SELF VERTEX SWITCHINGS OF DISCONNECTED UNICYCLIC GRAPHS

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Abstract

A vertex $v \in V(G)$ is said to be a ***self vertex switching*** of G if G is isomorphic to G^v, where G^v is the graph obtained from G by deleting all edges of G incident to v and adding all edges incident to v which are not in G. In [6], the author characterized connected unicyclic graphs each with a self vertex switching. In this paper, we characterize disconnected unicyclic graphs each with a self vertex switching.

Key words: Switching, Self vertex switching, unicyclic, $SS_1(G)$, $ss_1(G)$.

1. Introduction

For a finite undirected simple graph $G(V, E)$ with $|V(G)| = p$ and a set $\sigma \subseteq V$, the switching of G **by** σ is defined as the graph $G^\sigma(V, E')$, which is obtained from G by removing all edges between σ and its complement $V-\sigma$ and adding as edges all non edges between σ and $V-\sigma$. Switching has been defined by Seidel [2] and is also referred to as Seidel switching. When $\sigma = \{v\} \subset V$, we call the corresponding switching $G^{\{v\}}$ as ***vertex switching*** and denoted it as G^v [1]. A subset σ of $V(G)$ to be a ***self switching*** of G if $G \cong G^\sigma$. The set of all self switchings of G with cardinality k is denoted by $SS_k(G)$ and its cardinality by $ss_k(G)$. If k = 1, then we call the corresponding self switching as ***self vertex switching*** [1, 3].

A ***branch*** at v in G is a maximal connected subgraph B of G such that the intersection of B with the vertex v is v and $B-v$ is connected [3]. A branch B at v in G is said to be **self switching branch** if $B \cong B^v$. In G, two branches B_1 and B_2 at v are said to be **complementary switching branches** if there exist isomorphisms f_1 between B_1 and B_2^v and f_2 between B_2 and B_1^v such that $f_1(v) = f_2(v)$[3]. A simple graph in which each pair of distinct vertices is joined by an edge is called a complete graph. A complete graph on n vertices is denoted by K_n. A walk in a graph is a finite non-null sequence $v_0e_1v_1e_2v_2 \dots e_nv_n$ whose terms are alternatively vertices and edges such that e_i is incident with v_{i-1} and v_i. A path is a walk in which all the vertices are distinct. A path with n vertices is denoted by P_n. Two vertices u and v in G are said to be **interchange similar** if there is an automorphism α of G such that $\alpha(u) = v$ and $\alpha(v) = u$.

In [4], we characterized interchange similar vertices to be self vertex switchings. In [5], we characterized trees and forests, each with a self vertex switching. In [6], we characterized connected unicyclic graphs, each with a self vertex switching. In this paper, we characterize disconnected unicyclic graphs, each with a self vertex switching and we consider simple graphs only. Now consider the following results, which are required in the subsequent sections.

Theorem 1.1.[1] If v is a self vertex switching of a graph G of order p, then the degree of the vertex v in G is $d_G(v) = (p-1)/2$.

Theorem 1.2.[3] Let B_i be either a branch at v in G or the union of v and a component of G not containing v, $i = 1, 2, ..., k(G-v)$. Then $G = \bigcup_{i=1}^{k} B_i$ and $G^v = \bigcup_{i=1}^{k} B_i^v$ where $k = k(G-v)$, $k(G)$ is the number of components of G.

Lemma 1.3.[5] D is a component of a graph G not containing v if and only if $D+v$ is a branch at v in G^v.

Theorem 1.4.[5] Let v be any vertex of a nontrivial connected graph G. Then G^v is a tree if and only if $G-v$ is acyclic and $d_B(v) = |V(B)|-2$ for every branch B at v in G.

Theorem 1.5.[5] Let v be a vertex of a nontrivial graph G. Then G^v is a disconnected graph with k components if and only if G has at least k-1 branches at v and $d_B(v) = |V(B)|-1$ only for k-1 branches B's at v in G.

Theorem 1.6.[6] Let v be a non cutvertex of a graph G of order $p \geq 3$. Then G^v is connected and unicyclic if and only if either of the following holds:

(a) $G = K_2 \cup (p-2)K_1$ and v is one of the K_1's.

(b) G is connected, $G-v$ is acyclic and $d_G(v) = |V(G)|-3$.

(c) G is connected, $G-v$ is unicyclic and $d_G(v) = |V(G)|-2$.

(d) $G = D \cup (p-|V(D)|)K_1$, $G-v$ is unicyclic and $d_G(v) = |V(D)|-2$.

(e) $G = D \cup (p-|V(D)|)K_1$, $G-v$ is acyclic and $d_G(v) = |V(D)|-3$.

(f) $G = D \cup K_2 \cup (p{-}2{-}|V(D)|)K_1$, $G{-}v$ is acyclic and $d_G(v) = |V(D)|{-}2$ where $D \neq K_1, K_2$ is a component of G containing v.

2. Characterization of G^v to be disconnected and unicyclic

Let v be a vertex of a graph G. Let G^v be the switching of G by v. In [5], we gave a condition on vertex v of G such that G^v is disconnected with a given number of components. In this section, we characterize vertex v of G such that G^v is disconnected and unicyclic.

Theorem 2.1. Let G be a graph of order $p \geq 3$ and $D \neq K_1, K_2$ be a component of G containing v. Then G^v is disconnected and unicyclic with k components if and only if G has r branches at v, $d_B(v) = |V(B)|{-}1$ only for $k{-}1$ branches B's at v in G, $r \geq k{-}1$ and either of the following holds:

(a) G is connected, $G{-}v$ is unicyclic, $r = k{-}1$ and one $B{-}v$ is unicyclic.

(b) G is connected, $G{-}v$ is acyclic, $r > k{-}1$, $d_B(v) \in \{|V(B)|{-}2, |V(B)|{-}3\}$ for the remaining $r{-}k{+}1$ branches B's at v in G and $d_B(v) = |V(B)|{-}3$ only for one B.

(c) G is connected, $G{-}v$ is unicyclic, $r > k{-}1$, one $B{-}v$ is unicyclic and $d_B(v) = |V(B)|{-}2$ for the remaining $r{-}k{+}1$ branches B's at v in G.

(d) $G = D \cup K_2 \cup (p{-}2{-}|V(D)|)K_1$, $G{-}v$ is acyclic and $r = k{-}1$.

(e) $G = D \cup K_2 \cup (p{-}2{-}|V(D)|)K_1$, $G{-}v$ is acyclic, $r > k{-}1$ and $d_B(v) = |V(B)|{-}2$ for the remaining $r{-}k{+}1$ branches B's at v in G.

(f) $G = D \cup (p{-}|V(D)|)K_1$, $G{-}v$ is acyclic, $r > k{-}1$, $d_B(v) = |V(B)|{-}3$ only for one branch B at v in G and $d_B(v) = |V(B)|{-}2$ for the remaining $r{-}k$ branches B's at v in G.

(g) $G = D \cup (p{-}|V(D)|)K_1$, $G{-}v$ is unicyclic, $r = k{-}1$ and one $B{-}v$ is unicyclic.

(h) $G = D \cup (p{-}|V(D)|)K_1$, $G{-}v$ is unicyclic, one $B{-}v$ is unicyclic and $d_B(v) = |V(B)|{-}2$ for the remaining $r{-}k{+}1$ branches B's at v in G.

Proof. Let G^v be a disconnected and unicyclic graph with k components. Using Theorem 1.5, G has $r \geq k{-}1$ branches B's at v and $d_B(v) = |V(B)|{-}1$ only for $k{-}1$ branches B's at v in G. Let $B_1, B_2, \ldots, B_{k-1}$ be the branches at v in G with $d_{B_i}(v) = |V(B_i)|{-}1$, $1 \leq i \leq k{-}1$. This implies that for any branch $B \neq B_i$, $d_B(v) \leq |V(B)|{-}2$, $1 \leq i \leq k{-}1$. Since G^v is unicyclic,

$G-v$ is acyclic or unicyclic. Here G may be either connected or disconnected. If G is connected, $G-v$ is acyclic and $r = k-1$, then $G^v = K_1 \cup (\bigcup_{i=1}^{k-1}(B_i - v))$, v is K_1. This implies that G^v is not unicyclic since $(B_i - v)$'s are acyclic. Hence we consider the following seven cases.

Case 1. G is connected, $G-v$ is unicyclic and $r = k-1$.

In this case $G^v = K_1 \cup (\bigcup_{i=1}^{k-1}(B_i - v))$, v is K_1. Since G^v is unicyclic, one $B_i - v$ is unicyclic. Hence (a) is proved.

Case 2. G is connected, $G-v$ is acyclic and $r > k-1$.

Let G^* be the graph obtained from G by deleting the branches $B_1, B_2, \ldots, B_{k-1}$ except v. Then $G = G^* \cup (\bigcup_{i=1}^{k-1} B_i)$. Using Theorem 1.2, $G^v = G^{*v} \cup (\bigcup_{i=1}^{k-1}(B_i - v))$ since B_i^v is the union of the vertex v and $B_i - v$. Since G^v is unicyclic with k components and $G-v$ is acyclic, G^{*v} is unicyclic and connected. Let B_x be the unicyclic branch at v in G^{*v}. Let B^* be the branch at v in G corresponding to the branch B_x at v in G^v. Using Theorem 1.6(b) to B^*, $d_{B^*}(v) = |V(B^*)| - 3$. Also for any branch $B \neq B_i$ and B^*, we have $d_B(v) = |V(B)| - 2$, $1 \leq i \leq k-1$. Hence (b) is proved.

Case 3. G is connected, $G-v$ is unicyclic and $r > k-1$.

Since $G-v$ is unicyclic, one $B-v$ is unicyclic. Let B^* be the branch at v in G such that B^*-v is unicyclic. We consider the following two subcases with respect to B^*.

Case 3.a. $B^* = B_i$ for at least one i, $1 \leq i \leq k-1$.

As in Case-2, $G^v = G^{*v} \cup (\bigcup_{i=1}^{k-1}(B_i - v))$. This implies that G^{*v} is a tree. Using Theorem 1.4, we get $d_B(v) = |V(B)| - 2$ for any branch $B \neq B_i$ at v in G, $1 \leq i \leq k-1$.

Case 3.b. $B^* \neq B_i$, $1 \leq i \leq k-1$.

Now $d_{B^*}(v) \neq |V(B^*)| - 1$. Suppose $d_{B^*}(v) < |V(B^*)| - 2$. Then G^v has at least two cycles, one contains v and the other not. This is a contradiction to our assumption that G^v is unicyclic and hence $d_{B^*}(v) = |V(B^*)| - 2$. Also $d_B(v) = |V(B)| - 2$ for $B \neq B_i$, $1 \leq i \leq k-1$. Hence (c) is proved.

For proving (d) to (h), we assume that G is a disconnected graph with m components. Let the components be $D, D_1, D_2, \ldots, D_{m-1}$ and v be in

D. Let $D^* = D_1 \cup D_2 \cup \ldots \cup D_{m-1}$ so that $G = D \cup D^*$. Since G has r branches at v, D also has r branches at v.

Case 4. G–v is acyclic and $r = k$–1.

Here $D = \bigcup_{i=1}^{k-1} B_i$ so that $G = D^* \cup \bigcup_{i=1}^{k-1} B_i$. This implies that $G^v = (D^*+v) \cup (\bigcup_{i=1}^{k-1}(B_i - v))$. Since G^v is unicyclic and $(B_i - v)$'s are acyclic, D^*+v is unicyclic. Let $D_j \neq D$ be a nontrivial component of G for at least one j, $1 \leq j \leq m$–1. Then $D_j = K_2$ since otherwise G^v is not unicyclic. Moreover the remaining components are trivial graphs. Thus $G = D \cup K_2 \cup (p\text{–}2\text{–}|V(D)|)K_1$ and hence (d) is proved.

Case 5. G–v is acyclic and $r > k$–1.

Here $G = D \cup D^*$ and so $G^v = D^v \cup (D^*+v)$. Then D^v may be either acyclic or unicyclic.

When D^v is acyclic, one component of G is K_2 and others K_1's since G^v is unicyclic. This implies that $G = D \cup K_2 \cup (p\text{–}2\text{–}|V(D)|)K_1$. Also for any branch B at v in G such that $B \neq B_i$, we have B_i^v is a tree in G^v and hence $d_B(v) = |V(B)|$–2, $1 \leq i \leq k$–1. Hence (e) is proved.

When D^v is unicyclic, each component of G other than D is a trivial graph. This implies that $G = D \cup (p\text{–}|V(D)|)K_1$. Now D is connected, D^v is unicyclic and D–v is acyclic. Applying Case-2 to D, we get $d_B(v) = |V(B)|$–3 only for one B and for other r–k branches B's at v in G, $d_B(v) = |V(B)|$–2. Hence (f) is proved.

Case 6. G–v is unicyclic and $r = k$–1.

Suppose a component of G not containing v is unicyclic. Then G^v has more than two cycles, which is a contradiction and hence cycles are in the component D. Also the other components of G are trivial graphs. This implies that $G = D \cup (p\text{–}|V(D)|)K_1$. Since G^v is unicyclic, one $B_i - v$ is unicyclic, $1 \leq i \leq k$–1. Hence (g) is proved.

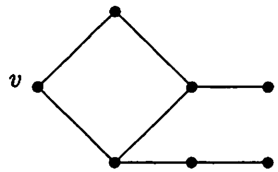
Case 7. G–v is unicyclic and $r > k$–1.

Clearly, each component of G other than D is a trivial graph. This implies that $G = D \cup (p\text{–}|V(D)|)K_1$. Applying Case-3 to D, we get one B–v is unicyclic and $d_B(v) = |V(B)|$–2 for the remaining r–k+1 branches B's at v in G. Hence (h) is proved.

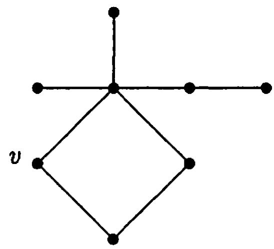
On the converse part of the theorem, using Theorem 1.5, G^v is disconnected with k components. Clearly, each case implies that G^v is unicyclic. This completes the proof. □

Note 2.2 [3] Consider a cycle $C_r = (v_1, v_2, \ldots, v_r)$(clock-wise). For our convenience we denote it by $C_{r(v_1)}$. Identifying an end vertex of paths P_m at v_i and P_s at v_j, then the resulting graph is denoted by $C_{r(v_1)}$ (0, ..., P_m, 0, ..., P_s, 0, ..., 0). Identifying an end vertex of paths P_m and P_s at the vertex v_j, then the resulting graph is denoted by $C_{r(v_1)}$ (0, ..., $P_m \cup P_s$, 0, ..., 0).

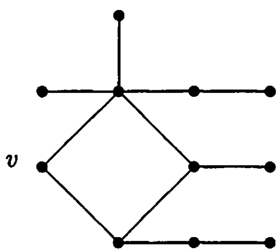
The graphs $C_{4(v)}$ (0, 0, P_2, P_3), $C_{4(v)}$ (0, 2 $P_2 \cup P_3$, 0, 0) and $C_{4(v)}$ (0, 2 $P_2 \cup P_3$, P_2, P_3) are given in Figure 2.1.



$C_{4(v)}$ (0, 0, P_2, P_3)



$C_{4(v)}$ (0, 2 $P_2 \cup P_3$, 0, 0)

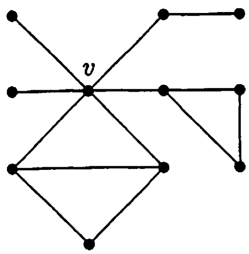


$C_{4(v)}$ (0, 2 $P_2 \cup P_3$, P_2, P_3)

Fig. 2.1

Note 2.3.[5] Let v be a cutvertex of a connected graph G. Let B_1, B_2, ..., B_k be the branches with $n_1, n_2, ..., n_k$ number of copies at v in G, respectively. In this case, we denote the graph G by $G(v;\ n_1B_1, n_2B_2, ..., n_kB_k)$.

Consider the graph G given in Figure 2.2. There are four distinct branches B_1, B_2, B_3 and B_4 at v in G and they are given Figure 2.3. Thus $G = G(v;\ 2B_1, B_2, B_3, B_4)$.



$G = G(v;\, 2\,B_1\,,\; B_2\,,\; B_3\,,\; B_4\,)$

Fig.2.2.

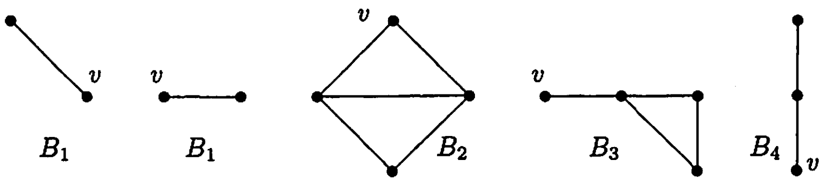


Fig.2.3.

3. Characterizing disconnected unicyclic graphs with a self vertex switching

Theorem 3.1. A disconnected and unicyclic graph G of order $p = 2n{+}1$ with k components has a self vertex switching v if and on ly if it is either $D \cup K_2 \cup (k{-}2)\,K_1$ where D is either $D(v;\ (n{-}2)\,P_2\,, K_3\,)$ or $D(v;\ (k{-}2)\,P_2\,,\ K_3\,,\ (n{-}k)\,P_3\,)$ according as $k = n$ or $k < n$ or $D \cup (k{-}1)\,K_1$ and $k = p{+}1{-}\mid V(D)\mid$ where D is either of the following: $D(v;\ C_4\,, K_{1,3}\,, (k{-}1)\,P_2\,, (n{-}k{-}2)\,P_3\,)$, $D(v;\ C_{3(w)}\,(\,P_2\,, 0, 0), P_4\,, (k{-}1)\,P_2\,, (n{-}k{-}2)\,P_3\,)$, $D(v;\ C_{4(w)}\,(\,P_2\,, 0, 0, 0), (k{-}1)\,P_2\,, (n{-}k{-}1)\,P_3\,)$ and $D(v;\ C_{3(w)}\,(\,P_3\,, 0, 0), (k{-}1)\,P_2\,, (n{-}k{-}1)\,P_3\,)$ where w is a vertex adjacent to v in G and for any branch B at v in G, $d_B\,(v) = 1$ or 2 according as B is a tree or unicyclic branch.

Proof. Let v be a self vertex switching of a disconnected unicyclic graph G. Using Theorem 1.1, $d_G(v) = n$. And using Theorem 2.1, G has $r \geq k-1$ branches at v and $d_B(v) = |V(B)|-1$ only for $k-1$ branches B's. The following five cases arise.

Case 1. $G = D \cup K_2 \cup (p-2-|V(D)|)K_1$, $G-v$ is acyclic and $r = k-1$ where $D \neq K_1, K_2$ is a component of G containing v.

$G-v$ is acyclic implies that v lies on the cycle in the cyclic branch, say B^*, at v in G. Since G is unicyclic and $d_G(v) = n$, there are $n-1$ branches at v in G and hence $k = n$. Since K_2 is a component of G, $K_2 + v = K_3$ is a branch at v in G^v and hence $B^* = K_3$. Since the other branches at v in G are trees and $d_B(v) = |V(B)|-1$, they are P_2's. These $n-2$ branches make only K_1's in G^v. Since $G \cong G^v$, $n-2 = p-2-|V(D)|$. Hence $G = D \cup K_2 \cup (k-2)K_1$ where D is $D(v;\ (n-2)P_2, K_3)$.

Case 2. $G = D \cup K_2 \cup (p-2-|V(D)|)K_1$, $G-v$ is acyclic, $r > k-1$ and $d_B(v) = |V(B)|-2$ for the remaining $r-k+1$ branches B's at v in G, where $D \neq K_1, K_2$ is a component of G containing v.

Let B^* be the unicyclic branch at v in G. As in Case-1, $B^* = K_3$ and there are $n-1$ branches at v in G. Clearly $d_{B^*}(v) = |V(B^*)|-1$. Since the other branches at v in G are trees, any branch B at v in G with $d_B(v) = |V(B)|-2$ is P_3 and with $d_B(v) = |V(B)|-1$ is P_2. Since the branches P_2's at v in G make only K_1's in G^v and $G \cong G^v$, we get $k-2 = p-2-|V(D)|$. Hence $G = D \cup K_2 \cup (k-2)K_1$ where D is $D(v;\ (k-2)P_2, K_3, (n-k)P_3)$. Also $n > k$ since $r = n-1$.

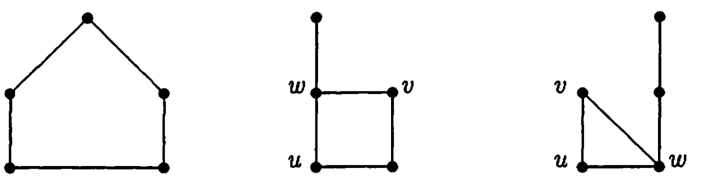
Case 3. $G = D \cup (p-|V(D)|)K_1$, $G-v$ is acyclic, $r > k-1$, $d_B(v) = |V(B)|-3$ only for one branch B at v in G and $d_B(v) = |V(B)|-2$ for the remaining $r-k$ branches B's at v in G where $D \neq K_1, K_2$ is a component of G containing v.

G is unicyclic and $G-v$ is acyclic implies that v lies on the cycle in the cyclic branch, say B^*, at v in G. Hence there are $n-1$ branches at v in G and $d_{B^*}(v) = 2$. If B^* has 3 vertices, then B^*-v is a component of G^v. Since $G \cong G^v$, G has a component K_2, which is a contradiction and hence $|V(B^*)| \geq 4$. Using Lemma 1.3, each component $K_1 \neq D$ in G becomes a branch P_2 at v in G^v. If B is a branch at v in G with $d_B(v) \neq |V(B)|-1$, then B^v is a branch at v in G^v. Using Lemma 1.3, $d_B(v) = |V(B)|-1$ for a branch B at v in G if and only if $B-v$ is a component of G^v. Since $G \cong G^v$ and $k-1$ branches B's at v in G with $d_B(v) = |V(B)|-1$, we get $k-1 = p-|V(D)|$. If B is a branch at v in G with $d_B(v) \neq |V(B)|-1$, then $|V(B)| \geq 3$. If B^* has order at least 6, then $p = |V(B^*)| + \sum_{d_B(v)=|V(B)|-1} |V(B)|$

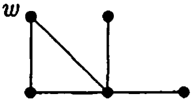
$+ \sum_{d_B(v)\neq |V(B)|-1, B\neq B^*} |V(B)|$ +(the number of K_1 's in G)$-r+1 \geq 6+2(k-1)+3(r-k)+p-|V(D)|-r+1 = 2r+4 = 2(n-1)+4 = 2n+2 > p$, which is a contradiction and hence $|V(B^*)| = 4$ or 5.

When $|V(B^*)| = 4$, there are only two unicyclic graphs on 4 vertices and they are C_4 and $C_{3(w)}(P_2, 0, 0)$. For any vertex v with degree 2 in C_4 and in $C_{3(w)}(P_2, 0, 0)$, the switching of them by v are $K_{1,3}$ and P_4, respectively, and in these degree of v is 1. Clearly C_4 and $K_{1,3}$ and $C_{3(w)}(P_2, 0, 0)$ and P_4 are complementary switching branches at v. For any branch $B \neq B^*$ and $B \ncong B^{*v}$, if $d_B(v) = |V(B)|-1$, then $B = P_2$ since otherwise $B-v \neq K_1$ is a component of G^v and hence G has a nontrivial component other than D and if $d_B(v) = |V(B)|-2$, then B has 3 vertices since otherwise G has more vertices than p and hence $B = P_3$. This implies that $G = D \cup (k-1) K_1$ where D is either $D(v;\ C_4, K_{1,3}, (k-1) P_2, (n-k-2) P_3)$ or $D(v;\ C_{3(w)}(P_2, 0, 0), P_4, (k-1) P_2, (n-k-2) P_3)$ where vertex w is adjacent to v in G.

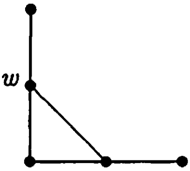
When $|V(B^*)| = 5$, there are only five unicyclic graphs on 5 vertices which are given in Figure 3.1. If B^* is either $C_{3(w)}(P_2, P_2, 0)$, $C_{3(w)}(0, 2P_2, 0)$ or C_5, then for any v in B^* with degree 2, B^{*v} is unicyclic and $B^{*v} \ncong B^*$. Hence B^* is either $C_{4(w)}(P_2, 0, 0, 0)$ or $C_{3(w)}(P_3, 0, 0)$ and has a self vertex switching in the cycle, which is adjacent to w and $d_{B^*}(v) = |V(B^*)|-3$. For any branch $B \neq B^*$, if $d_B(v) = |V(B)|-1$, then $B = P_2$ and if $d_B(v) = |V(B)|-2$, then $B = P_3$ since otherwise G has more than p vertices. This implies that $G = D \cup (k-1) K_1$, D is either $D(v;\ C_{4(w)}(P_2, 0, 0, 0), (k-1) P_2, (n-k-1) P_3)$ or $D(v;\ C_{3(w)}(P_3, 0, 0), (k-1) P_2, (n-k-1) P_3)$ where the vertex w is adjacent to v in G.



C_5 $\qquad$ $C_{4(w)}(P_2, 0, 0, 0)$ $\qquad$ $C_{3(w)}(P_3, 0, 0)$



$C_{3(w)}(0, 2P_2, 0)$



$C_{3(w)}(P_2, P_2, 0)$

Fig. 3.1

Case 4. $G = D \cup (p-|V(D)|)K_1$, $G-v$ is unicyclic, $r = k-1$ and one $B-v$ is unicyclic where $D \neq K_1, K_2$ is a component of G containing v.

Let B^* be the branch at v in G such that B^*-v is unicyclic. Since $d_{B^*}(v) = |V(B^*)|-1$, B^*-v is a component of G^v. Since $G \cong G^v$, $D \cong B^*-v$, which is a contradiction to the fact that B^*-v is a proper subgraph of D. In this case G cannot have a self vertex switching.

Case 5. $G = D \cup (p-|V(D)|)K_1$, $G-v$ is unicyclic, $r > k-1$, one $B-v$ is unicyclic and $d_B(v) = |V(B)|-2$ for the remaining $(r-k+1)$ branches B's at v in G where $D \neq K_1, K_2$ is a component of G containing v.

G and $G-v$ are unicyclic implies that v does not lie on the cycle in the unicyclic branch, say B^*, at v in G and hence v is an end vertex in B^* and so $|V(B^*)| > 3$. Since $d_{B^*}(v) = |V(B^*)|-1$ or $|V(B^*)|-2$, $d_{B^*}(v) > 1$, which is a contradiction to v is an end vertex in B^*. In this case also G cannot have a self vertex switching.

From cases (1), (2) and (3), we get the required graph G.

Conversely, if G is the graph as given in the theorem, then v is the self vertex switching of G. This completes the proof. □

Corollary 3.2. Let G be a disconnected unicyclic graph. Then $ss_1(G) =$ 0 or 1. And $ss_1(G) = 1$ if and only if G is the graph given in Theorem 3.1.

Example 3.3. The five disconnected unicyclic graphs G on $p = 2n+1 = 17$ vertices, each of which has v as the self vertex switching and with 3 components are given in Figures 3.2 to 3.6.

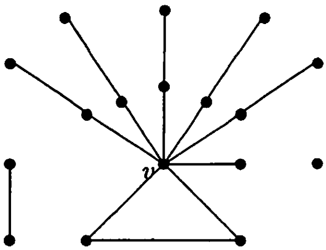


Fig. 3.2. G

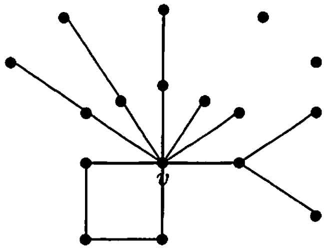


Fig. 3.3. G

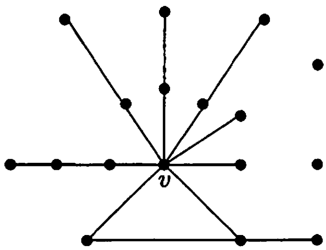


Fig. 3.4. G

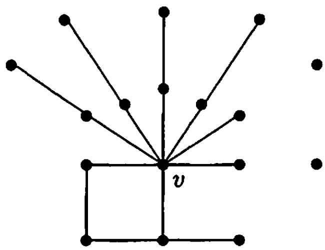


Fig. 3.5. G

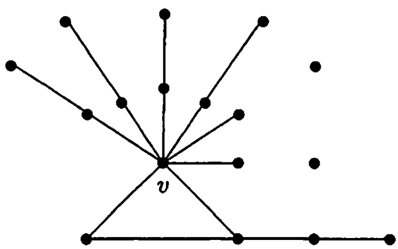


Fig. 3.6. G

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