a connected component of the graph $G[V(G)-V(C)]$, $N(V(H)) \cap V(C) = \{a_1, a_2, ..., a_l\}$ with $h_ia_i \in E$, where $h_i \in V(H)$ for each $i$, $1 \le i \le l$, and $a_1$, $a_2$, ..., $a_l$ are labeled in the order of the orientation of $C$. Since $G$ is $k$ ($k \ge 2$) - connected, $l \ge k$. Moreover since $G$ is $K_{1,5}$ - free, $G \notin \mathcal{K} - \mathcal{F}$. Thus $G \notin \mathcal{K}$. Hence Lemma 2 implies that $a_i^- a_i^+ \in E$ for each $i$, $1 \le i \le l$.

Notice first that $a_i a_{i+1}^- \notin E$ for each $i$, $1 \le i \le l$, where the index $(l+1)$ is regarded as 1. Otherwise $G$ has a cycle

$$h_i a_i \overleftarrow{C}[a_{i+1}^-, a_i^+] \overleftarrow{C}[a_i^-, a_{i+1}] h_{i+1} H h_i$$

which is longer than $C$. Moreover $a_i a_{i+1}^{--} \notin E$ for each $i$, $1 \le i \le l$. Otherwise $G$ has a cycle

$$h_i a_i \overleftarrow{C}[a_{i+1}^{--}, a_i^+] \overleftarrow{C}[a_i^-, a_{i+1}^+] a_{i+1}^- a_{i+1} h_{i+1} H h_i$$

which is longer than $C$. Let $b_i$ be the most far neighbor of $a_i$ along $\overrightarrow{C}[a_i^+, a_{i+1}^-]$ for each $i$, $1 \le i \le l$. Then $|\overrightarrow{C}[b_i^+, a_{i+1}^-]| \ge 2$ for each $i$, $1 \le i \le l$. Next will prove that any two distinct vertices in $\{a_1, b_1, a_2, b_2, ..., a_l, b_l\}$ are incomparable.

For each $i$ and $j$, $1 \le i < j \le l$, we have $a_j^+ \notin N(a_i)$. Otherwise $G$ has a cycle

$$h_i a_i \overrightarrow{C}[a_j^+, a_i^-] \overrightarrow{C}[a_i^+, a_j] h_j H h_i$$

which is longer than $C$. Similarly, $a_i^+ \notin N(a_j)$. Thus $a_i$ and $a_j$ are incomparable for each $i$ and $j$, $1 \le i < j \le l$.

Obviously, $b_i^+ \notin N(a_i)$ for each $i$, $1 \le i \le l$. Since $N(h_i) \cap V(C) \subseteq N(V(H)) \cap V(C)$, $h_i \notin N(b_i)$, for each $i$, $1 \le i \le l$. Thus $a_i$ and $b_i$ are incomparable for each $i$, $1 \le i \le l$.

For each $i$ and $j$, $1 \le i < j \le l$, we have $b_j^+ \notin N(a_i)$. Otherwise $G$ has a cycle

$$h_i a_i \overrightarrow{C}[b_j^+, a_i^-] \overrightarrow{C}[a_i^+, a_j^-] \overrightarrow{C}[a_j^+, b_j] a_j h_j H h_i$$

which is longer than $C$. Since $N(h_i)\cap V(C) \subseteq N(V(H))\cap V(C)$, $h_i \notin N(b_j)$ for each $i$ and $j$, $1 \le i < j \le l$. Thus $a_i$ and $b_j$ are incomparable for each $i$ and $j$, $1 \le i < j \le l$. Similarly, $a_i$ and $b_j$ are incomparable for each $i$ and $j$, $1 \le j < i \le l$.

Now we will prove that $b_s$ and $b_t$ are incomparable for each $s$ and $t$, $1 \le s \ne t \le l$. Suppose, to the contrary, that there exist $b_i$ and $b_j$ such that they are comparable. Then $N(b_i) \subseteq N(b_j) \cup \{b_j\}$ or $N(b_j) \subseteq N(b_i) \cup \{b_i\}$. We first consider the case that $N(b_i) \subseteq N(b_j) \cup \{b_j\}$. Without loss of generality, we assume that $i < j$. Since $N(b_i) \subseteq N(b_j) \cup \{b_j\}$, $a_i \in N(b_j)$ and $b_i^+ \in N(b_j)$. Since $a_i \in N(b_j)$, $b_j \ne a_j^+$ and $b_j \ne a_j^{++}$. Otherwise $G$ has cycles which are longer $C$. Thus $|\overrightarrow{C}[a_j^{++}, b_j^-]| \ge 2$.

Now we will prove that $G[b_j, a_i, b_i^+, a_j, b_j^-, b_j^+]$ is isomorphic to $K_{1,5}$ or $K_{1,5} + e$. Clearly, $a_i b_i^+ \notin E$. If $a_i b_j^- \in E$, then G has a cycle

$$h_i a_i \overleftarrow{C}[b_j^-, a_j^+] \overleftarrow{C}[a_j^-, a_i^+] \overleftarrow{C}[a_i^-, b_j] a_j h_j H h_i$$

which is longer than $C$, a contradiction. Thus $a_i b_j^- \notin E$.

If $a_i b_j^+ \in E$, then $G$ has a cycle

$$h_i a_i \overrightarrow{C}[b_j^+, a_i^-] \overrightarrow{C}[a_i^+, a_j^-] \overrightarrow{C}[a_j^+, b_j] a_j h_j H h_i$$

which is longer than $C$, a contradiction. Thus $a_i b_j^+ \notin E$.

If $b_i^+ a_j \in E$, then $G$ has a cycle

$$h_i H h_j a_j \overrightarrow{C}[b_i^+, a_j^-] \overrightarrow{C}[a_j^+, a_i^-] \overrightarrow{C}[a_i^+, b_i] a_i h_i$$

which is longer than $C$, a contradiction. Thus $b_i^+ a_j \notin E$.

If $b_i^+ b_j^+ \in E$, then $G$ has a cycle

$$h_i a_i \overleftarrow{C}[b_i, a_i^+] \overleftarrow{C}[a_i^-, b_j^+] \overrightarrow{C}[b_i^+, a_j^-] \overrightarrow{C}[a_j^+, b_j] a_j h_j H h_i$$

which is longer than $C$, a contradiction. Thus $b_i^+ b_j^+ \notin E$.

If $a_j b_j^- \in E$, then $G$ has a cycle

$$h_i a_i \overrightarrow{C}[b_j, a_i^-] \overrightarrow{C}[a_i^+, a_j^-] \overleftarrow{C}[a_j^+, b_j^-] a_j h_j H h_i$$

which is longer than $C$, a contradiction. Thus $a_j b_j^- \notin E$.

If $a_j b_j^+ \in E$, then $G$ has a cycle

$$h_i H h_j a_j \overrightarrow{C}[b_j^+, a_i^-] \overrightarrow{C}[a_i^+, a_j^-] \overrightarrow{C}[a_j^+, b_j] a_i h_i$$

which is longer than $C$, a contradiction. Thus $a_j b_j^+ \notin E$.

If $b_j^- b_j^+ \in E$, then $G$ has a cycle

$$h_i H h_j a_j b_j \overrightarrow{C}[b_i^+, a_j^-] \overrightarrow{C}[a_j^+, b_j^-] \overrightarrow{C}[b_j^+, a_i^-] \overrightarrow{C}[a_i^+, b_i] a_i h_i$$

which is longer than $C$, a contradiction. Thus $b_j^- b_j^+ \notin E$.

If $b_i^+ b_j^- \in E$, then $a_j^- b_j \notin E$. Otherwise $G$ has a cycle

$$h_i a_i \overleftarrow{C}[b_i, a_i^+] \overleftarrow{C}[a_i^-, b_j] \overleftarrow{C}[a_j^-, b_i^+] \overrightarrow{C}[b_j^-, a_j] h_j H h_i$$

which is longer than $C$. Since $N(h_j) \cap V(C) \subseteq N(V(H)) \cap V(C)$, $b_j \notin N(h_j)$. Thus $b_j \in N(a_j) - (N(h_j) \cup N(a_j^-))$. Since $C$ is a longest cycle in $G$, $d(h_j, a_j^-) = 2$ and $a_j \in N(h_j) \cap N(a_j^-)$. Since $G$ is an $L_1$ - graph, we have

$$|N(h_j) \cap N(a_j^-)| \geq |N(a_j) - (N(h_j) \cup N(a_j^-))| - 1 \geq |\{h_j, a_j^-, b_j\}| - 1 = 2.$$

Clearly, $N(h_j) \cap N(a_j^-) \cap (V(G) - V(C)) = \emptyset$. Otherwise $G$ has a cycle which is longer than $C$. Therefore $N(h_j) \cap N(a_j^-) \cap (V(C) - \{a_j\}) \neq \emptyset$. Again since $N(h_j) \cap V(C) \subseteq N(V(H)) \cap V(C)$, there exists a vertex $a_p (\neq a_j)$, $1 \leq p \leq l$, such that $a_p a_j^- \in E$ and therefore $G$ has a cycle which is longer than $C$, a contradiction. Thus $b_i^+ b_j^- \notin E$.

In view of what we have proved above, we have that $G[b_j, a_i, b_i^+, a_j, b_j^-, b_j^+]$ is isomorphic to $K_{1,5}$ or $K_{1,5} + e$, which is a contradiction. Similarly, we can arrive at a contradiction when $N(b_j) \subseteq N(b_i) \cup \{b_i\}$.

Since any two distinct vertices in $\{a_1, b_1, a_2, b_2, ..., a_l, b_l\}$ are incomparable, $2k \leq Dil(G)$, contradicting to the assumption that $Dil(G) \leq 2k - 1$. Therefore the proof of Theorem 1 is complete. QED.

**Remark.** Let $K_{1,4}^e$ denote the graph obtained by subdividing one edge in $K_{1,4}$. It is observed that $G[b_j, b_i^+, a_j, b_j^-, b_j^+, h_j]$ is isomorphic to $K_{1,4}^e$ in the above proof of Theorem 1. Therefore we have also the following theorem.

**Theorem 2** *Let $G$ be a $k$ $(k \geq 2)$ - connected $L_1$ - graph. If $G$ is $K_{1,4}^e$ - free and $Dil(G) \leq 2k - 1$, then $G$ is Hamiltonian or $G \in \mathcal{K}$.*

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