

# A Construction of Multi-receiver Authentication Codes with Dynamic Sender from Linear Codes \*

Shangdi Chen<sup>†</sup> Lizhen Chang<sup>‡</sup>

*College of Science, Civil Aviation University of China, Tianjin, 300300;  
Shanxi Technology And Business University, Taiyuan, 030006, P.R.China*

**Abstract** Multi-receiver authentication codes with dynamic sender (DMRA-codes) are extensions of traditional group communication system in which any member of a group can broadcast an authenticated message such that all other group members can individually verify its authenticity, and some malicious participants of group can not successfully impersonate the potential sender, or substitute a transmitted message. In this paper, a construction of DMRA-code will be given using linear code and its unconditional security is also guaranteed.

**Keywords** Multi-receiver; Authentication code; Dynamic; Linear code

**Mathematics Subject Classification 2000:** 05B25, 94A62

## 1 Introduction

With the development of information technology, traditional point-to-point message authentication systems have been extensively studied in the literature. In this paper, we consider authentication for group communication. As we all known, many schemes have been proposed by several researchers to provide authentication to secure group communication. Y.Desmedt and

\*The Project-sponsored by the National Natural Science Foundation of China(61179026), the Fundamental Research Funds of the Central Universities of China(3122014K015).

<sup>†</sup>Corresponding author. E-mail: 11csd@163.com; sdchen@cauc.edu.cn

<sup>‡</sup>Corresponding author. E-mail: clzscj.123@163.com; 362969673@qq.com

R.Safavi-Naini developed a series of authentication schemes in [1-2] which consider a single transmitter who is fixed before hand. Safavi-Naini and Wang extended the schemes in [3-4] and they relaxed the restriction that the sender is fixed before hand and introduced a dynamic sender concept in which any one of the users can become the sender. In [5], they dropped the restriction of a single dynamic sender and developed a scheme for the situation with  $t$  senders. In [6-7], Aparna and Amberker constructed some secure authentication codes with dynamic senders, which promoted the growth of group communication in further. In this paper, a new construction of multi-receiver authentication code with dynamic sender (DMRA-code) will be proposed, the parameters and maximum probabilities of success in various attacks are also computed.

In this paper, let  $GF(q)$  be the finite field with  $q$  elements, where  $q$  is a power of a prime. We use  $GF(q)^k$  to denote the  $k$ -dimensional row vector space over  $GF(q)$ . The set of all non-zero elements of  $GF(q)^k$  is denoted as  $GF(q)^{k*}$ .

The rest of the paper is organized as follows. In section 2 we describe the models of multi-receiver authentication codes with dynamic senders (DMRA-code). In section 3 we give the calculating formulas of the probability of attacks which are from a group of receivers who have access to part of the key information. We present, in section 4, a new construction of DMRA-code and the bounds. Finally, we conclude the paper.

## 2 The Models of DMRA-code

In this section, we study MRA-codes with dynamic senders. We consider the scenario where there is a group of  $k$  users  $U = \{U_1, U_2, \dots, U_k\}$  and a KDC who only runs the keys distribution of participants. In this model, because every user can be a sender as well as a receiver to other users, so the keys of each user have also dual functions which can not only encode but also decode messages. Let  $C_i = (S, E_i, M_i; f_i, g_{ij})$  ( $j \neq i$ ) be authentication codes of user  $U_i$  and  $U_j$ , where  $\{i, j\} \subset \{1, 2, \dots, k\}$ ,  $f_i : S \times E_i \rightarrow M_i$  be authentication algorithm of user  $U_i$ ,  $g_{ij} : S \times E_i \rightarrow M_j$  be verification algorithm of  $U_j$ . For authenticating a message, every user should comply with protocols:

(1)Key Distribution: The KDC randomly chooses a encoding rule  $e \in E$  and applies some key distribution algorithms to generate a key  $e_i$  for each user  $U_i$ , then secretly sends  $e_i$  to  $U_i$ . In addition, KDC also generates  $k$  distinct values  $a_i$  which are public knowledge as identity information for user  $U_i$ , ( $i = 1, 2, \dots, k$ );

(2)Broadcast: If a user  $U_i$  wants to send a source state  $s$  to others,  $U_i$  computes  $m_i = f_i(s, e_i)$  and sends  $(s, m_i)$  to others with his identity information  $a_i$ ;

(3)Verification: A user  $U_j$  ( $j \neq i$ ) uses his verification algorithm  $g_{ji}$  to accept or reject the received codeword. That is, he checks the authenticity by verifying whether  $m_i = g_{ji}(s, e_j)$  or not. If the equality holds, the message is authentic and is accepted. Otherwise, the message is rejected.

We assume that the system follows the Kerckhoff's principle which except the actual used keys of each user, the other information of the whole system is public. This includes the uniform probability distribution of the source states and the keys of users.

### 3 The calculation formulas

In the whole system, we assume  $U = \{U_1, U_2, \dots, U_k\}$  ( $k \geq 3$ ) are a group of users,  $S$  is the source state space,  $E_i$  is the encoding rules set of user  $U_i$  and  $M_i$  is the message space of user  $U_i$ , ( $1 \leq i \leq k$ ).

To assess the security, we consider the probabilities of success in various attacks. Attackers could be outsiders who do not have access to any key information, or insiders who have part of key information. We only need to consider the latter group as it is at least as powerful as the former. We consider the system to protect against the coalition of groups of up to a maximum size of users, and study impersonation and substitution attacks.

Let  $L$  be a subset of  $\{1, 2, \dots, k\}$  with  $|L| = w-1$  ( $w \leq k-1$ ). Without loss of generality, let  $L = \{1, 2, \dots, w-1\}$ , denote  $U_L = \{U_1, U_2, \dots, U_{w-1}\}$  and  $E_L = E_1 \times E_2 \times \dots \times E_{w-1}$ .

In the impersonation attack,  $U_L$  collude and try to launch an attack against a pair of users  $U_i$  and  $U_j$ , by generating a message such that  $U_j$  accepts it as authentic and as being sent from  $U_i$ . It can be expressed as

$$P_I(i, j) = \max_{e_L \in E_L, \{i, j\} \not\subset L} \left\{ \frac{\max_{m \in M_i} |\{e_j \in E_j \mid e_j \subseteq m, p(e_L, e_j) \neq 0\}|}{|\{e_j \in E_j \mid p(e_L, e_j) \neq 0\}|} \right\}.$$

$P_I$  is the best probability of all such attacks and is defined by  $P_I = \max_{\{i, j\}} P_I(i, j)$ , where  $L \cup \{i, j\}$  runs through all the  $w+1$ -subsets of  $\{1, 2, \dots, k\}$ .

In the substitution attack, after seeing a valid message  $m$  broadcasted by  $U_i$ , the collaborators  $U_L$  construct a new message  $m'$  ( $m' \neq m$ ) such that

$U_j$  will accept  $m'$  as being sent from  $U_i$ . We denote the success probability in this case by

$$P_S(i, j) = \max_{e_L \in E_L, \{i, j\} \not\subseteq L} \max_{e_i \in E_i} \max_{e_j \subseteq m} \left\{ \begin{array}{l} \max_{m \neq m' \in M_i} |\{e_j \in E_j | e_j \subseteq m, e_j \subseteq m', p(e_L, e_j) \neq 0\}| \\ |\{e_j \in E_j | e_j \subseteq m, p(e_L, e_j) \neq 0\}| \end{array} \right\},$$

and the best probability of all such attacks by  $P_S = \max_{\{i, j\}} P_S(i, j)$ , where  $L \cup \{i, j\}$  runs through all the  $w + 1$ -subsets of  $\{1, 2, \dots, k\}$ .

Notes: (1)  $p(e_L, e_j) \neq 0$  implies that any source state  $s$  encoded by  $e_L$  can be authenticated by  $e_j$ . (2)  $e_j \subseteq m$  implies that  $m$  can be verified to be authentic by  $e_j$ .

## 4 Construction and the Bounds

Safavi-Naini and Wang [3-4] gave two constructions of DMRA-code based symmetric polynomials, they also showed that the latter one is optimal and has the minimum number of keys set  $E_i$  for each user  $U_i$  and the shortest length of the authenticated message  $M_i$ . So far, there is no other optimal construction. In this section, a new construction of DMRA-code will be proposed based linear codes and that the construction would result in new optimal system.

Let the set of source states be  $S = GF(q) \setminus \{-1\}$ ; the set of  $i$ -th user's encoding rules  $E_i = \{e_i | e_i \in GF(q)^k \times GF(q)^{k^*}\}$ ; the set of the authenticated message  $M_i = \{m_i | m_i \in C\} \subseteq GF(q)^n$ , where  $C = [n, k]$  is a linear code over  $GF(q)$ . A  $k \times n$  matrix  $G$  over  $GF(q)$  is called a generator matrix of  $C$  if its row vectors generate the linear subspace  $C$  and  $G$  is publicly known.

Define the encoding map of each user  $U_i$  ( $i = 1, 2, \dots, k$ ) as

$$f_i : S \times E_i \longrightarrow M_i, f_i(s, e_i) = (\delta_i + s\gamma_i)G \quad (1 \leq i \leq k),$$

where  $e_i = (\delta_i, \gamma_i) \in E_i$ .

The decoding map of each user  $U_i$  with another user  $U_j$  as

$$g_{ij} : S \times E_i \longrightarrow M_j, g_{ij}(s, e_i) = [(\delta_i + s\gamma_i) + (1 + s)(a_j - a_i)]G,$$

where  $a_i, a_j \in GF(q)^k$  and  $i \neq j$ .

This code works as follows:

### 1. Key distribution phase

(1) The KDC randomly chooses an  $(u, v)$  of  $C \times C^*$  and assumes  $u = (u_1, u_2, \dots, u_n), v = (v_1, v_2, \dots, v_n)$ . Then he calculates  $(\alpha_1, \beta_1)$  satisfying  $\alpha_1 G = u, \beta_1 G = v$ , that is  $(\alpha_1, \beta_1) \in GF(q)^k \times GF(q)^k$ ;

(2) The KDC randomly selects  $k$  distinct elements  $b_1, b_2, \dots, b_k$  of  $GF(q)^{k^*}$  and computes  $\delta_i = \alpha_1 + b_i, \gamma_i = \beta_1 + b_i$  such that  $\gamma_i \neq 0, i = 1, 2, \dots, k$ . Then he privately transmits  $e_i = (\delta_i, \gamma_i)$  to user  $U_i$  for each  $1 \leq i \leq k$ , which consists of the secret key of  $U_i$ ;

(3) KDC also randomly chooses  $b_0 \in GF(q)^{k^*}$  and calculates values  $a_i = b_0 + b_i$  which is public knowledge and is used as identity information for user  $U_i, (i = 1, 2, \dots, k)$ .

**2. Broadcast phase** For  $1 \leq i \leq k$ , assume user  $U_i$  wants to construct an authenticated message for a source state  $s \in S$ .  $U_i$  computes  $m_i = f_i(s, e_i) = (\delta_i + s\gamma_i)G$  and sends  $(s, a_i, m_i)$  to all the other users.

**3. Verification phase** The user  $U_j (j \neq i)$  can verify the authenticity of the message in the following way.  $U_j$  accepts  $(s, a_i, m_i)$  as authentic being sent from  $U_i$  if  $g_{ji}(s, e_j) = [(\delta_j + s\gamma_j) + (1 + s)(a_i - a_j)]G = m_i$ , otherwise, he rejects it.

For the sake of simplicity, we assume that after the key distribution phase, each user can only send at most a single authenticated message.

Next, we will show that the above construction is a well defined DMRA-code.

**Lemma 4.1** Let  $C_i = (S, E_i, M_i; f_i)$ , then  $C_i (1 \leq i \leq k)$  is an A-code.

**Proof.** For any  $s \in S, e_i \in E_i$ , we assume that  $e_i = (\delta_i, \gamma_i)$ , then  $f_i(s, e_i) = (\delta_i + s\gamma_i)G = m_i \in M_i$ ; Conversely, for any  $m_i \in M_i$ , choose  $e_i = (\delta_i, \gamma_i) \in E_i$ , let  $f_i(s, e_i) = (\delta_i + s\gamma_i)G = m_i$ , then  $\delta_i G = m_i - s\gamma_i G$ . Because  $m_i$  is a codeword, so  $m_i - s\gamma_i G$  is also a codeword. Thus there must exist a  $\delta_i \in GF(q)^k$  satisfying  $f_i$ . It means that  $f_i$  is a surjection.

If  $s' \in S$  is another source state satisfying  $m_i = f_i(s', e_i)$ , then  $(\delta_i + s'\gamma_i)G = (\delta_i + s\gamma_i)G$ , thus  $(s - s')\gamma_i G = 0$ . As  $\gamma_i \neq 0, \gamma_i G \neq 0$  and  $s = s'$ . That is,  $s$  is the uniquely source state determined by  $e_i$  and  $m_i$ . So  $C_i (1 \leq i \leq k)$  is an A-code.

**Lemma 4.2** For any valid message  $m = (s, a_i, m_i)$  from user  $U_i, U_j (j \neq i)$  will accept it.

**Proof.** For any valid message  $m = (s, a_i, m_i)$  from user  $U_i$ , there must exist  $e_i = (\delta_i, \gamma_i) \in E_i$ , such that  $m_i = (\delta_i + s\gamma_i)G$ . According to the given protocol, we can get  $\delta_i = \alpha_1 + b_i, \gamma_i = \beta_1 + b_i$  and  $a_i = b_0 + b_i, 1 \leq i \leq k$ ,

where  $(\alpha_1, \beta_1) \in GF(q)^k \times GF(q)^k$  and  $(b_0, b_i) \in GF(q)^{k^*} \times GF(q)^{k^*}$ . Thus

$$\begin{aligned}
 m_i &= [(\alpha_1 + b_i) + s(\beta_1 + b_i)]G \\
 &= [(\alpha_1 + s\beta_1) + (1 + s)b_i]G \\
 &= [(\alpha_1 + b_j) + s(\beta_1 + b_j) + (1 + s)(b_i - b_j)]G \\
 &= [\delta_j + s\gamma_j + (1 + s)((a_i - b_0) - (a_j - b_0))]G \\
 &= [\delta_j + s\gamma_j + (1 + s)(a_i - a_j)]G,
 \end{aligned}$$

where  $e_j = (\delta_j, \gamma_j) \in E_j$  is the key of user  $U_j$ . It means that message  $m = (s, a_i, m_i)$  could be verified by user  $U_j$ . So  $U_j$  will accept it.

From Lemma 4.1 to Lemma 4.2, we can see this construction is well defined. Next, we will compute the parameters and the maximum probability of success in various attacks.

**Theorem 4.1** The parameters of constructed authentication code with dynamic sender are:  $|S| = q - 1$ ;  $|E_i| = q^k(q^k - 1)$ ;  $|M_i| = |C| = q^k$ .

*Proof.* The result is straightforward.

**Lemma 4.3** For any fixed  $e_L = \{(\delta_1, \gamma_1), (\delta_2, \gamma_2), \dots, (\delta_{w-1}, \gamma_{w-1})\} \in E_L$ , where  $(\delta_l, \gamma_l) \in GF(q)^k \times GF(q)^{k^*}$ ,  $(l = 1, 2, \dots, w - 1)$ , let the number of  $e_j$  which is incidence with  $e_L$  be  $a$ . Then  $a = q^k - (w + 1)$ .

*Proof.* For any fixed  $e_L = \{(\delta_1, \gamma_1), (\delta_2, \gamma_2), \dots, (\delta_{w-1}, \gamma_{w-1})\}$ , according to the given protocol, we can get  $\delta_l = \alpha_1 + b_l$ ,  $\gamma_l = \beta_1 + b_l$ ,  $(l = 1, 2, \dots, w - 1)$  for a common and fixed  $(\alpha_1, \beta_1) \in GF(q)^k \times GF(q)^k$ . That is,  $(\delta_l, \gamma_l)$  is only determined by  $b_l$  of  $GF(q)^{k^*}$ . Let  $e_j = (\delta_j, \gamma_j)$ , then  $e_j$  is incidence with  $e_L$  if and only if  $\delta_j = \alpha_1 + b_j$ ,  $\gamma_j = \beta_1 + b_j$ . As  $b_j \in GF(q)^{k^*}$ ,  $b_j \neq -\beta_1$  and  $b_j \neq b_l$ ,  $(l = 1, 2, \dots, w - 1)$ , the number of  $b_j$  is  $q^k - (w + 1)$ . It means that the number of  $e_j$  which is incidence with  $e_L$  is  $q^k - (w + 1)$ . So  $a = q^k - (w + 1)$ .

**Lemma 4.4** For any fixed  $e_L = \{(\delta_1, \gamma_1), (\delta_2, \gamma_2), \dots, (\delta_{w-1}, \gamma_{w-1})\} \in E_L$ , where  $(\delta_l, \gamma_l) \in GF(q)^k \times GF(q)^{k^*}$ ,  $(l = 1, 2, \dots, w - 1)$ ,  $m \in M_i$ , where  $m$  is generated by collaborators  $U_L$  who want to send it to  $U_j$  with the identity information of  $U_i$ . Let the number of  $e_j$  ( $j \neq i$ ) which is incidence with  $e_L$  contained in  $m$  be  $c$ . Then  $c = 1$ .

*Proof.* Let  $e_j = (\delta_j, \gamma_j)$ ,  $m = (s, a_i, m'_i)$  generated by collaborators  $U_L$ . For any fixed  $e_L = \{(\delta_1, \gamma_1), (\delta_2, \gamma_2), \dots, (\delta_{w-1}, \gamma_{w-1})\}$ , similarly, we can get  $\delta_l = \alpha_1 + b_l$ ,  $\gamma_l = \beta_1 + b_l$ ,  $(l = 1, 2, \dots, w - 1)$  for a common and fixed  $(\alpha_1, \beta_1) \in GF(q)^k \times GF(q)^k$ . If  $e_j$  is incidence with  $e_L$ , then

$$\delta_j = \alpha_1 + b_j, \quad \gamma_j = \beta_1 + b_j,$$

for some  $b_j \in GF(q)^{k^*}$ .

Again,  $e_j \subseteq m$ , then

$$[(\delta_j + s\gamma_j) + (1 + s)(a_i - a_j)]G = m'_i.$$

By combining the above equations, we can get

$$\begin{aligned} & [(\alpha_1 + b_j) + s(\beta_1 + b_j) + (1 + s)(a_i - a_j)]G \\ &= [(1 + s)b_j + (\alpha_1 + s\beta_1) + (1 + s)(a_i - a_j)]G = m'_i. \end{aligned}$$

As  $m'_i$  is a codeword, there must uniquely exist a  $\xi \in GF(q)^k$  satisfying that

$$(1 + s)b_j + (\alpha_1 + s\beta_1) + (1 + s)(a_i - a_j) = \xi,$$

thus

$$(1 + s)b_j = \xi - (\alpha_1 + s\beta_1) - (1 + s)(a_i - a_j).$$

Because  $(a_i, a_j)$ ,  $(\alpha_1, \beta_1)$  and  $(s, \xi)$  are fixed and  $s \neq -1$ , so  $b_j$  is only determined by them. That is, the number of  $e_j$  which is incidence with  $e_L$  contained in  $m$  is only one. Then  $c = 1$ .

**Lemma 4.5** For any fixed  $e_L = \{(\delta_1, \gamma_1), (\delta_2, \gamma_2), \dots, (\delta_{w-1}, \gamma_{w-1})\} \in E_L$ , where  $(\delta_l, \gamma_l) \in GF(q)^k \times GF(q)^{k^*}$ ,  $(l = 1, 2, \dots, w-1)$ ,  $m \in M_i$ , where  $m$  is sent by user  $U_i$  who want to broadcast it to other users. Let the number of  $e_j$  which is incidence with  $e_L$  contained in  $m$  be  $d$ . Then  $d = q^k - (w + 2)$ .

**Proof.** Let  $e_j = (\delta_j, \gamma_j)$ ,  $m = (s, a_i, m_i)$  sent by user  $U_i$ . For any fixed  $e_L = \{(\delta_1, \gamma_1), (\delta_2, \gamma_2), \dots, (\delta_{w-1}, \gamma_{w-1})\}$ , from Lemma 4.3, we can get  $\delta_l = \alpha_1 + b_l$ ,  $\gamma_l = \beta_1 + b_l$ ,  $(l = 1, 2, \dots, w-1)$  for a common and fixed  $(\alpha_1, \beta_1) \in GF(q)^k \times GF(q)^{k^*}$ . That is,  $(\delta_l, \gamma_l)$  is only determined by  $b_l$  of  $GF(q)^{k^*}$ . If  $e_j$  is incidence with  $e_L$ , then

$$\delta_j = \alpha_1 + b_j, \quad \gamma_j = \beta_1 + b_j,$$

for some  $b_j \in GF(q)^{k^*}$ .

Again, we have known that  $e_j \subseteq m$  and  $m$  is encoded by  $e_i$ , the key of user  $U_i$ , which means that  $e_j$  is incidence with  $e_i$ . Let  $e_i = (\delta_i, \gamma_i)$ , similarly, we can see that  $\delta_i = \alpha_1 + b_i$ ,  $\gamma_i = \beta_1 + b_i$ .

By combining the above conclusions, it is easily to get  $b_j \neq -\beta_1$ ,  $b_j \neq b_i$  and  $b_j \neq b_l$ ,  $(l = 1, 2, \dots, w-1)$ . Also,  $b_j \in GF(q)^{k^*}$ , then the number of  $b_j$  satisfying the above requirements is  $q^k - (w + 2)$ . That is,  $d = q^k - (w + 2)$ .

**Lemma 4.6** For any fixed  $e_L = \{(\delta_1, \gamma_1), (\delta_2, \gamma_2), \dots, (\delta_{w-1}, \gamma_{w-1})\} \in E_L$ , where  $(\delta_l, \gamma_l) \in GF(q)^k \times GF(q)^{k^*}$ ,  $(l = 1, 2, \dots, w-1)$ ,  $m \in M_i$ ,  $m' \in M_i$  ( $m' \neq m$ ), where  $m$  is sent by user  $U_i$  and  $m'$  is generated

by collaborators  $U_L$  who want to send it to user  $U_j$  with the identity information of  $U_i$ . Let the number of  $e_j$  which is incidence with  $e_L$  contained both in  $m$  and  $m'$  be  $f$ . Then  $f = 1$ .

**Proof.** Let  $e_j = (\delta_j, \gamma_j)$ ,  $m = (s, a_i, m_i)$  and  $m' = (s', a_i, m'_i)$  ( $s \neq s'$ ). For any fixed  $e_L = \{(\delta_1, \gamma_1), (\delta_2, \gamma_2), \dots, (\delta_{w-1}, \gamma_{w-1})\}$ , similarly, we can get  $\delta_l = \alpha_1 + b_l$ ,  $\gamma_l = \beta_1 + b_l$ , ( $l = 1, 2, \dots, w-1$ ) for a common and fixed  $(\alpha_1, \beta_1) \in GF(q)^k \times GF(q)^k$ . It means that  $(\delta_l, \gamma_l)$  is only determined by  $b_l \in GF(q)^k$ . If  $e_j$  is incidence with  $e_L$ , then

$$\delta_j = \alpha_1 + b_j, \quad \gamma_j = \beta_1 + b_j,$$

for some  $b_j \in GF(q)^k$ . Again,  $e_j \subseteq m$  and  $e_j \subseteq m'$ , then from the given protocol, we can see that

$$[(\delta_j + s\gamma_j) + (1 + s)(a_i - a_j)]G = m_i$$

and

$$[(\delta_j + s'\gamma_j) + (1 + s')(a_i - a_j)]G = m'_i.$$

By combining all the above conclusions, we can get

$$[(\alpha_1 + b_j) + s(\beta_1 + b_j) + (1 + s)(a_i - a_j)]G = m_i$$

and

$$[(\alpha_1 + b_j) + s'(\beta_1 + b_j) + (1 + s')(a_i - a_j)]G = m'_i.$$

Because both  $m_i$  and  $m'_i$  are codewords, so there must exist two fixed values  $(\sigma_1, \sigma_2)$  satisfying that

$$\alpha_1 + b_j + s(\beta_1 + b_j) + (1 + s)(a_i - a_j) = \sigma_1$$

and

$$\alpha_1 + b_j + s'(\beta_1 + b_j) + (1 + s')(a_i - a_j) = \sigma_2,$$

where  $(\sigma_1, \sigma_2) \in GF(q)^k \times GF(q)^k$ . Hence,  $(s - s')(\beta_1 + b_j + a_i - a_j) = \sigma_1 - \sigma_2$ . As  $s \neq s'$ ,  $\beta_1 + b_j + a_i - a_j = (s - s')^{-1}(\sigma_1 - \sigma_2)$ . Also,  $a_i$ ,  $a_j$  and  $\beta_1$  are fixed, so  $b_j$  is only defined. That is, the number of  $e_j$  satisfying all the above requirements is only one. Then  $f = 1$ .

**Theorem 4.2** In this authentication code with dynamic sender, if the encoding rules of users are chosen according to a uniform probability distribution, then the largest probabilities of success for different types of deceptions are  $P_I = \frac{1}{q^k - (w+1)}$ , and  $P_S = \frac{1}{q^k - (w+2)}$ .

**Proof.** (1) From Theorem 4.3 and Lemma 4.4, we know that the largest probability of  $w-1$  malicious users' successful impersonation attack



is

$$\begin{aligned}
 P_I(i, j) &= \\
 \max_{e_L \in E_L, \{i, j\} \not\subseteq L} & \left\{ \begin{array}{l} \max_{m \in M_i} |\{e_j \in E_j | e_j \subseteq m, p(e_L, e_j) \neq 0\}| \\ |\{e_j \in E_j | p(e_L, e_j) \neq 0\}| \end{array} \right\} \\
 &= \frac{c}{a} = \frac{1}{q^k - (w+1)}.
 \end{aligned}$$

(2) From Lemma 4.5 and Lemma 4.6, we know that the largest probability of  $w - 1$  malicious users' successful substitution attack is

$$\begin{aligned}
 P_S(i, j) &= \\
 \max_{e_L \in E_L, \{i, j\} \not\subseteq L} & \max_{e_i \in E_i} \max_{e_i \subseteq m} \left\{ \begin{array}{l} \max_{m \neq m' \in M_i} |\{e_j \in E_j | e_j \subseteq m, e_j \subseteq m', p(e_L, e_j) \neq 0\}| \\ |\{e_j \in E_j | e_j \subseteq m, p(e_L, e_j) \neq 0\}| \end{array} \right\}, \\
 &= \frac{f}{d} = \frac{1}{q^k - (w+2)}.
 \end{aligned}$$

Obviously, both  $P_I(i, j)$  and  $P_S(i, j)$  are constants, so  $P_I = P_I(i, j) = q^k - \frac{1}{(w+1)}$  and  $P_S = P_S(i, j) = q^k - \frac{1}{(w+2)}$ .

Compared with the construction of R.Safavi-Naini and H.Wang[4], we see that in this model the size of each user's key is  $|E_i| = q^k(q^k - 1) > q^{2w}$  for all  $1 \leq i \leq k$ , and the size of codewords is  $|M_i| = |C| = q^k > q^w|S|$ . At the same time, it is easily to see that  $P_I < \frac{1}{q}$  and  $P_S < \frac{1}{q}$ . That is, the beat chances of success in the corresponding attacks are more reduced than [4].

## 5 Conclusion

Multi-receiver Authentication codes with dynamic sender (DMRA-code) are interesting and important cryptographic primitive in secure group communication. In this paper, we mainly gave a new construction of DMRA-code using linear code and derived related bounds. In addition, according to the above results, we can see that it is more optimal than the result of [4]. Also, there are many applications for such system, such as group communication of conference system, airline travel, network security etc., where members of a group want to broadcast messages such that every other group members can verify the authenticity of the received messages. Of course, they are interesting open problems, which need us to do further research.

## References

- [1] Y.Desmedt, Y.Frankle and M.Yung. Multi-receiver / Multi-sender network security: efficient authenticated multicast / feedback.IEEE Infocom'92, 1992, pp.2045-2054.
- [2] R.Safavi-Naini and H.Wang, Bounds and Constructions for Multireceiver Authentication Codes, Lecture Notes in Computer Science, Vol.(1514/1998), 1998, pp.242-256.
- [3] R.Safavi-Naini and H.Wang, New Results on Multireceiver Authentication codes, In Advances in Cryptology-Eurocrypt'98,LNCS,1438(1998), pp.527-541.
- [4] R.Safavi-Naini and H.Wang, Multi-receiver authentication codes: Models, Bounds, Constructions and Extensions, Information and Computation, 151, 1999, pp.148-172.
- [5] R.Safavi-Naini and H.Wang, Broadcast Authentication for Group Communication, Theoretical Computer Science, 269(1-2), 2001, pp.1-21.
- [6] R.Aparna and B.B.Amberker, Multi-Sender Multi-Receiver Authentication For Dynamic Secure Group Communication, IJCSNS International Journal of Computer Science and Network Security, VOL.7, No.10, 2007, pp.310-315.
- [7] R.Aparna and B.B.Amberker, Authenticated Secure Group Communication using Broadcast Encryption Key Computation, Fifth International Conference on Information Technology: New Generations, 2008, pp.348-353.