Trees with a given order and matching number that have maximum general sum-connectivity index

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Abstract. The general sum-connectivity index is defined as $\chi_\alpha(G) = \sum_{uv\in E(G)}(d_G(u) + d_G(v))^\alpha$. Let $\mathscr{T}(n,\beta)$ be the class of trees of order n with given matching number β. In this paper, we characterize the structure of the trees with a given order and matching number that have maximum general sum-connectivity index for $0 < \alpha < 1$ and give sharp upper bound for $\alpha \geq 1$.

Keywords: general sum-connectivity index; tree; matching

AMS subject classification: 05C69, 05C05

1. Introduction

Let $G = (V(G), E(G))$ be a connected simple graph with $|V(G)| = n$ and $|E(G| = m$. If $m = n + c - 1$, then G is called a c-cyclic graph. Specially, if $c = 0$ or 1, then G is called a tree or a unicyclic graph, respectively. Let $N_G(v)$ denote the neighbor set of vertex v in G, then $d_G(v) = |N_G(v)|$ is named the degree of v in G. Let $\Delta(G)$ be the maximum degree of G. Let P_n and S_n be respectively the path and the star with n vertices. A pendent path in G is a path having one end-vertex of degree at least 3, the other is of degree 1 and the intermediate vertices are of degree 2. An internal path of G is defined as a walk $v_0v_1\dots v_s (s \geq 1)$ such that the vertices $v_0, v_1, \dots, v_s$ are distinct, $d_G(v_0) > 2, d_G(v_s) > 2$ and $d_G(v_i) = 2$, whenever $0 < i < s$. For a path P_n, denote by $|P_n|$ the length of P_n. A matching M of the graph G is a subset of $E(G)$ such that no two edges in M share a common vertex. A matching M of G is said to be maximum, if for any other matching M' of G, $|M'| \leq |M|$. The matching number β of G is the number of edges of a maximum matching in G. If M is a matching

This project is supported supported by the Foundation of State Ethnic Affairs (14ZNZ023) and the Natural Science Foundation of Hubei Province (2014CFC1118, 2014CFB657) (E-mail: zzxun73@mail.scuec.edu.cn; w.zhang@mail.ccnu.edu.cn)

of a graph G and vertex $v \in V(G)$ is incident with an edge of M, then v is said to be M-saturated, otherwise be M-unsaturated.

The well-known Randić connectivity index $R(G)$ of G, is defined as: $R(G) = \sum_{uv \in E(G)} (d_G(u) d_G(v))^{-\frac{1}{2}}$, which is proposed by Randić in 1975 [1], has received intensive attention since its successful applications in QSPR and QSAR [2]. Its mathematical properties as well as those of its generalizations have been studied extensively as summarized in the books [3, 4]. Recently, a closely related variant of Randić connectivity index called the sum-connectivity index [5], denoted by $\chi(G)$, is defined as: $\chi(G) = \sum_{uv \in E(G)} (d_G(u) + d_G(v))^{-\frac{1}{2}}$.

It has been found that the sum-connectivity index and the Randić connectivity index correlate well among themselves and with π-electronic energy of benzenoid hydrocarbons [5]. The ordinary Randić connectivity index has been extended to the general Randić connectivity index [4] defined as: $R_\alpha(G) = \sum_{uv \in E(G)} (d_G(u) d_G(v))^{\alpha}$, where α is a real number. Motivated by this, the general sum-connectivity index [6] is defined as: $\chi_\alpha(G) = \sum_{uv \in E(G)} (d_G(u) + d_G(v))^{\alpha}$.

Some properties of $\chi_\alpha(G)$, such as for trees and unicyclic graphs, have already been established [6, 7]. But by the results of experiments, B. Zhou and N. Trinajstić [7] have pointed out that the optimum exponent of the sum-connectivity index depends on the property considered and the properties of the general sum-connectivity index warrant further studies. In this paper, we will study the trees of order n with given matching number β, and characterize the structure of the trees which have maximal general sum-connectivity index for $0 < \alpha < 1$ and give sharp upper bound for $\alpha \geq 1$ in $\mathscr{T}(n, \beta)$, where $\mathscr{T}(n, \beta)$ is the class of trees of order n with given matching number β.

2. Trees with maximal general sum-connectivity index in $\mathscr{T}(n, \beta)$

For convenience, let T^* be the extremal tree in $\mathscr{T}(n, \beta)$ which has maximal general sum-connectivity index for $\alpha > 0$, $V^* = \{v \in V(T^*), d(v) \geq 3\}$, and M^* a fixed maximum matching of T^*. Note that if $\beta = 1$, then $T^* \cong S_n$; if $\beta = 2, n = 4$, $T^* \cong P_4$. Hence we can only consider the case for $\beta \geq 2$ and $n \geq 5$ in the following discussion. Note that we have $\Delta(T^*) \geq 3$. Otherwise, if $\Delta(T^*) \leq 2$, then T^* must be a path. Let $T^* = v_0 v_1 \ldots v_{n-1}$ and $T' = T^* - \{v_1 v_2\} + \{v_1 v_3\}$, it is easy to see that T' and T have the same matching number and $\chi_\alpha(T') - \chi_\alpha(T^*) = 5^\alpha - 3^\alpha > 0$, for $n = 5$, and for $n \geq 6$ $\chi_\alpha(T') - \chi_\alpha(T^*) = 2 \cdot 5^\alpha - 2 \cdot 4^\alpha > 0$, it is contradict to maximality of T^*.

Lemma 2.1. *Let P be a pendent path of T^*, then $|P| \leq 2$.*

Proof. Assume to the contrary that $P = v_0v_1 \ldots v_l$ $(l \geq 3)$ is a pendent path of T^* with $d(v_0) = t$ $(t \geq 3)$, $d(v_l) = 1$, $d(v_1) = \ldots = d(v_{l-1}) = 2$. Denote $N_0 = N_{T^*}(v_0) \setminus \{v_1\}$. Let $T' = T^* - \{v_{l-2}v_{l-1}\} + \{v_0v_l\}$. Note that if $v_{l-1}v_l \in M^*$, it must have $v_{l-2}v_{l-1} \notin M^*$, then M^* is also a maximal matching of T'. If $v_{l-1}v_l \notin M^*$, it must have $v_{l-2}v_{l-1} \in M^*$, then $M^* - \{v_{l-2}v_{l-1}\} + \{v_{l-1}v_l\}$ is a maximal matching of T'. Hence $T' \in \mathscr{T}(n, \beta)$.

Case 1. If $l \geq 4$, then $\chi_\alpha(T') - \chi_\alpha(T^*) > [(t+3)^\alpha - 4^\alpha] - [4^\alpha - 3^\alpha] \geq [6^\alpha - 4^\alpha] - [4^\alpha - 3^\alpha]$. By the Lagrange mean-value theorem, we have $[6^\alpha - 4^\alpha] - [4^\alpha - 3^\alpha] = \alpha(2 \cdot \eta_1^{\alpha-1} - \eta_2^{\alpha-1})$, where $\eta_1 \in (4,6), \eta_2 \in (3,4)$. If $\alpha \geq 1$, it is obvious that $2 \cdot \eta_1^{\alpha-1} - \eta_2^{\alpha-1} > 0$. If $\alpha \in (0,1)$, $2 \cdot \eta_1^{\alpha-1} - \eta_2^{\alpha-1} > 2 \cdot 6^{\alpha-1} - 3^{\alpha-1}$, and $\frac{2 \cdot 6^{\alpha-1}}{3^{\alpha-1}} = \frac{2}{2^{1-\alpha}} > 1$, then $2 \cdot \eta_1^{\alpha-1} - \eta_2^{\alpha-1} > 0$. Hence $\chi_\alpha(T') > \chi_\alpha(T^*)$, a contradiction.

Case 2. If $l = 3$, then

$$\begin{aligned} \chi_\alpha(T') - \chi_\alpha(T^*) &= \sum_{u \in N_0} (t+1+d(u))^\alpha + (t+3)^\alpha + (t+2)^\alpha - \\ &\quad [\sum_{u \in N_0} (t+d(u))^\alpha + (t+2)^\alpha + 4^\alpha] > 0, \end{aligned}$$

also a contradiction. This completes the proof. □

Lemma 2.2. *Let P be an internal path of T^*, then $|P| \leq 1$.*

Proof. Assume to the contrary that $P = v_0v_1 \ldots v_l (l \geq 2)$ is an internal path of T^* with $d(v_0) = t (t \geq 3)$, $d(v_l) = s (s \geq 3)$, $d(v_1) = \ldots = d(v_{l-1}) = 2$. Let $N_0 = N_{T^*}(v_0) \setminus \{v_1\}$ and $N_1 = N_{T^*}(v_l) \setminus \{v_{l-1}\}$.

Case 1. $l = 2$.

Subcase 1.1. If v_1 is M^*-unsaturated, then v_0 and v_2 are both M^*-saturated. Let $T' = T^* - \{v_1v_2\} + \{v_0v_2\}$, we have $T' \in \mathscr{T}(n, \beta)$ (as shown in Figure 1).

$$\begin{aligned} &\chi_\alpha(T') - \chi_\alpha(T^*) \\ = &\sum_{u \in N_0} (t+1+d(u))^\alpha + (t+2)^\alpha + (t+s+1)^\alpha + \sum_{u \in N_1} (s+d(u))^\alpha \\ &-[\sum_{u \in N_0} (t+d(u))^\alpha + (t+2)^\alpha + (s+2)^\alpha + \sum_{u \in N_1} (s+d(u))^\alpha] > 0, \end{aligned}$$

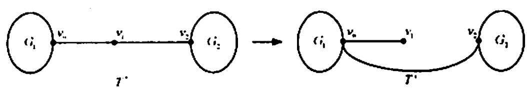

Figure 1: The graphs T^* and T' in Subcase 1.1.

a contradiction.

Subcase 1.2. If v_1 is M^*-saturated, without loss of generality, let $v_0v_1 \in M^*$. Using the same transformation as in subcase 1.1, it is contradict to the assumption of T^*.

Case 2. $l = 3$.

Subcase 2.1. If $v_1v_2 \in M^*$, let $T' = T^* - \{v_2v_3\} + \{v_0v_3\}$, we have $T' \in \mathscr{T}(n, \beta)$ (as shown in Figure 2).

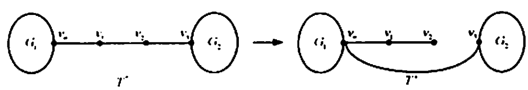

Figure 2: The graphs T^* and T' in Subcase 2.1.

$$
\begin{aligned}
\chi_\alpha(T') - \chi_\alpha(T^*) &> [(t+s+1)^\alpha - (s+2)^\alpha] - [4^\alpha - 3^\alpha] \\
&= \alpha \cdot [(t-1)\eta_1^{\alpha-1} - \eta_2^{\alpha-1}],
\end{aligned}
$$

where $\eta_1 \in (s+2, s+t+1), \eta_2 \in (3,4)$. If $\alpha \geq 1$, $(t-1)\eta_1^{\alpha-1} - \eta_2^{\alpha-1} > 0$ since $s, t \geq 3$. If $\alpha \in (0,1)$, $(t-1)\eta_1^{\alpha-1} - \eta_2^{\alpha-1} > (t-1)(s+t+1)^{\alpha-1} - 3^{\alpha-1} > 2 \cdot 7^{\alpha-1} - 3^{\alpha-1} > 2 \cdot 6^{\alpha-1} - 3^{\alpha-1} > 0$ since $s, t \geq 3$. Then $\chi_\alpha(T') > \chi_\alpha(T^*)$, a contradiction.

Subcase 2.2. If $v_1v_2 \notin M^*$, let $T' = T^* - \{v_1v_2\} + \{v_0v_3\}$, we have $T' \in \mathscr{T}(n, \beta)$ and $\chi_\alpha(T') - \chi_\alpha(T^*) > 0$, a contradiction.

Case 3. $l = 4$.

Subcase 3.1. If v_2 is M^*-unsaturated, then v_1 and v_3 must be M^*-saturated. Let $T' = T^* - \{v_1v_2, v_2v_3\} + \{v_0v_2, v_1v_3\}$, we have $T' \in \mathscr{T}(n, \beta)$ (as shown in Figure 3). $\chi_\alpha(T') - \chi_\alpha(T^*) = [\sum_{u \in N_0}(t+1+d(u))^\alpha -$

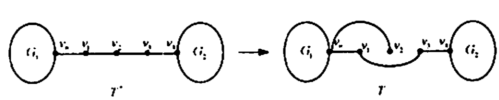

Figure 3: The graphs T^* and T' in Subcase 3.1.

$\sum_{u \in N_0}(t + d(u))^\alpha] + [(t+3)^\alpha - 4^\alpha] > 0$, since $t \geq 3$, a contradiction.

Subcase 3.2. If v_2 is M^*-saturated, without loss of generality, let $v_1v_2 \in M^*$, let $T' = T^* - \{v_2v_3\} + \{v_0v_3\}$, we have $T' \in \mathscr{T}(n,\beta)$. $\chi_\alpha(T') - \chi_\alpha(T^*) > [(t+3)^\alpha - 4^\alpha] - [4^\alpha - 3^\alpha] > [6^\alpha - 4^\alpha] - [4^\alpha - 3^\alpha] > 0$, since $t \geq 3$, a contradiction.

Case 4. $l \geq 5$. Let $T' = T^* - \{v_1v_2, v_3v_4\} + \{v_0v_2, v_1v_4\}$, we have $T' \in \mathscr{T}(n,\beta)$(as shown in Figure 4), we also have $\chi_\alpha(T') - \chi_\alpha(T^*) > 0$, a contradiction. □

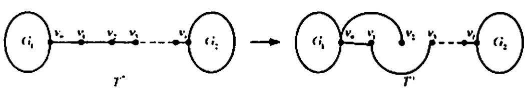

Figure 4: The graphs T^* and T' in Case 4.

Lemma 2.3. $|V^*| \leq 2$.

Proof. Assume that $|V^*| \geq 3$, then by Lemma 2.2, there exist $v_0, v_1, v_2 \in V^*$ such that $v_0v_1, v_1v_2 \in E(T^*)$. Let $d(v_0) = r, d(v_1) = s, d(v_2) = t$. Let $N_0 = N_{T^*}(v_0) \setminus \{v_1\}$, $N_1 = N_{T^*}(v_1) \setminus \{v_0, v_2\}$ and $N_2 = N_{T^*}(v_2) \setminus \{v_1\}$, respectively.

Case 1. Only one vertex of $\{v_0, v_1, v_2\}$ is M^*-saturated. Then it must be v_1. Let $T' = T^* - \cup_{v\in N_2}\{vv_2\} + \cup_{v\in N_0}\{vv_1\}$, obviously, M^* is also a maximum matching of T'. $\chi_\alpha(T') - \chi_\alpha(T^*) > (r+s+t-1)^\alpha - (r+s)^\alpha > 0$, a contradiction.

Case 2. Exactly two vertices of $\{v_0, v_1, v_2\}$ are M^*-saturated.

Subcase 2.1. If v_0, v_1 are M^*-saturated, then every vertex in N_2 is M^*-saturated. Let $T' = T^* - \cup_{v\in N_2}\{vv_2\} + \cup_{v\in N_2}\{vv_1\}$, obviously, M^* is also a maximum matching of T'. Similar to Case 1, we also obtain a contradiction.

Subcase 2.2. If v_0, v_2 are M^*-saturated, assume $v_2x \in M^*$. Let $T' = T^* - \cup_{v\in N_2}\{vv_2\} + \cup_{v\in N_2}\{vv_1\}$, obviously, $M^* - \{v_2x\} + \{v_1v_2\}$ is also a maximum matching of T'. Similar to Case 1, we also have a contradiction.

Subcase 2.3. If v_1, v_2 are M^*-saturated, then every vertex in N_0 is M^*-saturated. Let $T' = T^* - \cup_{v\in N_0}\{vv_2\} + \cup_{v\in N_0}\{vv_1\}$, obviously, M^* is also a maximum matching of T'. $\chi_\alpha(T') - \chi_\alpha(T^*) > (r+s+t-1)^\alpha - (s+t)^\alpha > 0$, a contradiction.

Case 3. $\{v_0, v_1, v_2\}$ are all M^*-saturated.

Subcase 3.1. $v_0v_1 \in M^*$ or $v_1v_2 \in M^*$. Without loss of generality, let $v_0v_1 \in M^*$, then there exists $w \in V(T^*) \setminus \{v_0, v_1\}$ such that $v_2w \in M^*$.

let $T' = T^* - \cup_{v \in N_0}\{vv_0\} + \cup_{v \in N_0}\{vv_1\}$, obviously, M^* is also a maximum matching of T'. Similar to Subcase 2.3, a contradiction to the assumption of T^*.

Subcase 3.2. $v_0v_1, v_1v_2 \notin M^*$. Then there exist $x \in N_0, y \in N_1, z \in N_2$ such that $v_0x, v_1y, v_2z \in M^*$. If $d(x) \geq 3$, we choose x, v_0, v_1 instead of v_0, v_1, v_2 for consideration, and back to Subcase 1.1, we get a contradiction. If $d(x) = 2$, by Lemma 2.1, we know that the degree of the vertex which is adjacent to x other than v_0 is 1. It is easy to see that $M^* - \{v_0x\} + \{ux\}$ is a maximum matching of T^*. At the same time, there are exactly two vertices of $\{v_0, v_1, v_2\}$ that are saturated by $M^* - \{v_0x\} + \{ux\}$. then back to Case 1, we also get a contradiction. So we have $d(x) = 1, d(y) = 1, d(z) = 1$. Let $T' = T^* - \cup_{v \in (N_0 - x)}\{vv_0\} - \cup_{v \in (N_2 - z)}\{vv_2\} + \cup_{v \in (N_0 - x) \cup (N_2 - z)}\{vv_1\}$.

$$\begin{aligned} \chi_\alpha(T') - \chi_\alpha(T^*) \quad > \quad & (r+s+t-3)^\alpha + 2(r+s+t-2)^\alpha + 2 \cdot 3^\alpha - \\ & [(r+1)^\alpha + (s+1)^\alpha + (t+1)^\alpha + (r+s)^\alpha + (s+t)^\alpha]. \end{aligned}$$

Let $F(r,s,t) = (r+s+t-3)^\alpha + 2(r+s+t-2)^\alpha + 2 \cdot 3^\alpha - [(r+1)^\alpha + (s+1)^\alpha + (t+1)^\alpha + (r+s)^\alpha + (s+t)^\alpha]$. If $\alpha = 1$, it is easy to see that $F(r,s,t) = r + t - 2 > 0$, a contradiction. If $\alpha > 1$, we have $\frac{\partial F(r,s,t)}{\partial r}, \frac{\partial F(r,s,t)}{\partial s}, \frac{\partial F(r,s,t)}{\partial t} > 0$. Hence, $F(r,s,t) \geq F(3,3,3) > 0$, where $\eta_1 \in (6,7)$ and $\eta_2 \in (3,4)$, also a contradiction.

If $0 < \alpha < 1$, without loss of generality, let $r \leq s \leq t$ and $T' = T^* - v_0x + xy$. $\chi_\alpha(T') - \chi_\alpha(T^*) > [(r+s-1)^\alpha - (r+s)^\alpha] + [3^\alpha - (r+1)^\alpha] - [(s+1)^\alpha - (s+2)^\alpha] \geq \alpha(\eta_1^{\alpha-1} + \eta_2^{\alpha-1} - \eta_3^{\alpha-1})$, where $\eta_1 \in (r+s-1, r+s), \eta_2 \in (3, r+1)$ and $\eta_3 \in (s+1, s+2)$. Obviously, $\eta_2 < \eta_3$, then $\eta_2^{\alpha-1} - \eta_3^{\alpha-1} > 0$. Hence $\chi_\alpha(T') > \chi_\alpha(T^*)$, it is a contradiction. This completes the proof. □

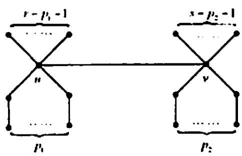

Figure 5: The structure of the graph in Theorem 2.4 (ii).

By Lemmas 2.1-2.3, we have the following theorem.

Theorem 2.4. *Let T^* be the extremal tree in $\mathscr{T}(n, \beta)$ which has maximal general sum-connectivity index for $\alpha > 0$, $V^* = \{v \in V(T^*), d(v) \geq 3\}$.*

(i) If $|V^*| = 1$ and $V^* = \{w\}$, then the attached parts of w are either pendent paths of length 1 or pendent paths of length 2;

(ii) If $|V^*| = 2$ and $V^* = \{u, v\}$, then $uv \in E(T^*)$. Besides u and v, the attached parts of u and v are either pendent paths of length 1 or pendent paths of length 2 (as shown in Figure 5).

Lemma 2.5. *If $V^* = \{u, v\}$, let $d_{T^*}(u) = r, d_{T^*}(v) = s (r, s \geq 3)$, then there exist pendent vertices in $N_{T^*}(u)$ and $N_{T^*}(v)$, respectively.*

Proof. We prove it by contradiction. Without loss of generality, suppose there are no pendent vertices in $N_{T^*}(u)$. Let t be the number of pendent paths of length 2 attached at v, and $T' = T^* - \sum_{w\in(N_{T^*}(v)-\{u\})} vw + \sum_{w\in(N_{T^*}(v)-\{u\})} uw$, obviously, $T' \in \mathscr{T}(n, \beta)$. $\chi_\alpha(T') - \chi_\alpha(T^*) = (r - 1)[(r+s+1)^\alpha - (r+2)^\alpha] + t[(r+s+1)^\alpha - (s+2)^\alpha] + (s-t-1)[(r+s)^\alpha - (s+1)^\alpha] > 0$, since $r, s \geq 3$. It is contradict to the maximality of T^*. □

Lemma 2.6. *If $\alpha \geq 1$, $|V^*| = 1$.*

Proof. Suppose to the contrary that $V^* = \{u, v\}$. Assume $d_{T^*}(u) = r, d_{T^*}(v) = s$ and $r \geq s \geq 3$. Let p_1 be the number of paths of length 2 attached at u and p_2 the number of paths of length 2 attached at v. By Lemma 2.5, there exists $z \in N_{T^*}(v)$ with $d_{T^*}(z) = 1$, and $r - p_1 - 1 \geq 1$. Let $T' = T^* - \sum_{w\in(N_{T^*}(v)-\{u,z\})} wv + \sum_{w\in(N_{T^*}(v)-\{u,z\})} wu$. Then $\chi_\alpha(T') - \chi_\alpha(T^*) \geq (r+s-1)^\alpha - (r+1)^\alpha + 3^\alpha - (s+1)^\alpha$. Let $F(r, s) = (r+s-1)^\alpha - (r+1)^\alpha + 3^\alpha - (s+1)^\alpha$. If $\alpha \geq 1$, we have $\frac{\partial F(r,s)}{\partial r}, \frac{\partial F(r,s)}{\partial s} > 0$. Then $F(r, s) \geq F(3, 3) = (5^\alpha - 4^\alpha) - (4^\alpha - 3^\alpha) = \eta_1^{\alpha-1} - \eta_2^{\alpha-1} > 0$, since $\eta_1 \in (4, 5)$ and $\eta_2 \in (3, 4)$. Hence $\chi_\alpha(T') > \chi_\alpha(T^*)$, it is contradict to the maximality of T^*. □

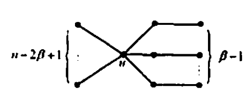

Figure 6: The graph $T(n, \beta)$.

Theorem 2.7. *For $\alpha \geq 1$, let $T \in \mathscr{T}(n, \beta)$, then we have $\chi_\alpha(T) \leq (\beta - 1)(n - \beta + 2)^\alpha + (\beta - 1)3^\alpha + (n - 2\beta + 1)(n - \beta + 1)^\alpha$. The equality holds if and only if T has the structure of Figure 6.*

Proof. If $n = 2\beta$ or $n \geq 2\beta + 2$, by Theorem 2.4 and Lemma 2.6, the structure of extremal tree T^* is unique and showed in Figure 6.

$$\chi_\alpha(T^*) = (\beta - 1)(n - \beta + 2)^\alpha + (\beta - 1)3^\alpha + (n - 2\beta + 1)(n - \beta + 1)^\alpha.$$

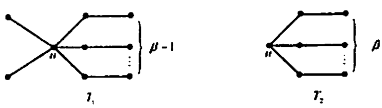

Figure 7: The graphs T_1 and T_2.

If $n = 2\beta + 1$, then there are two possible structures of extremal trees showed in Figure 7. By direct calculation, we have

$$\begin{aligned} \chi_\alpha(T_1) &= (\beta-1)(\beta+3)^\alpha + 2(\beta+2)^\alpha + (\beta-1)3^\alpha, \\ \chi_\alpha(T_2) &= \beta(\beta+2)^\alpha + \beta \cdot 3^\alpha, \end{aligned}$$

then,

$$\chi_\alpha(T_1) - \chi_\alpha(T_2) = (\beta-2)[(\beta+3)^\alpha - (\beta+2)^\alpha] + (\beta+3)^\alpha - 3^\alpha > 0.$$

This completes the proof. □

Remark: For $T \in \mathscr{T}(n,\beta)$ ($n \geq 5$, $\beta \geq 2$), if $\alpha \geq 1$, we have obtained the maximum value of the general sum-connectivity index and characterized the corresponding extremal tree. But for the case $0 < \alpha < 1$, it is rather complicated, we only have characterized the structure of the corresponding extremal trees in Theorem 2.4. It would be interesting to find the exact extremal graphs for $0 < \alpha < 1$.

References

[1] M. Randić, On characterization of molecular branching. J. Am. Chem. Soc. 97(1975), 6609-6615.

[2] R. García-Domenech, J. Gálvez, de Julian-Ortiz, J. V. and L. Pogliani, Some new trends in chemical graph theory. Chem. Rev, 108(2008), 1127-1169.

[3] I. Gutman, B. Furtula (Eds.), Recent Results in the Theory of Randić Index, Univ. Kragujevac, Kragujevac, 2008.

[4] X. Li, I. Gutman, Mathematical Aspects of Randić-Type Molecular Structure Descriptors, Univ. Kragujevac, Kragujevac, 2006.

[5] B. Zhou, N. Trinajstić, On a novel connectivity index, J. Math. Chem. 46(2009), 1252-1270.

[6] B. Zhou, N. Trinajstić, On general sum-connectivity index. J. Math. Chem. 47(2010), 210-218.

[7] Z. Du, B. Zhou, N. Trinajstić, Minimum general sum-connectivity index of unicyclic graphs, J. Math. Chem. 48(2010), 697-703.