The crossing number of pancake graph P_4 is six *

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Abstract

The *crossing number* of a graph G is the smallest number of pairwise crossings of edges among all the drawings of G in the plane. The pancake graph is an important network topological structure for interconnecting processors in parallel computers. In this paper, we prove the exact crossing number of pancake graph P_4 is six.

Keywords: *Crossing number*; *Drawing* ;*Pancake graph*;

1 Introduction

The notion of *crossing number* is a central one for Topological Graph Theory with long history, which means the minimum possible number of edge crossings among all the drawings of graph G in the plane. Because of its various applications, such as VLSI theory and wiring layout problems, the crossing number problem has been studied extensively by mathematicians including Erdős, Guy, Turán and Tutte, et al (see [9, 11, 14, 15]). However, the investigation on the crossing number problem is extremely difficult. In 1973, Erdős and Guy wrote, "Almost all questions that one can ask about crossing numbers remain unsolved." Actually, Garey and Johnson in [10] proved that computing the crossing number is NP-complete.

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Thereby, it's not surprising that the exact crossing numbers are known only for a few families of graphs (see [7, 13]). In most cases, to give the upper and lower bounds is a more practical way (see [12, 16, 17]). As to a nice drawing of a graph with the number of crossings that can hardly be decreased, it is very difficult to prove that the number of crossings in this drawing is indeed the crossing number of the graph we studied.

The pancake graph was proposed by Akers and Krishnameurthy in [8] as a special case of Cayley graphs. It not only possesses several attractive features just like hypercubes, such as symmetry properties and high fault tolerant, but also offers three significant advantages over hypercubes: a lower degree, a smaller diameter and average diameter. Therefore, there are more and more research about pancake graphs recently. In [1], Lin, Huang and Hsu proved that the n-dimensional pancake graph P_n is super connected if and only if $n \neq 3$. In addition, Deng and Zhang proved that the automorphism group of the pancake graph P_n is the left regular representation of the symmetric group S_n for $n \geq 5$ in [3]. More research about pancake graph can be found in [2–6].

In [18], Sýkora and Vrt'o proved an approximative value of the crossing number of the $n-$dimensional pancake graph. However, their results are valuable only when the dimension n is large enough. Yet there is little study of the exact crossing number of pancake graphs when n is small, which is of theoretical importance and practical value. In this paper, we prove that the crossing number of pancake graph P_4 is exactly six.

2 Notations and basic lemmas

Let G be a simple connected graph with vertex set $V(G)$ and edge set $E(G)$. For $S \subseteq E(G)$, let $[S]$ be the subgraph of G induced by S. Let $P_{v_1 v_2 \cdots v_n}$ be the path with n vertices $v_1, \cdots, v_n$ and let $C_{v_1 v_2 \cdots v_n v_1}$ be the cycle with n vertices $v_1, \cdots, v_n$.

A drawing of G is said to be a *good* drawing, provided that no edge crosses itself, no adjacent edges cross each other, no two edges cross more than once, and no three edges cross in a point. It is well known that the crossing number of a graph G, denoted by $cr(G)$, is attained only in the good drawings of the graph. So, we always assume that all drawings throughout this paper are good drawings.

For a drawing D of a graph G, let $\nu(D)$ be the number of crossings in D. In a drawing D, if an edge is not crossed by any other edge, we say that it is *clean* in D.

For two disjoint subsets of an edge set E, say A and B, the number of the crossings formed by an edge in A and another edge in B is denoted by $\nu_D(A,B)$ in a drawing D. The number of the crossings that involve a pair of edges in A is denoted by $\nu_D(A)$. Then $\nu_D(A \cup B) = \nu_D(A) + \nu_D(B) + \nu_D(A,B)$ and $\nu(D) = \nu_D(E)$.

Definition 2.1. (*Pancake Graph*) *The n-dimensional pancake graph, denoted by P_n, is a graph consisting of $n!$ vertices labelled by distinct permutations on $\{1, 2, \cdots, n\}$. There is an edge from vertex v_i to vertex v_j if and only if v_j is a permutation of v_i such that $v_i = i_1 i_2 \cdots i_k i_{k+1} \cdots i_n$ and $v_j = i_k \cdots i_2 i_1 i_{k+1} \cdots i_n$, where $2 \leq k \leq n$.*

The pancake graphs P_2, P_3 and P_4 are shown in Figure 2.1 for illustration.

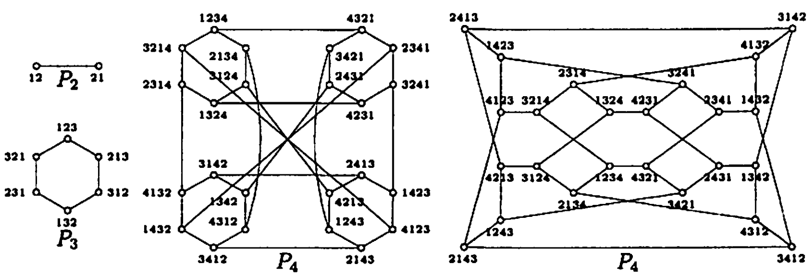

Figure 2.1: Some drawings of P_n

There are four 6-cycles $C_i (1 \leq i \leq 4)$ in P_4. For $1 \leq i \leq 4$, the subgraph of P_4 induced by $V(P_4) - V(C_i)$ is homeomorphic to graph G_{12} shown in Figure 2.2.

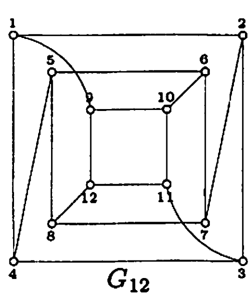

Figure 2.2: A drawing of G_{12}

Lemma 2.1. $cr(G_{12}) = 2$.

Proof. We give a drawing of G_{12} with two crossings in Figure 2.2 and then we have $cr(G_{12}) \le 2$. Let m be the smallest number of the edges of G_{12} whose deletion from G_{12} results in a planar subgraph G_{12}^* of G_{12}. G_{12}^* has 12 vertices and $18 - m$ edges. Let D_{12}^* be a planar drawing of G_{12}^* and p denote the number of faces in D_{12}^*. Then, according to the Euler Polyhedron Formula,

$$\begin{aligned} 12 - (18 - m) + p &= 2, \\ p &= 8 - m. \end{aligned}$$

Since all cycles in G_{12} have length at least six except for three disjoint 4-cycles, and $cr(G_{12}) \le 2$, we could remove at most two 4-cycles in G_{12}. By considering the number of 4-cycles remaining in G_{12}^*, we have three cases:

$$\begin{aligned} \text{three 4-cycles:} \quad 3 \times 4 + (8 - m - 3) \times 6 &\le |E(G_{12}^*)| = 2 \times (18 - m), \\ \text{two 4-cycles:} \quad 2 \times 4 + (8 - m - 2) \times 6 &\le |E(G_{12}^*)| = 2 \times (18 - m), \\ \text{one 4-cycle:} \quad 1 \times 4 + (8 - m - 1) \times 6 &\le |E(G_{12}^*)| = 2 \times (18 - m). \end{aligned}$$

It follows

$$\begin{aligned} \text{three 4-cycles:} \quad 4m &\ge 6, \\ \text{two 4-cycles:} \quad 4m &\ge 8, \\ \text{one 4-cycle:} \quad 4m &\ge 10. \end{aligned}$$

Hence, we know $m \ge 2$. With $cr(G_{12}) \le 2$, we have $cr(G_{12}) = 2$. □

For $i = 1, 2, 3$, let $C_i^4 = C_{v_{4i-3}v_{4i-2}v_{4i-1}v_{4i}v_{4i-3}}$.

Lemma 2.2. *Let D be a drawing of G_{12}, where at least one pair of 4-cycles crosses each other, then $\nu(D) \ge 3$.*

Proof. By contradiction. Suppose $\nu(D) \le 2$. Then there is exact one pair of 4-cycles, say C_1^4 and C_2^4, crossing each other. Since $\nu(D) \le 2$, there is no 4-cycle crossing itself, and edges $v_1v_9, v_3v_{11}, v_6v_{10}$ and v_8v_{12} are all clean. Consider the sub-drawing S of C_1^4 and C_3^4 in D. By hypothesis, there is no crossing in S. Assume that C_1^4 is inner the region defined by C_3^4 in D. Hence, C_1^4, C_3^4 and the edges v_6v_{10} and v_8v_{12} define 4 regions of the plane: $R_1 = (5, 6, 7, 8)$, $R_2 = (9, 10, 11, 12)$, $R_3 = (5, 6, 10, 11, 12, 8)$, $R_4 = (7, 8, 12, 9, 10, 6)$ in a first case (see Figure 2.3 (1)), or $R_3 = (5, 6, 10, 9, 12, 8)$, $R_4 = (7, 8, 12, 11, 10, 6)$ in a second case (see Figure 2.3 (2)). By hypothesis, since that v_3v_{11} and v_1v_9 are not crossed in D, vertex v_3 belongs to R_3 and vertex v_1 belongs to R_4 in the first case, while vertex v_3 belongs to R_4 and vertex v_1 belongs to R_3 in the second case. Since there are 2 edge disjoint paths in C_1^4 joining vertices v_1 to v_3, there are at least four crossings along the edges v_5v_6, v_5v_8, v_6v_7 and v_7v_8, a contradiction. □

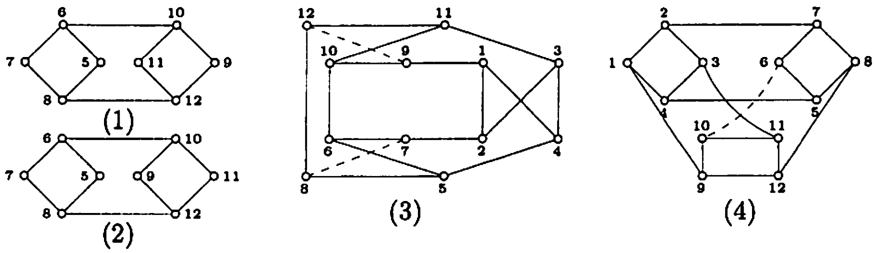

Figure 2.3: Some drawings of G_{12}

Lemma 2.3. *Let D be a drawing of G_{12}, where any pair of 4-cycles does not cross each other and any two 4-cycles lie in the same side of the third 4-cycle, then $\nu(D) \geq 3$.*

Proof. By contradiction. Suppose $\nu(D) \leq 2$.

Case 1. There is at least one 4-cycle, say C_1^4, crossing itself. Without loss of generality, we may assume that v_1v_4 crosses v_2v_3. We show this situation in Figure 2.3 (3). Since $\nu(D) \leq 2$, at least one of the edge disjoint cycles $C_{v_1v_9v_{10}v_6v_7v_2v_1}$ and $C_{v_3v_{11}v_{12}v_8v_5v_4v_3}$, say cycle $C_{v_1v_9v_{10}v_6v_7v_2v_1}$, does not cross itself. And at least one of the cycles $C_{v_4v_3v_{11}v_{10}v_9v_1v_4}$ and $C_{v_3v_4v_5v_6v_7v_2v_3}$, say cycle $C_{v_4v_3v_{11}v_{10}v_9v_1v_4}$, does not cross itself, since they only have one common edge. By considering the possible locations of vertex v_{12}, we could find at least one edge of $\{v_{12}v_{11}, v_{12}v_9, v_{12}v_8, v_8v_7\}$ is crossed, since edges $v_{11}v_{12}$, v_9v_{12} and path $P_{v_{12}v_8v_7}$ can not be in the same region. Hence, cycle $C_{v_3v_4v_5v_6v_7v_2v_3}$ can not cross itself. Since $\nu(D) \leq 2$, path $P_{v_5v_8v_{12}v_{11}}$ can be crossed at most once, and edge v_8v_{12} has to lie outside of cycle $C_{v_5v_4v_3v_{11}v_{10}v_6v_5}$. It follows edges $v_{12}v_9$ and v_8v_7 are both crossed, and $\nu(D) \geq 3$, a contradiction.

Case 2. There is no 4-cycle crossing itself. Since $\nu(D) \leq 2$, at least one of all the three pairs of 4-cycles, say C_1^4 and C_2^4, satisfies the following conditions: the edges between that pair of 4-cycles do not cross each other, and they do not cross the pair of 4-cycles either. By symmetry, we may assume v_3, v_6 lie inside of cycle $C_{v_1v_2v_7v_8v_5v_4v_1}$ (see Figure 2.3 (4)). Since any pair of 4-cycles does not cross each other and any two 4-cycles lie in the same side of the third 4-cycle, 4-cycle C_3^4 has to lie outside of cycle $C_{v_2v_7v_6v_5v_4v_3v_2}$ or inside of cycle $C_{v_2v_7v_6v_5v_4v_3v_2}$. By symmetry, we may assume 4-cycle C_3^4 lies outside of cycle $C_{v_2v_7v_6v_5v_4v_3v_2}$. Then edges v_3v_{11} and v_6v_{10} are crossed. Since $\nu(D) \leq 2$, edges v_1v_9 and v_8v_{12} are clean. It follows at least one of edges v_3v_{11} and v_6v_{10} is crossed at least twice, and $\nu(D) \geq 3$, a contradiction. □

3 Crossing number of P_4

In Figure 3.1, we show a drawing of P_4 with 6 crossings. Hence, we have:

Lemma 3.1. $cr(P_4) \leq 6$.

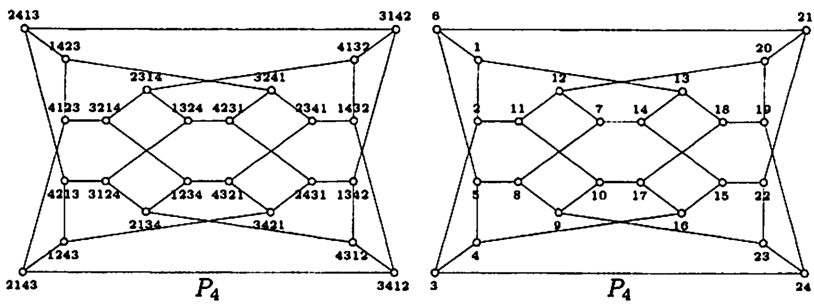

Figure 3.1: A good drawing of P_4 with 6 crossings

In the rest of this section, we shall prove that the value of $cr(P_4)$ is exactly equal to 6. We rename the vertices of P_4 as shown in Figure 3.1.

For $i = 1, 2, 3, 4$, let

$$\begin{aligned}
C^i_{p_4} &= C_{v_{6i-5}v_{6i-4}v_{6i-3}v_{6i-2}v_{6i-1}v_{6i}v_{6i-5}}, \\
V^i_{p_4} &= V(C^i_{p_4}), \\
E^i_{p_4} &= E(C^i_{p_4}), \\
E^{i,j}_{p_4} &= \{uv : u \in V^i_{p_4} \wedge v \in V^j_{p_4}\}, \\
E'^i_{p_4} &= E^i_{p_4} \cup \bigcup_{(1 \leq j \leq 4) \wedge j \neq i} E^{i,j}_{p_4}, \\
\overline{E'^i_{p_4}} &= E(P_4) - E'^i_{p_4}.
\end{aligned}$$

For convenience, we abbreviate

$$\begin{aligned}
C_i &= C^i_{p_4}, V_i &= V^i_{p_4}, E_i &= E^i_{p_4}, \\
E_{i,j} &= E^{i,j}_{p_4}, E'_i &= E'^i_{p_4}, \overline{E'_i} &= \overline{E'^i_{p_4}}.
\end{aligned}$$

Since $[\overline{E'_i}]$ is homeomorphic to G_{12} (see Figure 3.2), by Lemmas 2.1 - 2.3, we have

Lemma 3.2. *For* $i = 1, 2, 3, 4$,
1) *Let* D *be an arbitrary drawing of* $[\overline{E'_i}]$, *then* $\nu(D) \geq 2$.
2) *Let* D *be a drawing of* $[\overline{E'_i}]$, *where at least one pair of* 6*-cycles crosses*

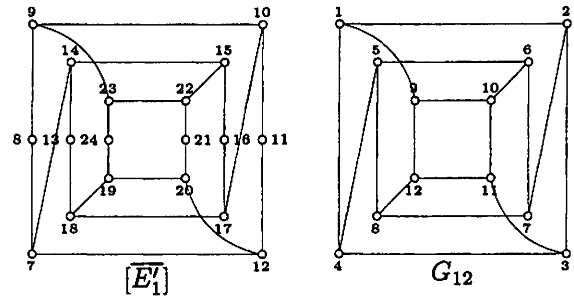

Figure 3.2: $[\overline{E'_i}]$ is homeomorphic to G_{12}

each other, then $\nu(D) \geq 3$.
3) *Let* D *be a drawing of* $[\overline{E'_i}]$, *where any pair of* 6*-cycles does not cross each other and any two 6-cycles lie in the same side of the third* 6*-cycle, then* $\nu(D) \geq 3$.

Proof. 1) Because $[\overline{E'_i}]$ is homeomorphic to G_{12}, by Lemma 2.1, we have $cr([\overline{E'_i}]) = cr(G_{12}) = 2$. It follows $\nu(D) \geq 2$.

2) Because $[\overline{E'_i}]$ is homeomorphic to G_{12}, and every 6-cycle in $[\overline{E'_i}]$ is homeomorphic to a 4-cycle in G_{12}, by Lemma 2.2, we have $\nu(D) \geq 3$.

3) For the same reason as mentioned above, by Lemma 2.3, we have $\nu(D) \geq 3$. □

Lemma 3.3. *Let* D *be a drawing of* P_4, *where at least two pairs of* 6*-cycles cross each other, then* $\nu(D) \geq 6$.

Proof. By contradiction. Suppose $\nu(D) \leq 5$. Since each pair of 6-cycles crossing each other will produce at least two crossings, there are at most two pairs of 6-cycles crossing each other. By symmetry, there are two cases:

Case 1. C_1 crosses C_2 and C_3. By Lemma 3.2, $\nu_D(\overline{E'_1}) \geq 2$. It follows $\nu(D) \geq 2 + 4 = 6$, a contradiction.

Case 2. C_1 crosses C_2, and C_3 crosses C_4. Notice that the four regions formed by C_1 and C_2 are topologically equivalent (see Figure 3.3 (1)), we only need to consider the case that C_3 and C_4 lie in the outer region. By Lemma 3.2, $\nu_D(\overline{E'_1}) \geq 3$. Since $\nu(D) \leq 5$, C_1 does not cross itself, and any edge of $\bigcup_{j=2,3,4} E_{1,j}$ should be clean. It follows edges v_1v_{13} and v_4v_{16} are clean. Now at least one edge of $E_{1,4}$ is crossed, which contradicts any edge of $\bigcup_{j=2,3,4} E_{1,j}$ is clean (see Figure 3.3 (2)). Hence $\nu(D) \geq 6$, a contradiction. □

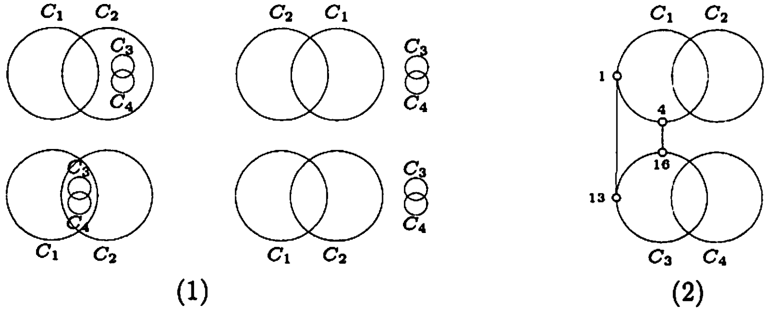

Figure 3.3: Some drawings of P_4, where just two pairs of 6-cycles cross each other

Lemma 3.4. *Let D be a drawing of P_4, where just one pair of 6-cycles crosses each other, then $\nu(D) \geq 6$.*

Proof. Without loss of generality, let C_1 and C_2 be the cycles crossing each other. We prove this lemma by contradiction. Suppose $\nu(D) \leq 5$. By symmetry, we only need to consider the case that C_3 lies outside of C_1 and C_2 (see Figure 3.4 (1)). There are three cases depending on C_4's position:

Case 1. C_4 lies inside of C_3 (see Figure 3.4 (2)). Then each edge of $E_{4,1}$ crosses the edges of E_3 at least once, and each edge of $E_{4,2}$ crosses the edges of E_3 at least once. It follows $\nu(D) \geq 2+2+2=6$, a contradiction.

Case 2. C_4 lies inside of C_2 (see Figure 3.4 (3) and (4)). Then each edge of $E_{4,3}$ crosses the edges of E_2 at least once. By Lemma 3.2, $\nu_D(\overline{E_2'}) \geq 2$. It follows $\nu(D) \geq 2+2+2=6$, a contradiction.

Case 3. C_4 lies outside of C_1, C_2 and C_3 (see Figure 3.4 (5)). By Lemma 3.2, $\nu_D(\overline{E_1'}) \geq 3$. Since $\nu(D) \leq 5$, C_1 does not cross itself, and any edge of $\bigcup_{j=2,3,4} E_{1,j}$ is clean. It follows v_1v_{13} and v_4v_{16} are clean. Now at least one edge of $E_{1,4}$ is crossed, which contradicts any edge of $\bigcup_{j=2,3,4} E_{1,j}$ is clean.

By Cases 1-3, we have $\nu(D) \geq 6$. □

Lemma 3.5. *Let D be a drawing of P_4, where there is no pair of 6-cycles: C_i and C_j such that C_i crosses C_j, then $\nu(D) \geq 6$.*

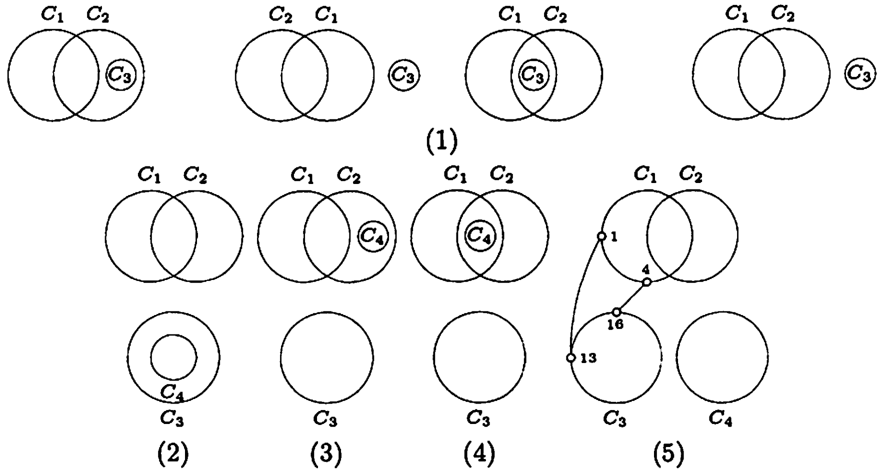

Figure 3.4: Some drawings of P_4,
where just one pair of 6-cycles crosses each other

Proof. By contradiction. Suppose $\nu(D) \leq 5$.

Case 1. C_2 lies inside of C_1, and both of C_3 and C_4 lie outside of C_1.

Case 1.1. C_4 lies inside of C_3. Then each edge of $E_{2,4}$ crosses the edges of E_1 and E_3 at least once, respectively. Meanwhile, each edge of $E_{2,3}$ crosses the edges of E_1 at least once, and each edge of $E_{4,1}$ crosses the edges of E_3 at least once. It follows $\nu(D) \geq 4 + 2 + 2 = 8$ (see Figure 3.5 (1)).

Case 1.2. C_4 lies outside of C_3. Then each edge of $E_{2,3}$ and $E_{2,4}$ crosses the edges of E_1 at least once. By Lemma 3.2, $\nu_D(\overline{E_1'}) \geq 3$. It follows $\nu(D) \geq 3 + 4 = 7$, a contradiction (see Figure 3.5 (2)).

Case 2. each C_i lies outside of the other three C_j $(1 \leq j \leq 4, j \neq i)$.

Case 2.1. C_3 does not cross itself. Since $\nu(D) \leq 5$ and $\nu_D(\overline{E_3'}) \geq 3$, $\nu_D(E_3') + \nu_D(E_3', \overline{E_3'}) \leq 2$. It follows that $(\nu_D(E_{1,3}) + \nu_D(E_3, E_{1,3})) + (\nu_D(E_{2,3}) + \nu_D(E_3, E_{2,3})) + (\nu_D(E_{3,4}) + \nu_D(E_3, E_{3,4})) \leq 2$. Without loss of generality, we may assume $\nu_D(E_{1,3}) + \nu_D(E_3, E_{1,3}) = 0$ (see Figure 3.5 (3)). Then at least one edge of $E_{3,2}$ crosses one edge of $E_1 \cup E_3 \cup E_{1,3}$. Additionally, at least one edge of $E_{3,4}$ crosses one edge of $E_1 \cup E_3 \cup E_{1,3}$ and at least one edge of $E_{3,4}$ crosses one edge of $E_2 \cup E_3 \cup E_{2,3}$. It follows $\nu_D(E_3') + \nu_D(E_3', \overline{E_3'}) \geq 3$, which contradicts $\nu_D(E_3') + \nu_D(E_3', \overline{E_3'}) \leq 2$.

Case 2.2. C_3 crosses itself. Since $\nu(D) \leq 5$ and $\nu_D(\overline{E_3'}) \geq 3$, $\nu_D(E_3') +$

$\nu_D(E'_3, \overline{E'_3}) \leq 2$. It follows that $(\nu_D(E_{1,3}) + \nu_D(E_3, E_{1,3})) + (\nu_D(E_{2,3}) + \nu_D(E_3, E_{2,3})) + (\nu_D(E_{3,4}) + \nu_D(E_3, E_{3,4})) \leq 1$ since C_3 crosses itself. Without loss of generality, we may assume $\nu_D(E_{1,3}) + \nu_D(E_3, E_{1,3}) = \nu_D(E_{2,3}) + \nu_D(E_3, E_{2,3}) = 0$. If C_1 does not cross itself, then the two edges of $E_{1,4}$ will be crossed at least three times in total (see Figure 3.5 (4)). If C_1 crosses itself, then each edge of $E_{1,4}$ will cross the edges of $E_2 \cup E_3 \cup E_{2,3}$ at least once (see Figure 3.5 (5)). By Lemma 3.2, $\nu_D(\overline{E'_1}) \geq 3$. It follows $\nu(D) \geq 3 + 3 = 6$, a contradiction. □

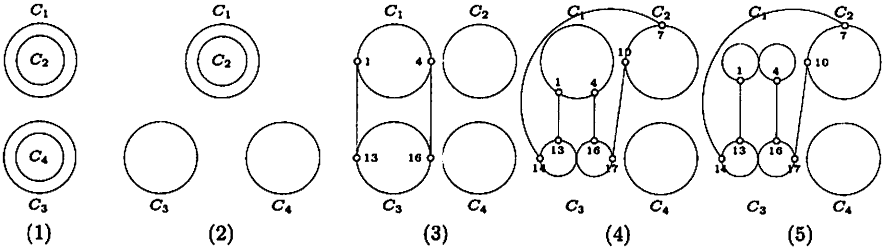

Figure 3.5: Some drawings of P_4,
where any pair of 6-cycles does not cross each other

By Lemmas 3.1 and 3.3 - 3.5, we have

Theorem 3.1. $cr(P_4) = 6$.

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