A method to compute the number of regular reversible Rosenberg hypergroup

Morteza Jafarpour[a], Irina Cristea[b] *, Ali Tavakoli[a]
[a] *Vali-e-Asr University of Rafsanjan, Rafsanjan, Iran*;
[b] *CSIT, University of Nova Gorica, Slovenia*
[b] *DICA, University of Udine, Italy*

Abstract

The purpose of this note is the study of the hypergroups associated with binary relations. New types of matrices, called i-very good and regular reversible matrices, are introduced in order to give some properties of the Rosenberg hypergroups related to them. A program written in *MATLAB* computes the number of these hypergroups up to isomorphism.

Keywords: hypergroup; binary relation; regular reversible hypergroup; Boolean matrix
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1 Introduction

Various combinatorial aspects of hypergroup theory have been investigated till now, principally in connection with ordered sets [4; 20; 21], graphs [31] and hypergraphs [8], lattices [24; 25], binary and n-ary relations [4; 9; 10; 11; 14; 16; 15; 18; 33; 35], and so on.

Several algorithms have been created to compute the number of finite hyperstructures having certain properties. This research has been initiated by Migliorato [29] who has determined the structure of all non-isomorphic hypergroups of order 3 and of total regular abelian hypergroupoids. Bayon and Lygeros have dedicated many papers to the computation of the number of finite hypergroups or H_v-groups [1; 2; 3].

*Corresponding author. Email: irinacri@yahoo.co.uk

Further, after the introduction of some hypergroups associated with a binary relation by Rosenberg [33] and Corsini [10], other programs written in $C\#$ or Mathematica or MS Visual Basic have been presented by Spartalis and Mamaloukas [34], Massouros and Tsitouras [26; 27], Cristea et al. [13] in order to calculate the number of non-isomorphic hypergroups determined by a binary relation.

This paper deals with regular reversible hypergroups associated with binary relations in the sense of Rosenberg [33] and called here regular reversible Rosenberg hypergroups. The regular hypergroups have been introduced at the beginning of the history of hypergroup theory by Dresher and Ore [19]. Later on, Corsini [5; 6] have determined other properties of them, in particular he characterized their heart and introduced the notions of weak hyperring and weak hypermodule based on the definition of a regular hypergroup.

After some basic notions concerning hypergroups and collected in the second section, we study the regular reversible Rosenberg hypergroups using the representation of binary relations by Boolean matrices. In the last part of this work, we present an algorithm based on the results obtained in Section 3, that enumerates the non isomorphic finite regular reversible Rosenberg hypergroups. Moreover the program computes how many of the Rosenberg hypergroups are i-Rosenberg hypergroup (i.e. their associated matrix is idempotent and very good). The paper ends with some concluding remarks.

2 Preliminaries

Let us briefly recall some basic notions and results about hypergroups; for a comprehensive overview of this subject, the reader is refereed to [7; 12].

For a nonempty set H, we denote by $\mathcal{P}^*(H)$ the set of all nonempty subsets of H.

Definition 2.1. A nonempty set H, endowed with a mapping, called *hyperoperation*, $\circ : H^2 \longrightarrow \mathcal{P}^*(H)$ is named *hypergroupoid*. A hypergroupoid which verifies the following conditions:

(i) $(x \circ y) \circ z = x \circ (y \circ z)$, for all $x, y, z \in H$ (the *associativity*)

(ii) $x \circ H = H = H \circ x$, for all $x \in H$ (the *reproduction axiom*)

is called *hypergroup*. In particular, an associative hypergroupoid is called a *semihypergroup* and a hypergroupoid that verifies the reproduction axiom is called a *quasihypergroup*.

If A and B are nonempty subsets of H, then $A \circ B = \bigcup\limits_{\substack{a \in A \\ b \in B}} a \circ b$.

Definition 2.2. Let $(H, \circ)$ be a hypergroupoid.

(i) An element $e \in H$ is called an *identity* or *unit* if

$$x \in e \circ x \cap x \circ e,$$

for every $x \in H$. The set of all identities of H is denoted by $E(H)$.

(ii) An element $x' \in H$ is called an *inverse* of $x \in H$ if there exists $e \in E(H)$ such that

$$e \in x \circ x' \cap x' \circ x.$$

Definition 2.3. (i) A hypergroup $(H, \circ)$ is *regular* if it has at least one identity and each element has at least one inverse.

(ii) A regular hypergroup $(H, \circ)$ is called *reversible* if, for any $(x, y) \in H^2$, it satisfies the following conditions:
(1) if $y \in a \circ x$, then there exists an inverse a' of a, such that $x \in a' \circ y$;
(2) if $y \in x \circ a$, then there exists an inverse a'' of a, such that $x \in y \circ a''$.

Example 2.4. *Every commutative hypergroup provided with an identity is regular.*

Example 2.5. *The following hypergroup is regular, but not reversible.*

H	a	b	c	d
a	a	b	c,d	d
b	b	a,b	c,d	c,d
c	c	c,d	a,b	a,b
d	c,d	c,d	a,b	a,b

Example 2.6. *The following hypergroup is regular and reversible.*

H	e	a_1	a_2	a_3
e	e, a_1	e, a_1	a_2, a_3	a_2, a_3
a_1	e, a_1	e, a_1	a_2, a_3	a_2, a_3
a_2	a_2, a_3	a_2, a_3	e, a_1	e, a_1
a_3	a_2, a_3	a_2, a_3	e, a_1	e, a_1

Definition 2.7. Let $(H, \circ)$ and $(H', \circ')$ be two hypergroups. A function $f : H \longrightarrow H'$ is called a *homomorphism* if it satisfies the condition: for any $x, y \in H$,

$$f(x \circ y) \subseteq f(x) \circ' f(y).$$

f is a *good homomorphism* if, for any $x, y \in H$, $f(x \circ y) = f(x) \circ' f(y)$. We say that the two hypergroups are *isomorphic* if there is a good homomorphism between them which is also a bijection.

Till now, various hyperoperations have been defined using a binary relation ρ on a nonempty set H. We recall here that introduced by Rosenberg [33].

Let ρ be a binary relation on a nonempty set H. The sets

$$\mathbb{D}(\rho) = \{x \in H \mid \exists y \in H : (x, y) \in \rho\},$$
$$\mathbb{R}(\rho) = \{y \in H \mid \exists x \in H : (x, y) \in \rho\}$$

are called the *domain* and the *range* of the relation ρ, respectively.

Rosenberg [33] has associated a partial hypergroupoid $\mathbb{H}_\rho = (H, \circ_\rho)$ with a binary relation ρ defined on a set H, in the following way: for any $x, y \in H$,

$$x \circ_\rho x = \{z \in H \mid (x, z) \in \rho\} \quad \text{and} \quad x \circ_\rho y = x \circ_\rho x \cup y \circ_\rho y. \tag{2.1}$$

Definition 2.8. [33] An element $x \in H$ is called *outer element* of ρ if there exists $h \in H$ such that $(h, x) \notin \rho^2$.

Necessary and sufficient conditions have been determined for the particular hypergroupoid $\mathbb{H}_\rho$ such that it is a hypergroup. We recall them here.

Theorem 2.9. (Proposition 2. [33]) *$\mathbb{H}_\rho$ is a hypergroup if and only if*

(i) *ρ has full domain: $\mathbb{D}(\rho) = H$;*

(ii) *ρ has full range: $\mathbb{R}(\rho) = H$;*

(iii) *$\rho \subset \rho^2$;*

(iv) *If $(a, x) \in \rho^2$ then $(a, x) \in \rho$, whenever x is an outer element of ρ.*

Corsini [9] has investigated when $\mathbb{H}_\rho$ is a regular reversible hypergroup, obtaining the following results.

Theorem 2.10. (Theorem 1.3 [9]) *If $\mathbb{H}_\rho$ is a hypergroup, then*

(i) *ρ^2 is transitive.*

(ii) *If, moreover, ρ is symmetric, then ρ^2 is an equivalence relation on H.*

We need the following notations.

Set $P = \{x \in H \mid x \notin x \circ_\rho x\}$ and $K = \{e \in H \mid P \subset e \circ_\rho e\}$, where e is an identity for H.

Theorem 2.11. (Theorem 1.1 [9]) *$\mathbb{H}_\rho$ is regular if and only if $K \neq \emptyset$.*

Theorem 2.12. (Theorem 1.8 [9]) *If $K \neq \emptyset$ and ρ is symmetric, then $\mathbb{H}_\rho$ is a regular reversible hypergroup.*

3 Regular reversible Rosenberg hypergroups

In this section we discuss some properties of the regular reversible Rosenberg hypergroups necessary for the algorithm described in the next section.

To do so, we use the connection between the binary relations defined on a nonempty set and the Boolean matrices. Every binary relation ρ on a finite set H of cardinality n, may be represented by a Boolean matrix $M(\rho)$ and conversely, every Boolean matrix of order n defines on H a binary relation. Indeed, let $H = \{a_1, ..., a_n\}$; a Boolean matrix of order n is constructed in the following way: the element in the position (i, j) of the matrix is 1, if $(a_i, a_j) \in \rho$ and it is 0 if $(a_i, a_j) \notin \rho$ and vice versa. Hence, on every set with n elements, 2^{n^2} partial hypergroupoids can be defined. Furthermore, recall that in a Boolean algebra the following properties hold: $0+1 = 1+0 = 1+1 = 1$, while $0+0 = 0$, and $0 \cdot 0 = 0 \cdot 1 = 1 \cdot 0 = 0$, $1 \cdot 1 = 1$. If ρ is a binary relation on H, then $M(\rho^2) = M^2(\rho)$. Other properties of the Boolean matrices which represent binary relations may be found in [34].

In what follows we use some notations we explain here. An $n \times 1$ matrix (one column and n rows) is called a column vector and for a given matrix $M = (a_{ij})$, $i, j \in \{1, 2, \dots, n\}$, M_j is the column vector (a_{nj}) and M_j^2 is the j-column vector of the matrix $M(\rho^2)$. In particular, (0) is the column vector with all elements equal to 0, and (1) is the column vector with all elements equal to 1. The transpose of a matrix M is the matrix M^T, formed by turning rows into columns and vice versa.

Definition 3.1. (See [13]) The matrix $M(\rho)$ is called *very good* if and only if $\mathbb{H}_\rho$ is a hypergroup.

We introduce the following notions.

Definition 3.2. A Rosenberg hypergroup $\mathbb{H}_\rho$ is called *i-Rosenberg hypergroup* if and only if $M(\rho)$ is an idempotent very good matrix (i.e. $M(\rho)^2 = M(\rho)$).

Definition 3.3. (i) A very good matrix $M(\rho)$ is called *i-very good* if and only if $\mathbb{H}_\rho$ is an i-Rosenberg hypergroup.

(ii) A very good matrix $M(\rho)$ is called *regular* if and only if $\mathbb{H}_\rho$ is a regular hypergroup.

(iii) A regular matrix $M(\rho)$ is called *reversible* if and only if $\mathbb{H}_\rho$ is a regular reversible hypergroup.

Example 3.4. *The matrix* $M = (a_{ij})$, *with* $a_{ij} = 1$, *for any* $i, j \in \{1, 2, \ldots, n\}$, *is a very good matrix and the corresponding hypergroup is the total hypergroup which is a regular reversible Rosenberg hypergroup.*

Example 3.5. *We consider on* $H = \{x, y, z\}$ *the binary relation*

$$\rho = \{(x,x), (x,y), (y,x), (z,y), (z,z)\}.$$

The associated Rosenberg hypergroupoid $\mathbb{H}_\rho$ *is represented by the following table*

$\mathbb{H}_\rho$	x	y	z
x	x, y	x, y	H
y	x, y	x	H
z	H	H	y, z

We notice that $E(H) = \{x, z\}$ *and* $i(a) = H$, *for any* $a \in H$ *and thus* $\mathbb{H}_\rho$ *is a regular reversible hypergroup.*
On the other hand, the associated Boolean matrix is

$$M(\rho) = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$

which is therefore a reversible matrix, but it is not a i-very good one.

Example 3.6. *Let* ρ *be the following binary relation on* $H = \{x, y, z\}$:

$$\rho = \{(x,x), (x,y), (y,x), (y,y), (z,z).$$

Then the corresponding matrix

$$M(\rho) = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

is an idempotent matrix. The associated Rosenberg hypergroupoid with the following table

$\mathbb{H}_\rho$	x	y	z
x	x, y	x, y	H
y	x, y	x, y	H
z	H	H	z

is a regular reversible hypergroup, thus $M(\rho)$ *is a i-very good matrix.*

Theorem 3.7. (Theorem 4.2 [13]) *A matrix $M = M(\rho)$ is a very good matrix if and only if, for any j, with $1 \le j \le n$, the following assertions hold:*

(i) $M_j^T \neq (0)$;

(ii) $M_j \neq (0)$;

(iii) *if* $M_j^2 \neq (1)$, *then* $M_j = M_j^2$.

As an immediate consequence of Theorem 3.7 we obtain the following characterization of an idempotent very good matrix.

Proposition 3.8. *An idempotent matrix $M = M(\rho)$ (i.e. $M^2 = M$) is a very good matrix if and only if, for any j, with $1 \le j \le n$, the following assertions hold:*

(i) $M_j^T \neq (0)$;

(ii) $M_j \neq (0)$.

It is important to know when two Rosenberg hypergroups are isomorphic. A necessary and sufficient condition is given by the following result.

Theorem 3.9. (See [13]) *Let $H = \{a_1, ..., a_n\}$ be a finite set, ρ and ρ' be two binary relations on H and $M(\rho) = (t_{ij}), M(\rho') = (t'_{ij})$ be their associated matrices. The hypergroups $\mathbb{H}_\rho$ and $\mathbb{H}_{\rho'}$ are isomorphic if and only if $t_{ij} = t'_{\sigma(i)\sigma(j)}$, for σ a permutation of the set $\{1, 2, \dots, n\}$.*

The next theorem constructs a new idempotent very good matrix obtained from other two idempotent very good matrices.

Theorem 3.10. *Let $M = (t_{ij})_{n \times n}$, $M' = (t'_{ij})_{m \times m}$ be two idempotent very good matrices. Then $M \oplus M' = (m_{ij})_{k \times k}$, where $k = n + m$, and*

$$m_{ij} = \begin{cases} t_{ij}, & \text{if } i \le n, \ j \le n \\ t'_{ij}, & \text{if } n < i, \ n < j \\ 0 & \text{elsewhere} \end{cases}$$

is an idempotent very good matrix.

Proof. Since $M \oplus M' = \begin{pmatrix} M & O' \\ O & M' \end{pmatrix}_{k \times k}$, where O and O' are matrices that all entries are zero, we have $(M \oplus M')^2 = M^2 \oplus M'^2 = M \oplus M'$. Moreover, $(M \oplus M')_j \neq (0) \neq (M \oplus M')_j^T$ and hence, by Proposition 3.8, it follows that $M \oplus M'$ is an idempotent very good matrix. □

Now we check when a very good matrix is regular.

Theorem 3.11. *Let $M(\rho) = (t_{ij})_{n\times n}$ be a very good matrix. Then $M(\rho)$ is regular if and only if there exists k, $1 \le k \le n$, such that, for all i, $1 \le i \le n$, we have $t^c_{ii} \le t_{ki}$, where $t^c_{ij} = 0$ if and only if $t_{ij} = 1$.*

Proof. Using Theorem 2.11 we prove that $K \ne \emptyset$ if and only if there exists k, $1 \le k \le n$ such that, for all i, $1 \le i \le n$, we have $t^c_{ii} \le t_{ki}$. To this end we have $t_{ii} = 0$ if and only if $a_i \notin a_i \circ_\rho a_i$. Also $t_{ki} = 1$ if and only if $a_k \in a_i \circ_\rho a_i$. Now let $a_i \in P$ and $a_k \in K$; thus $a_i \notin a_k \circ_\rho a_k$ equivalently with $t_{ki} = 0$ and therefore

$$K \ne \emptyset \Leftrightarrow \exists k, 1 \le k \le n \text{ such that } \forall i, 1 \le i \le n, \quad t^c_{ii} \le t_{ki}.$$

□

Let's see now another idempotent very good matrix constructed with two idempotent very good matrices.

Theorem 3.12. *Let $M = (t_{ij})_{n\times n}$, $M' = (t'_{ij})_{m\times m}$ be two idempotent very good matrices. Then $M \boxplus M' = (m_{ij})_{k\times k}$, where $k = n + m$, and*

$$m_{ij} = \begin{cases} t_{ij}, & \text{if } i \le n, \quad j \le n \\ t'_{ij}, & \text{if } n < i, \quad n < j \\ 1, & \text{if } i \le n, \quad j > n \\ 0 & \text{if } n < i, \quad j \le n \end{cases}$$

is an idempotent very good matrix. Moreover, if M is regular then $M \boxplus M'$ is regular, too.

Proof. Since $M \boxplus M' = \begin{pmatrix} M & U \\ O & M' \end{pmatrix}_{k\times k}$, where O is a $m \times n$ matrix with all entries equal to zero and U is a $n \times m$ matrix that all entries are one. We have $(M \boxplus M')^2 = M^2 \boxplus M'^2 = M \boxplus M'$. Also $(M \boxplus M')_j \ne (0) \ne (M \boxplus M')^T_j$, hence $M \boxplus M'$ is an idempotent very good matrix.

Suppose M is regular, so

$$\exists k, 1 \le k \le m \text{ such that } \forall i, 1 \le i \le m, \quad t^c_{ii} \le t_{ki}.$$

Since $m_{ij} = 1$, for every $i \le n$, and $j > n$ we have $m^c_{ii} \le m_{ki}$, hence $M \boxplus M'$ is regular too. □

Theorem 3.13. *A regular matrix $M = M(\rho)$ is reversible if $M = M^T$.*

Proof. It easy to see that $M = M^T$ if and only if ρ is symmetric. Now by Theorem 2.10 our assertion holds. □

Since we are interested in obtaining all non isomorphic regular reversible Rosenberg hypergroups, we introduce the following concept.

Definition 3.14. We call that two matrices are *isomorphic* if the Rosenberg hypergroups obtained by them are isomorphic.

4 Application: the number of regular reversible Rosenberg hypergroups

Let $n \in \{2, 3, 4\}$. In order to calculate the number of $n \times n$ i-very good, regular and reversible matrices we formulate the following program, written in MATLAB. It is based on some procedures that return matrices that verify the requested conditions of Proposition 3.8 and Theorems 3.7, 3.11 and 3.13. First we create in $B1$ all very good matrices and then we create all very good matrices up to isomorphism in $B2$; after that, in $B3$ and $B4$ we check the i-very good matrices and i-very good matrices up to isomorphism and finally, in $B5$ and $B6$, we verify the conditions of Theorems 3.11 and 3.13. At the end of every procedure we list the output file for the 2×2 matrices.

In the following code the matrices $B3$ and $B4$ generate all i-very good matrices and i-very good matrices up to isomorphism, respectively.

```
A(:,:,1)=zeros(n);

B1(:,:,1)=A;

m1=1;

for u=1:n

        for v=1:n

          [l1,l2,l3]=size(A);

          for w=1:l3

               A(:,:,l3+w)=A(:,:,w);

                A(u,v,l3+w)=1;

               s1=0;

               for e=1:n

                    if (norm(A(:,e,l3+w))==0)||(norm(A(e,:,l3+w))==0)
```

```
            s1=s1+1;
            break
        end
    end
    if s1==0
        A2=A(:,:,l3+w)*A(:,:,l3+w);
         for r=1:n
            for s=1:n
                if A2(r,s)¿1
                    A2(r,s)=1;
                end
            end
        end
b=1;
s2(1)=0;
for e=1:n
        if norm(A2(:,e)-ones(n,1)) =0
            s2(b)=e;
            b=b+1;
        end
end
```

```
if s2(1) =0
        f=0;
        for a=1:length(s2)
            if norm(A2(:,s2(a))-A(:,s2(a),l3+w))==0
                f=f+1;
            end
        end
        if f==length(s2)
            B1(:,:,m1)=A(:,:,l3+w);
            m1=m1+1;
        end
else
        B1(:,:,m1)=A(:,:,l3+w);
        m1=m1+1;
end
clear('s2')
  end
    end
  end
end 0/0for u
p=zeros(1,n);
for i=1:n
```

```
        p(i)=i;
end
per=perms(p);
pm(:,:,length(per))=zeros(n);
0/0 The "pm" matrices generate all of the permutation matrices.
B2=B1;
Bh=B2;
k=1;
while k> 0
        [l1, l2, l3]=size(B2);
        if k==l3
          break
        end
        for q=1:length(per)
          for i=1:n
                for j=1:n
                      pm(i,j,q)=B2(per(q,i),per(q,j),k);
                end
          end
          end0/0q
        s=0;
```

```
        for r=k+1:l3
          for q=1:length(per)
                if norm(B2(:,:,r)-pm(:,:,q))==0
               Bh(:,:,r-s)=[ ];
              s=s+1;
              break
              end
          end0/0q
        end0/0for r
        k=k+1;
        B2=Bh;
end0/0for k
m2=l3;
B3(:,:,1)=zeros(n);
m3=1;
for r=1:l3
        s=0;
        for k=1:n
          for i=1:n
                if 1<=B2(k,i,r)+B2(i,i,r)
             s=s+1;
```

```
        end
      end0/0for i
      if s==n
        B3(:,:,m3)=B2(:,:,r);
        m3=m3+1;
        break
      end
    end0/0for k
end0/0for r
B4(:,:,1)=zeros(n);
m4=1;
for i=1:m3-1
    if norm(B3(:,:,i)-B3(:,:,i)')==0
      B4(:,:,m4)=B3(:,:,i);
      m4=m4+1;
    end
end
```

The output files $B3$ and $B4$, for $n = 2$, contain the following 2×2 matrices, respectively:
$1001, 1101, 1111, 1110$ and $1001, 1111, 1110$.

In the following code the matrices $B5$ and $B6$ generate the regular matrices and reversible matrices, respectively .

```
A(:,:,1)=zeros(n);
```

```
m1=1;m3=1;
B1(:,:,1)=A;
for u=1:n
        for v=1:n
                [l1, l2, l3]=size(A);
          for w=1:l3
                A(:,:,l3+w)=A(:,:,w);
                A(u,v,l3+w)=1;
                **********************************
                s1=0;
                for e=1:n
                     if (norm(A(:,e,l3+w))==0)||(norm(A(e,:,l3+w))==0)
                          s1=s1+1;
                          break
                     end
                end
                if s1==0
                     A2=A(:,:,l3+w)*A(:,:,l3+w);
                     for r=1:n
                          for s=1:n
                               if A2(r,s)¿1
```

```
                A2(r,s)=1;
            end
        end
    end
    if norm(A(:,:,l3+w)-A2)==0
        B5(:,:,m1)=A(:,:,l3+w);
        m1=m1+1;
    end
  end0/0if s1=0
 end0/0for w
end0/0for v
end0/0for u
p=zeros(1,n);
for i=1:n
    p(i)=i;
end
per=perms(p);
pm(:,:,length(per))=zeros(n);
0/0 The "pm" matrices generate all of the permutation matrices.
B6=B5;
Bh=B6;
```

```
k=1;
while k>0
    [l1, l2, l3]=size(B6);
    if k==l3
        break
    end
    for q=1:length(per)
        for i=1:n
            for j=1:n
                pm(i,j,q)=B6(per(q,i),per(q,j),k);
            end
        end
    end0/0q
    s=0;
    for r=k+1:l3
        for q=1:length(per)
            if norm(B6(:,:,r)-pm(:,:,q))==0
                Bh(:,:,r-s)=[];
                s=s+1;
                break
            end
```

```
        end0/0q
      end0/0for r
      k=k+1;
      B6=Bh;
  end0/0for k
  m2=l3;
```

The output files $B5$ and $B6$, for $n = 2$, contain the following 2×2 matrices, respectively:
$1001, 1101, 1111, 1011$ and $1001, 1111, 1101$.

5 Conclusions

In this paper, the class of all regular reversible hypergroups obtained from a binary relation in the sense of Rosenberg and called regular reversible Rosenberg hypergroups, has been investigated in order to determine the non-isomorphic such hypergroups. Having applied a program written in *MATLAB*, we have determined the number of these regular reversible hypergroups. The results of this computation (for $n = 2, 3$, or 4) are summarized in the following table:

N=	2	3	4
Number of very good matrices	6	149	9729
Number of very good matrices up to isomorphism	4	33	501
Number of very good idempotent matrices	4	35	559
Number of very good idempotent matrices up to isomorphism	3	10	44
Number of regular matrices up to isomorphism	4	30	400
Number of reversible matrices up to isomorphism	3	9	30

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