Some Notes on Combination Graphs

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Abstract: We introduce a theorem on bipartite graphs, and some theorems on chains of two and three complete graphs, considering when they are combination or non-combination graphs, present some families of combination graphs. We give a survey for trees of order ≤ 10, which are all combination graphs.

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Mathematics Subject Classification: 05C78

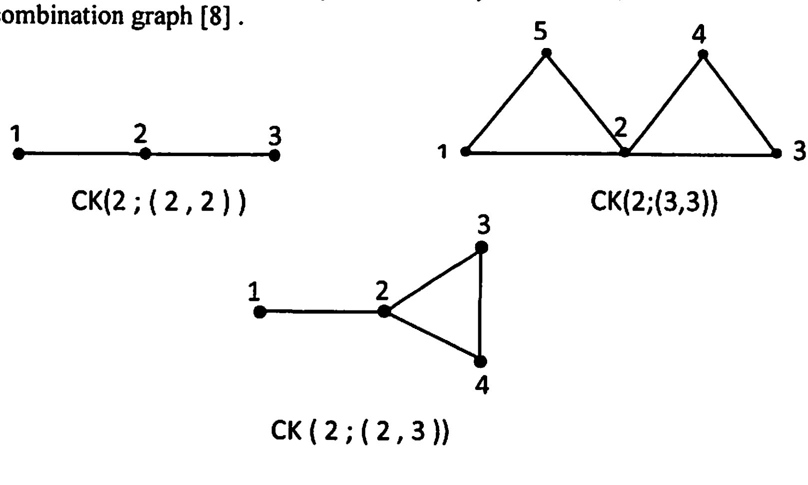
0 Introduction

Hegde and Shetty [2 , 4] define a graph G with n vertices to be a permutation graph if there exists an injection f from the vertices of G to the set {1, 2, 3,…,n }, such that the induced edge function g_f defined as $g_f(uv) = f(u)!/|f(u)-f(v)|!$, $f(u) > f(v)$ is injective .They say a graph G with n vertices is a combination graph if there exists an injection f from the vertices of G to the set { 1,2,3,…,n } such that the induced edge function g_f defined as $g_f(uv) = f(u)!/|f(u)-f(v)|!\,f(v)!$, $f(u) > f(v)$ is injective . We call a graph G non-combination if it is not a combination graph. They prove : K_n is a combination graph if and only if $n \leq 2$; C_n is a combination graph for $n > 3$, $K_{n,n}$ is a combination graph if and only if $n \leq 2$;W_n is a not a combination graph for $n \leq 6$, and a necessary condition for a (p , q)–graph to be a combination graph is that $4q \leq p^2$ if p is even, and $4q \leq p^2 - 1$ if p is odd . They strongly believe that W_n is a combination graph for n > 6 and all trees are combination graphs .Seoud and Anwar [7] give the number of edges in any maximal combination graph $G(n,q)$ if n is even or if n is odd , $n > 3$. They show that $K_{m,n}$ is a combination graph if and only if $n, m \leq 2$ or m=1 . They give a survey of all maximal combination graphs on n vertices and q edges such that $n \leq 6$. Also they give a necessary condition for a strong k-combination graph.

Seoud and Al-Harere [5] presented two Theorems:(1) A graph $G(n,q)$ having at least 6 vertices , such that 3 vertices are of degree $1, n-1, n-2$ is not a combination graph .(2) A graph $G(n,q)$ having at least 6 vertices , such that there exist 2 vertices of degree $n-3$, two vertices of degree 1 and one vertex of degree n-1 is not a combination graph . Second ,they show that the following families are combination graphs : Two copies of C_n sharing a common edge , the graph consisting of two cycles of the same order joined by a path of l vertices, the union of three cycles of the same order, the wheel W_n $n \geq 7$, what Hegde and shetty believed , the corona $T_n \odot K_1$, where T_n is the triangular snake , the graph obtained from the gear G_m , by attaching n pendent vertices to each vertex which is not joined to the center of the gear , and some corollaries .

Seoud and Al-Harere [6], prove: the graph $G(n,q)$, n≥3 is a non-combination graph if it has more than one vertex of degree $n-1$; and the following graphs are non-combination graphs; G_1+G_2 if n_1 or $n_2 > 2, n_1, n_2 \neq 1$; the double Fan $\overline{K_2}+P_n$; $K_{l,m,n}$; $K_{k,l,m,n}$; $P_2[G]$; $P_3[G]$; $C_3[G]$; $C_4[G]$; $K_m[G]$; $W_m[G]$; the splitting graph of K_n, $S'(K_n)$, $n \geq 3$; $K_n - e$, $n \geq 4$; $K_n - 3e$, $n \geq 5$; $K_{n,n} - e$, $n \geq 3$. Barrientos [1] define a chain graph as one with blocks $B_1, B_2, \ldots, B_m$ such that for every i, B_i and B_{i+1} have a common vertex in such a way that the block cut-point graph is a tree .

We will denote the chain graph with m blocks by $CK(m;(a_1,a_2,\ldots,a_m))$,where the sequence of m blocks is the complete graphs $K_{a_1}, K_{a_2}, \ldots, K_{a_m}$. We will assume that all $a_i \geq 2$. If $a_1 = a_2 = \cdots = a_m = 2$ then $CK(m;(2,2,\ldots,2)) = P_{m+1}$ .It is well known that P_m is a combination graph . If $a_1 = a_2 = \cdots = a_m = 3$ then $CK(m;(3,3,\ldots,3))$ is the triangular snake which is a combination graph [8] .



Figure(1)

Here, we introduce a theorem on bipartite graphs, and some theorems on chains of two and three complete graphs considering when they are combination or non- combination graphs. We show that some families of graphs are combination graphs. Finally we give a survey for trees of order≤10, which are all combination graphs.

Any notion or definition which is not found here could be found in [3].

1 General results

Lemma1.1.[5] In a combination graph the vertex of degree $n-1$ receives label 1 or 2.

Remark 1.2.[5] 1. The vertex v in the combination graph $G(n,q)$ could be labeled by k if $d(v) \leq \left\lfloor \frac{k}{2} \right\rfloor + n - k \ , k = 1,2,\dots,n.$

2. The graph $G(n,q)$ is a non- combination graph if it has no vertex of degree $\leq \left\lfloor \frac{n}{2} \right\rfloor$.

Theorem 1.3.[5]A graph $G(n,q)$ having at least 6 vertices, such that 3 vertices are of degree $n-1 \ , n-2 \ , 1$ is not a combination graph.

Theorm1.4.[5] The graph $G(n,q)$ having 2 vertices of degree 1, 2 vertices of degree $n-3$, 1 vertex of degree $n-1$ is not a combination graph.

Theorem 1.5 . If G is a bipartite graph, both of its sets has n elements, such that $\frac{n}{2}$ elements of each set has degree n, then G is a non-combination graph, n≥ 6 .

Proof. Let A and B be the sets of labels of the two bipartite sets of G and have n elements.Let $A = \{1, x_1, x_2, \dots, x_{n-1}\}$,where $x_1 < x_2 < \cdots < x_{n-1}$. $As\ C_1^{x+1} = C_x^{x+1}$ note that $1, x \in A$ implies $1 + x \notin B$. Now $1 + x_{n-2} \notin B,$ therfore $1 + x_{n-2} \in A,$ which implies that $1 + x_{n-2} = x_{n-1}$ .Similarly $1 + x_{n-3} \in A$ implies that $1 + x_{n-3} = x_{n-2}$,so that $A = \{1, x_1, x_1 + 1, x_1 + 2, \dots, x_1 + n - 2\}$, $B = \{y_1, y_2, \dots, y_n\}$, with $y_1 < y_2 < \cdots < y_n$. We will choose a labeling for this graph from the following four cases according to the degree of their vertices and Remark 1.2

Case 1 . $1 < x_1 < x_1 + 1 < x_1 + 2 < \cdots < x_1 + n - 2 < y_1 < y_2 < \cdots < y_n$. $A = \{1,2,\dots,n\}, B = \{n+1, n+2, \dots, 2n\}$. Clearly $\forall\, n+i \ , i = 1,\dots,n-1$,we get $\binom{n+i}{n} = \binom{n+i}{i}$, so the vertices labeled by $n+i \ , i = 1,\dots,n-1$ are not joined with all vertices in A .

Case 2 . $1 < y_1 < y_2 < \cdots < y_n < x_1 < x_1 + 1 < x_1 + 2 < \cdots < x_1 + n - 2$. $A = \{1, n+2, n+3 \dots, 2n\}, B = \{2,3,\dots,n+1\}$. $\forall\, n + i \epsilon A \ , i = 1,\dots,n$, we get $\binom{n+i}{n} = \binom{n+i}{i}$, so the vertices labeled by $n + i \epsilon A \ , i = 1,\dots,n$ are not joined with all vertices in B.

Case 3 . There exists k, 0 < k < n, such that

$1 < y_1 < \cdots < y_k < x_1 < x_1 + 1 < x_1 + 2 < \cdots < x_1 + n - 2 < y_{k+1} < y_{k+2} \cdots < y_n$.

All $y_i \ , i = 1,\dots,k$ can join all vertices in A, since $y_i < x_j \ \forall\, i = 1,\dots,k \ , j = 1,\dots,n-1$.

i) $k \leq \frac{n}{2}$

When $k < \frac{n}{2}$ all the vertices labeled by y_{k+i} $, i = 1, \ldots, n-k-1$ have degrees greater than $\left\lfloor \frac{y_{k+i}}{2} \right\rfloor$, such that the vertices labeled y_{k+i} cannot join the vertices labeled y_{k+i+1}, $i = 1, \ldots, n-k-1$ and $\binom{y_n}{x_1} = \binom{y_n}{x_1 + n - 2}$, so we have repeated edge labels. Now let $k = \frac{n}{2}$,the vertices labeled x_j are joined with all vertices of B if $x_j \geq 2y_k$, $2y_k = 2\left(\frac{n}{2} + 1\right) = n + 2$ $, n + 2 \leq x_j \leq 3\frac{n}{2}$, so the values $\binom{x_j}{y_i}$, $i = 1, \ldots, k$ are different labels , since $\binom{n}{r}$, $r = 1, \ldots, \left\lfloor \frac{n}{2} \right\rfloor$. Therefore the number of vertices in A which can join all vertices in B is $3\frac{n}{2} - (n+1) + 1 = \frac{n}{2}$, $1 \epsilon A$, but we have $\binom{x_2}{y_1} = \binom{x_2}{y_k}$.

ii) If $k > \frac{n}{2}$, the vertices in the set A which can be joined with all vertices in the set B are less than $\frac{n}{2}$.

Therefore G is a non-combination graph.

For a graph G , the splitting graph of G , $S'(G)$, is obtained from G by adding for each vertex v of G a new vertex v', so that v' is adjacent only to every vertex that is adjacent to v, so we have :

Corollary 1.6. The graph $S'(K_{n,n}), n \geq 3$ is a non- combination graph.

Figure (2) shows the graph $S'(K_{3,3})$.

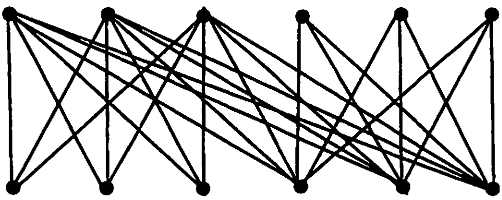


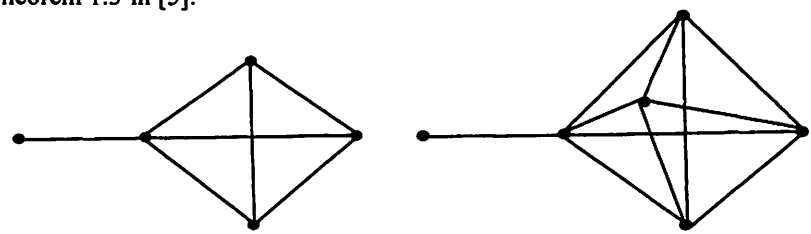
Figure (2)

2 Chain graphs with two blocks

If m = 2 , then $CK(2; (a_1, a_2))$ is the one-point union of two complete graphs , we assume $a_1 \leq a_2$.

Theorem 2.1 . For m = 2 and $a_1 = 2$, $CK(2; (a_1, a_2))$ is a non-combination graph if and only if $a_2 \geq 4$.

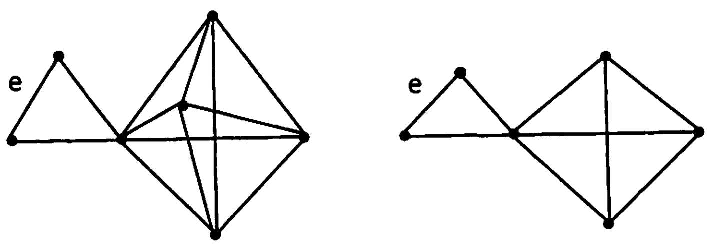
Proof . If $a_2 = 2$, then $CK(2; (2,2))$is a combination graph. If $a_2 = 3$ then $CK(2; (2,3))$is the dragon(Figure(1)), which is a combination graph .For $a_2 \geq 4$ the graph $CK(2; (a_1, a_2))$ has three vertices of degrees n , n-1 , 1, respectively . When $a_2 = n$, the graph is a non-combination graph according to Theorem 1.3 in [5].



CK (2 ; (2 , 4))   CK (2 ; (2 , 5))

Figure (3)

Theorem 2.2 . For m=2 and $a_1 = 3$ the graph $CK(2;(a_1,a_2))$ is a non-combination graph if and only if $a_2 \geq 4$.
Proof . If $a_2 = 3$, then it is a triangular snake, which is a combination graph . For $a_2 \geq 4$, let n+2 be the number of vertices of $CK(2;(a_1,a_2))$ .If we delete the edge e which is adjacent to the two vertices of degree 2 of k_{a_1} , we get the graph $CK(2;(a_1,a_2)) - e$ which has the following vertices : one vertex of degree n+1 , two vertices of degrees n-1,1 respectively , and according to Theorem 1.4 in [5] the graph is a non-combination graph. Since $CK(2;(a_1,a_2)) - e$ is a subgraph of $CK(2;(a_1,a_2))$ with the same number of vertices, it follows that $CK(2;(a_1,a_2))$ is a non-combination graph, $a_2 \geq 4$.



CK (2 ; (3 , 5))   CK (2 ; (3 , 4))

Figure (4)

Theorem 2.3. For m = 2,$CK(2;(a_1,a_2))$ is a non-combination graph if $a_1 = a_2 \geq 4$.
Proof . The number of the vertices of $CK(2;(a_1,a_2))$ is 2n – 1, and the number of edges is n (n – 1) . Since we have a vertex in common between K_n and K_n , so this vertex will be of degree 2n – 2.

Let v be this common vertex and let the vertices of K_{a_1} be v_i , $i = 1,2,\dots,n-1$, the vertices of K_{a_2} be u_j , $j = 1,2,\dots,n-1$. According to Lemma 1.1 in [5] we have two cases :

Case 1 . $f(v) = 1$, if we label v_1 by k , $k < 2n-1$ then we must label u_1 by $k+1$ since $\binom{k+1}{1} = \binom{k+1}{k}$, therefore the even labels will be in K_{a_1} and the odd labels in K_{a_2} ,or vice versa and this is a non-combination graph ,since $\binom{6}{4} = \binom{6}{2}$, $n \geq 4$.

Case 2 . $f(v) = 2$ .Without any loss of generality, let f $(v_1) = 1$. Since $\binom{3}{1} = \binom{3}{2}$, the label 3 will be in Ka_2 .Since 2 + 3 = 5 , so the label 5 will be in K_{a_1} hence label 4 is in K_{a_2} .If the label 6 in K_{a_1} , then $\binom{6}{1} = \binom{6}{5}$ .If it is in K_{a_2} we get also $\binom{6}{4} = \binom{6}{2}$.Hence the result.

3 Chain graphs with three blocks

There are several cases to consider for chain graphs with three blocks . Since the graph $CK(3;(a_1,a_2,a_3))$ is isomorphic to $CK(3;(a_3,a_2,a_1))$,we will consider one case instead of both .We will order the sequence by lexico graphical order.

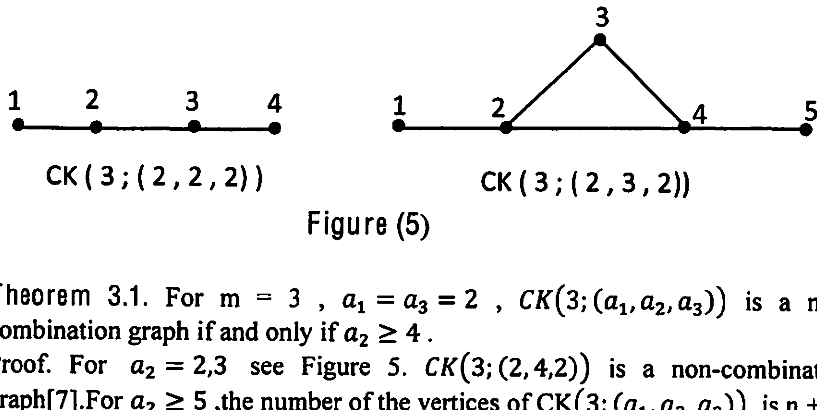
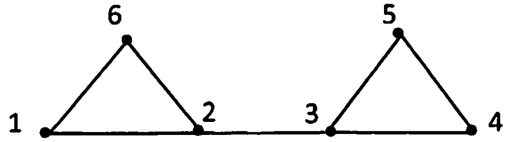


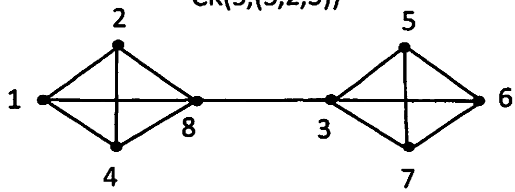
Figure (5)

Theorem 3.1. For m = 3 , $a_1 = a_3 = 2$, $CK(3;(a_1,a_2,a_3))$ is a non-combination graph if and only if $a_2 \geq 4$.

Proof. For $a_2 = 2,3$ see Figure 5. $CK(3;(2,4,2))$ is a non-combination graph[7].For $a_2 \geq 5$,the number of the vertices of $CK(3;(a_1,a_2,a_3))$ is n + 2 , according to Remark 1.2 in [5] we can label one of the vertices of degree one by n + 2 only , since the remaining vertices are of degrees > $\left\lfloor\frac{n+2}{2}\right\rfloor$. Also for the label n + 1 , we can label the second vertex of degree one by n + 1 only , since the remaining vertices are of degrees > $\left\lfloor\frac{n+1}{2}\right\rfloor$, so the vertices of k_{a_2} will be labeled 1, 2 , ..., n . (K_n is a combination graph if and only if $n \leq 2$ [4]) , and therefore $CK(3;(a_1,a_2,a_3))$ is a non-combination graph .



CK(3;(3,2,3))



CK (3 ; (4, 2 , 4))

Figure (6)

Theorem 3.2 . For m = 3,$a_2 = 2, CK(3; (a_1, a_2, a_3))$ is a non-combination graph if $a_1 = a_3 \geq 5$.

Proof .When $a_1 = a_3 = 3{,}4$ the graph is a combination graph(Figure (6)). For $a_1 = a_3 \geq 5$, let A and B be the set of vertices in the complete graph of vertices a_1 and a_3 respectively. Without loss of generality , let $1 \epsilon A$. Now , if $2n \epsilon A$ n≠1, then there exists $x \epsilon A$,such that $x \neq 2, n,$ then $x - 1 , x + 1 , 2n - x \in B$, but $2n - x + x - 1 = 2n - 1$ and $2n - 1 \in B$,so $\binom{2n-1}{2n-x} = \binom{2n-1}{x-1}$, and this is a non-combination graph, so $2n \notin A$ implies $2n \in B$. Now if $2n - 1 \in A$, n≠1, there exists $x \in A$ such that $x \neq 2$, implies $x - 1 , x + 1 , 2n - 1 - x \in B$,but $2n - 1 - x + x - 1 = 2n - 2$ and $2n - 2 \in B$ and we get $\binom{2n-2}{2n-1-x} = \binom{2n-2}{x-1}$ .Continuing in this procedure we get $B = \{2n, 2n - 1, \ldots, 2n - (n - 1)\}$. And this means $A = \{1,2, \ldots, n\}$, and it is clear that labeling of K_{a_1} is a non-combination labeling (K_n is a combination graph if and only if $n \leq 2$).Hence the result.

Theorem 3.3.The union of two complete graphs $K_n \cup K_n$,$n \geq 5$ is a non-combination graph.

Proof .Use the same idea of the proof of Theorem 3.2 .

4 Some combination families

Theorem 4.1. The C_4 -snake is a combination graph.

Proof . Let the C_4 –snake be described as in Figure (7).

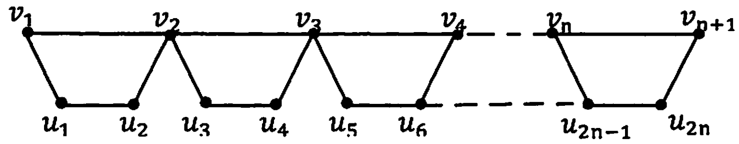


Figure (7)

The graph C_4 -snake is a graph of order $3n+1$, n is the number of the cycles C_4 .
We define the function:
$f : V(C_4 - snake) \rightarrow \{1,2,\dots,3n+1\}$ as follows :
$f(v_i) = i$, $i = 1,\dots,n+1$, $f(u_i) = n+1+j$, $j = 1,\dots,2n$.
The edge labels will be as follows:
$q_1 = \{2,3,\dots,n+3\}$, $q_2 = \{n+5, n+7,\dots,3n+1\}$,
$q_3 = \{ \binom{n+3}{2}, \binom{n+4}{2}, \binom{n+5}{3}, \binom{n+6}{3},\dots, \binom{3n-1}{n}, \binom{3n}{n} \}$, $q_4 = \{ \binom{3n+1}{n+1} \}$.
The labels in each of the previous sets are increasing.
Figure (8) shows a combination labeling of a C_4 -snake , n=4.

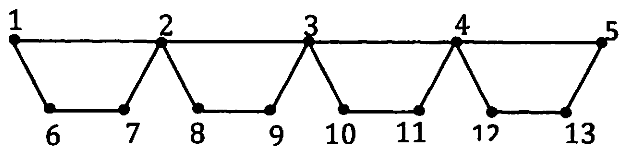


Figure (8)

A double triangular snake consists of two triangular snakes that have a common path.
Theorem 4.2. A double triangular snake is a combination graph ,n≥3.
Proof. The graph is shown in Figure (9)

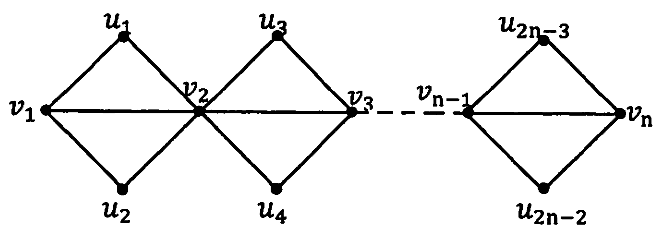


Figure (9)

We define the labeling function
$f : V(G) \rightarrow \{1,2,\dots,3n-2\}$ as follows:
$f(v_i) = i, i = 1,\dots n$, $f(u_j) = n+j$, $j = 1,\dots,2n-2$.

The edge labels will be as follows:

$q_1 = \{2, 3, \ldots, n+2\}$

$q_{i+1} = \{ \binom{n+2i-1}{i+1}, \binom{n+2i}{i+1}, \binom{n+2i+1}{i+1}, \binom{n+2i+2}{i+1} \}$, $i = 1, \ldots, n-2$.

$q_n = \{ \binom{3n-3}{n}, \binom{3n-2}{n} \}$. The labels in each of the previous sets are increasing.

$q_1 \cap q_j = \emptyset$,$j = 2, \ldots n$,since every label in q_1 is less than every label in q_j for every j=2,…,n . ,$q_i \cap q_j = \emptyset, i \neq j, i,j = 2, \ldots, n$, since $\binom{n+2i+2}{i+1} < \binom{n+2i+1}{i+2}$, $i = 1, \ldots, n-2$, $n \geq 4$.

Figure (10) shows a combination labeling of a double triangular snake , n=3 .

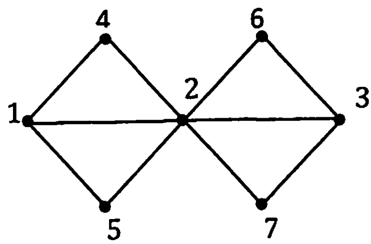


Figure (10)

A caterpillar is a tree for which, if all leaves (vertices of degree 1 and their associated edges) were removed, the result is a path.

Theorem 4.3. All caterpillars are combination graphs.

Proof: Let $v_1, v_2, \ldots, v_n$ be the vertices of the path and $u_1, u_2, \ldots, u_n$ the vertices of the leaves . We define the label function as follows : $f(v_i) = i$, $i = 1, \ldots, n$,$f(u_i) = n + i$, $i = 1, \ldots, m$. All labels are given from left to right.

The fan F_n is defined to be the graph $P_1 + K_n$.

Theorem 4.4 . The graph F_n is a combination graph if and only if $n \geq 6$.

Proof. The number of vertices of F_n is n+1 and the number of edges is $2n-1$, according to Theorem 3 in [4] , F_n is a non-combination graph , $n \leq 5$. When p is odd $4q > p^2 - 1$ implies $4(2n-1) > (n+1)^2 - 1$, i.e. $n^2 - 6n + 4 < 0$,$n \leq 4$.

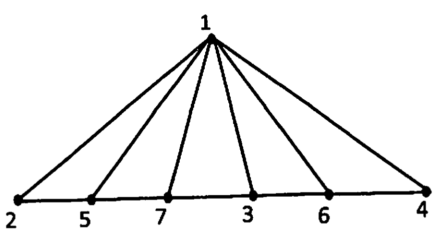


Figure (11)

When p is even, since $4q > p^2$ implies $4(2n-1) > (n+1)^2$,i.e. $n^2 - 6n + 5 < 0$, n<5.
For n =5 we have the number of edges = 9 > 8 , hence F_5 is a non-combination graph .The labeling of F_n when n=6 is as follows :
For n ≥ 7 , F_n is a subgraph of the wheel W_n with the same number of vertices and since W_n is a combination graph , n ≥ 7 [5], F_n is a combination graph , n ≥ 6 .
Theorem 4.5. The graph $4C_n$ is a combination graph for n≥3.
Proof. Figure (12) shows the graph.

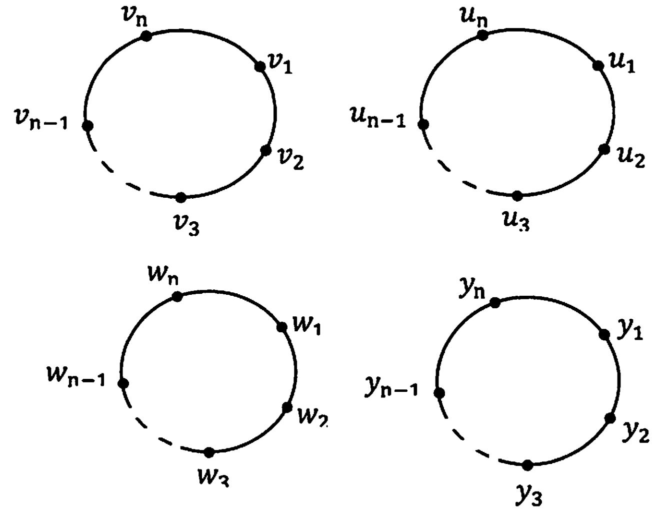
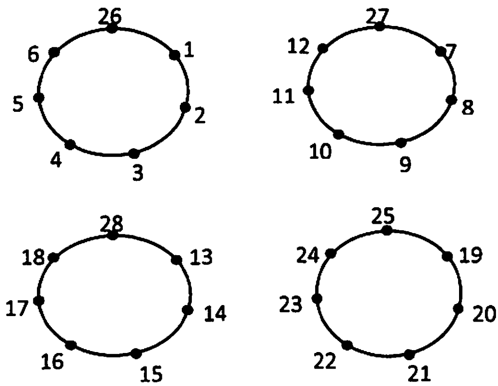


Figure (12)

We define the function $f : V(G) \to \{1,2,\ldots,4n\}$ as follows:
$f(v_i) = i$, $i = 1,\ldots,n-1$, $f(v_n) = 4n-2$, $f(u_i) = n-1+i$, $i = 1,\ldots,n-1$, $f(u_n) = 4n-1$, $f(w_i) = 2n-2+i$, $i = 1,\ldots,n-1$, $f(w_n) = 4n$, $f(y_i) = 3n-3+i$, $i = 1,\ldots,n-1$, $f(y_n) = 4n-3$.
The edge labels can be described as follows:
$q_1 = \{2,3,\ldots,n-1\}$, $q_2 = \{n+1,\ldots,2n-2\}$,
$q_3 = \{2n,\ldots,3n-3\}$, $q_4 = \{3n-1,\ldots,4n-3,4n-2\}$
$q_5 = \left\{\binom{4n-3}{n-1}, \binom{4n-2}{n-1}, \binom{4n-1}{n}, \binom{4n-1}{2n-2}, \binom{4n}{3n-3}, \binom{4n}{2n-1}\right\}$,
All edge labels are different; we need to notice only that:
$\binom{4n}{3n-3} \neq \binom{4n-1}{2n-2}$ and $\binom{4n-1}{n} \lneqq \binom{4n}{3n-3}$.

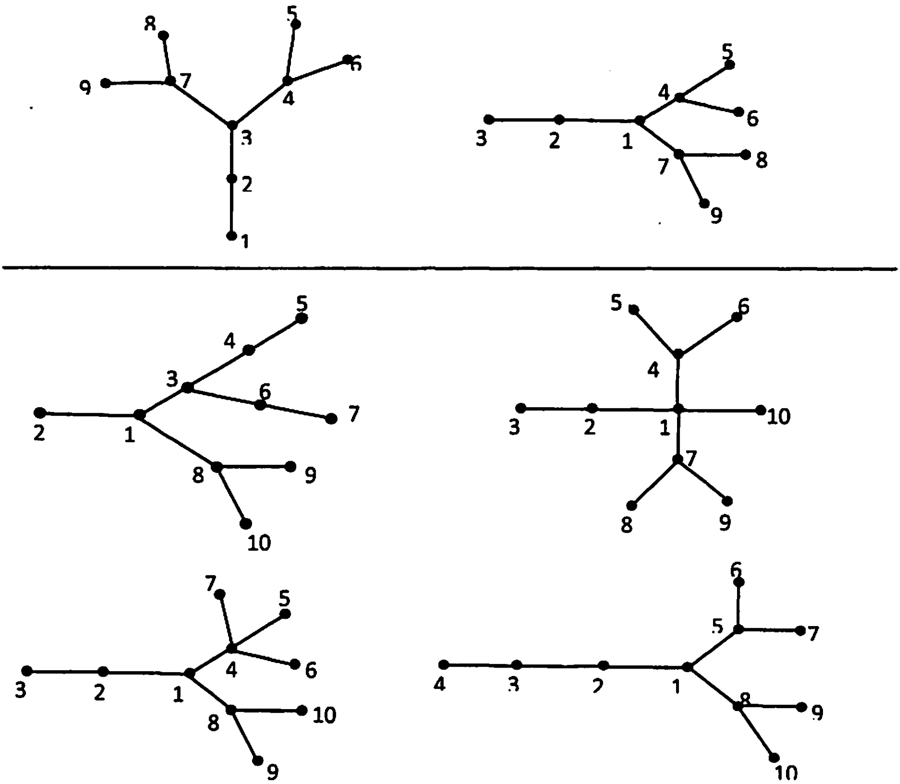
Example 4.6. Figure (13) shows a combination labeling of $4C_7$.



Figure(13)

Definition 4.7.
(1) A regular or a complete binary tree is a binary tree that meets the following conditions:
a) There is exactly one vertex of degree two , namely the root .
b) All vertices other than the root have degree one or three.
(c) All vertices of degree one are at the same distance from the root.
(2)Let $n \geq 4$, and consider all ternary trees, i.e. on n vertices, where "internal" vertices have degree three and "external " vertices have degree one.
(3) Now we consider the graph resulting from identifying the pendent vertices of the S_m with the paths P_{n_i} , for some n_i , $1 \leq n_i \leq$ m .
Theorem 4.8. (i) The trees described in Definition 4.7 are combination graphs.
(ii)The trees T_n , n $\leq$ 10, obtained by joining the centers of two trees by a path in Definition 4.7,(3) are combination graphs.
Proof. (i) According to Definition 4.7, we have:
(1) We will introduce a labeling of a full binary tree by using the Breath _First Algorithm. We label the root by 1and label the vertices that are adjacent to the root by 2 and 3, and then label the vertices that are adjacent to these vertices by 4,5 and 6,7 respectively and so on.
(2) Method of vertex labeling: The center of the star S_m is labeled by 1, and then label the vertices of distance 1 by 2 ,3 , 4, the vertices of distance 2 by 5,..., 10, the vertices of distance 3 by 11,...,22 and so on.
(3) We label the center of the star by 1, then we label the first branch by 2,3,...,n_1 ,the second branch by n_1+1, n_1+2,..., n_1+n_2 , where n_1 and n_2 are the number of vertices of the first and second branches respectively ,and so on.
(ii)We label the path P_n by $1,2,\dots,n$, then we label the branches as in(3).
Survey 4.9.We label all trees of order $\leq$10 as combination trees.
1) Paths and stars can be easily labeled.
2) All caterpillars (Theorem4.3).
3) Trees in Theorem4.8,(2)and(3).

4) The remaining trees are labeled in the following manner.



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