Supereulerian graphs and Chvátal-Erdös type conditions*

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Abstract: In 1972, Chvátal and Erdös showed that the graph G with independence number $\alpha(G)$ no more than its connectivity $\kappa(G)$ (i.e. $\kappa(G) \geq \alpha(G)$) is hamiltonian. In this paper, we consider a kind of Chvátal and Erdös type condition on edge-connectivity ($\lambda(G)$) and matching number (edge independence number). We show that if $\lambda(G) \geq \alpha'(G) - 1$, then G is either supereulerian or in a well-defined family of graphs. Moreover, we weaken the condition $\kappa(G) \geq \alpha(G) - 1$ in [11] to $\lambda(G) \geq \alpha(G) - 1$ and obtain the similar characterization on non-supereulerian graphs. We also characterize the graph which contains a dominating closed trail under the assumption $\lambda(G) \geq \alpha'(G) - 2$.

Keywords: Supereulerian graphs, Matching number, Chvátal-Erdös condition, Edge-connectivity

1 Introduction

Motivated by the Chinese Postman Problem, Boesch et al. [2] proposed the supereulerian graph problem: determine when a graph has a spanning eulerian subgraph. They indicated that this might be a difficult problem. Pulleyblank [15] showed that such a decision problem, even when restricted

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to planar graphs, is NP-complete. We refer the readers to [4, 9] for the supereulerian graph problem.

We use [1] for terminology and notation not defined here, and consider simple finite graphs only. In particular, we use $\alpha(G)$ and $\alpha'(G)$ to denote the *independence number* and the *matching number* of a graph G, respectively. We denote by $\kappa(G), \lambda(G), \delta(G)$ the connectivity, edge-connectivity and the minimum degree of G. It is known that $\kappa(G) \leq \lambda(G) \leq \delta(G)$ for any G. Chvátal and Erdös obtained the following well-known sufficient condition for a graph to be hamiltonian.

Theorem 1 ([10]). *If $\kappa(G) \geq \alpha(G)$, then G is hamiltonian.*

Many extensions of this well-known condition have been reported, see [12] for the details. Note that hamiltonian cycle is a closed spanning path. This stimulates us to consider a kind of Chvátal and Erdös type condition on discussing spanning closed trails of graphs. In the next section, we shall characterize the non-supereulerian graphs under the condition $\lambda(G) \geq \alpha'(G)-1$.

Recently, Han et al. [11] gave an extension of Chvátal-Erdös condition to consider supereulerian graphs as follows.

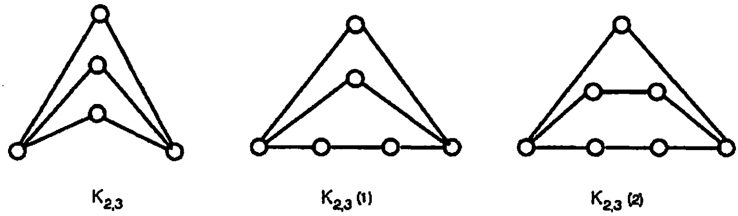

Figure.1 The graphs $K_{2,3}, K_{2,3}(1), K_{2,3}(2)$.

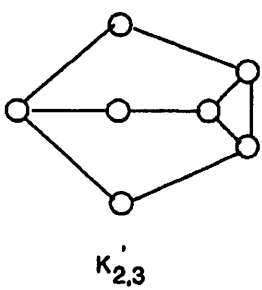

Figure.2 The graph $K'_{2,3}$.

Theorem 2 ([11]). *Let G be a 2-connected simple graph with $\kappa(G) \geq \alpha(G) - 1$. If G is not supereulerian, then either*

(a) G *is in* {*the petersen graph*, $K_{2,3}, K_{2,3}(1), K_{2,3}(2), K'_{2,3}$}, *or*

(b) G *is one of the two 2-connected graphs obtained from* $K_{2,3}$ *and* $K_{2,3}(1)$ *by replacing a vertex whose neighbors have degree three in* $K_{2,3}$ *and* $K_{2,3}(1)$ *with a complete graph of order at least three.*

Note that $\lambda(G) \geq \kappa(G)$. In the third section, we shall weaken the condition $\kappa(G) \geq \alpha(G) - 1$ to $\lambda(G) \geq \alpha(G) - 1$ and obtain a similar characterization for non-supereulerian graphs. To complete the characterization, Catlin's reduction method is needed.

For a graph G, let $O(G)$ denote the set of odd degree vertices of G. A graph G is *eulerian* if G is connected with $O(G) = \emptyset$, and G is *supereulerian* if G has a spanning eulerian subgraph. Given a subset R of $V(G)$, a subgraph Γ of G is called an R-subgraph if both $O(\Gamma) = R$ and $G - E(\Gamma)$ is connected. A graph G is *collapsible* if for any even subset R of $V(G)$, G has an R-subgraph. Note that when $R = \emptyset$, a spanning connected subgraph H with $O(H) = \emptyset$ is a spanning eulerian subgraph of G. Thus every collapsible graph is supereulerian. Catlin [3] showed that any graph G has a unique subgraph H such that every component of H is a maximally connected collapsible subgraph of G and every non-trivial connected collapsible subgraph of G is contained in a component of H. For a subgraph H of G, the graph G/H is obtained from G by identifying the two ends of each edge in H and then deleting the resulting loops. The contraction G/H is called the *reduction* of G if H is the maximal collapsible subgraph of G, i.e. there is no non-trivial collapsible subgraph in G/H. We use G' to denote the reduction of G. A vertex in G' is *trivial* if the vertex is obtained by contracting a trivial collapsible subgraph of G (K_1). A graph G is *reduced* if it is the reduction of itself. The following summarizes some of the previous results concerning collapsible graphs.

In particular, a trail in G is a dominating trail if each edge of G is incident with at least one internal vertex of the trail. Clearly, a spanning trail a dominating trail.

2 The graphs with $\lambda(G) \geq \alpha'(G) - 1$

In this section, we shall characterize the non-supereulerian graphs under the condition $\lambda(G) \geq \alpha'(G) - 1$. The following result is due to Jaeger [13].

Theorem 3 ([13]). *A 4-edge connected graph is supereulerian.*

By Theorem 3, we may assume the graphs with $\lambda(G) \leq 3$ from now on. This implies $\alpha'(G) \leq 4$. Chen [7] showed the following.

Theorem 4 ([7]). *If G is a 3-edge connected simple graph with matching number at most 5, then G is supereulerian if and only if G is not contractible to the Petersen graph.*

If a graph can be contracted to the Petersen graph, then its matching number is at least 5. Thus, a 3-edge connected graph with $\alpha'(G) = 4$ is supereulerian by Theorem 4. Since a graph containing cut-edges is not supereulerian, we may assume $\lambda(G) = 2$ and $\alpha'(G) \leq 3$ from now on.

Let m, n be two positive integers, and $H_1 \cong K_{2,m}, H_2 \cong K_{2,n}$ be two complete bipartite graphs. Let u_1, v_1 be the vertices of degree m in H_1, and u_2, v_2 be the vertices of degree n in H_2. Let $S_{n,m}$ denote the graph obtained from H_1 and H_2 by identifying v_1 and v_2, and adding a new edge u_1u_2. Note that $S_{1,1}$ is a 5-cycle. Define $K_{1,3}(1,1,1)$ to be the graph obtained from a 6-cycle $C = u_1u_2u_3u_4u_5u_6u_1$ by adding one vertex u and three edges uu_1, uu_3 and uu_5.

We first introduce several special graphs as follows.

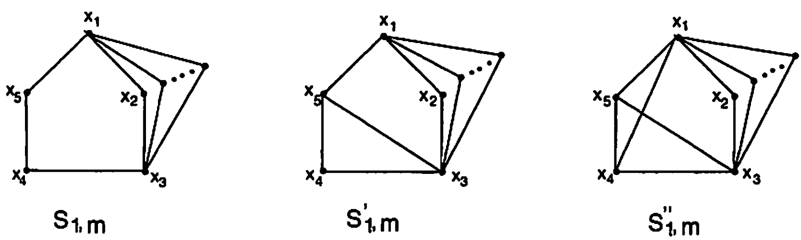

Figure 3. The case $n = 1$.

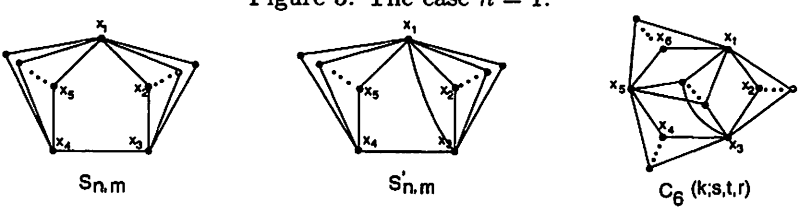

Figure 4. The case $m \geq n \geq 2$.

Let $S^*_{1,m} = S_{1,m}+x_1x_3, S'^*_{1,m} = S'_{1,m}+x_1x_3, S''^*_{1,m} = S'_{1,m}+x_1x_3, S''_{n,m} = S'_{n,m}+x_1x_4$. Similarly, we denote by $C^1_6(k;s,t,r), C^2_6(k;s,t,r), C^3_6(k;s,t,r)$ the graphs obtained from $C_6(k;s,t,r)$ by adding edges $\{x_1x_3\}$, $\{x_1x_3, x_3x_5\}$ and $\{x_1x_3, x_3x_5, x_5x_1\}$, respectively. We define $\mathcal{C} = \{C^1_6(k;s,t,r), C^1_6(k;s,t,r), C^2_6(k;s,t,r), C^3_6(k;s,t,r)\}$ and $\mathcal{S} = \{S_{1,m}, S'_{1,m}, S''_{1,m}, S^*_{1,m}, S'^*_{1,m}, S''^*_{1,m}, S_{n,m}, S'_{n,m}, S''_{n,m}, K_{2,t}, K'_{2,t}\}$.

Lemma 5. *Let G be a graph with $\delta(G) \geq 2$ and $\alpha'(G) \leq 3$. Then G is in $\mathcal{F} = \{G : |V(G)| \leq 7\} \cup \mathcal{S} \cup \mathcal{C}$.*

Proof. Suppose C is the longest cycle of G with length l. As $\alpha'(G) \leq 3$, one can see that $l \leq 7$ and $|V(G)| = 7$ if $l = 7$. Thus $G \in \mathcal{F}$ if $l = 7$.

If $l = 3$, then $G \in \{K_3, H, H', H''\}$, where H denotes the hourglass, H', H'' denote the two different graphs obtained from a hourglass and a triangle by identifying a vertex of the hourglass and a vertex of the triangle respectively. Each of the cases implies $|V(G)| \leq 7$ and then $G \in \mathcal{F}$. Thus we may assume $4 \leq l \leq 6$.

Case 1. $l = 6$

Let $C = x_1x_2x_3x_4x_5x_6x_1$ be a longest cycle of G. Clearly, C is a dominating cycle of G. Suppose $V(G-C) \neq \emptyset$. Since $\alpha'(G) \leq 3$ and $l = 6$, at most one end of an edge in $E(C)$ has neighbors in $V(G-C)$. Thus at most three pairwise nonadjacent vertices of $V(C)$ have neighbors in $V(G-C)$ and assume that they are x_1, x_3, x_5. Suppose there are k vertices in $V(G-C)$. It is easy to see that $G \in \mathcal{C}$.

Case 2. $l = 5$

Let $C = x_1x_2x_3x_4x_5x_1$ be a longest cycle of G. Note that $l = 5$ and $\delta(G) \geq 2$. Then if C is not a dominating cycle of G, then G is the graph obtained from a cycle C_5 (a cycle of length 5) and a triangle by identifying a vertex of the C_5 and a vertex of the triangle and adding some edges on the C_5. In this case, we have $|V(G)| \leq 7$. Suppose C is a dominating cycle of G. It is easy to see that G contains an $S_{n,m}$ as a subgraph for some m, n. If $m \geq n \geq 2$, then $G \in \{S_{n,m}, S'_{n,m}, S''_{n,m}\}$. If $n = 1$, then $G \in \{S_{1,m}, S'_{1,m}, S''_{1,m}\}$, see Figure 4.

Case 3. $l = 4$

Let $C = x_1x_2x_3x_4x_1$ be a longest cycle of G. Similarly, if C is not a dominating cycle, then $|V(G)| \leq 7$. If C is a dominating cycle, we have $G \cong K_{2,t}$ or $G \cong K^*_{2,t}$ for some integer t, where $K^*_{2,t}$ is the graph obtained form $K_{2,t}$ by adding an edge between the two vertices of degree t. $\square$

Note that $S^*_{1,m}, S'^*_{1,m}, S''^*_{1,m}$, and $S''_{n,m}$ are supereulerian. If $G \cong C_6(k; s, t, r)$, $k = 1, s, t, r \geq 1$ and the parities of s, t, r are the same, then G is not supereulerian. In fact, G has 4 vertices of odd degree and only one edge can be removed. So G is not supereulerian. If $k \geq 2$, the $k-1$ vertices of degree 3 can be used to adjust the parities of s, t, r such that one of them is different from others, then the resulting is supereulerian clearly. So we have the following theorem.

Theorem 6. *Let G be a graph with $\lambda(G) = 2 \geq \alpha'(G) - 1$. Then G is not supereulerian if and only if one of the following holds:*

(1) *If $G \cong S_{n,m}, m \geq n \geq 1$, then one of n, m is an even number;*

(2) *If $G \in \{S'_{1,m}, S''_{1,m}\}$, then m is an even number;*

(3) *If $G \cong S'_{n,m}, m \geq n \geq 2$, then x_4 is a vertex of odd degree.*

(4) *If $G \cong C_6(k; s, t, r)$, then $k = 0$ or 1. Moreover, if $k = 1$, then the parities of s, t, r are the same; If $k = 0$, then the parities of s, t, r are different.*

Note that if G contains vertices of degree 1, then G is not supereulerian. If we assume $\alpha'(G) \leq 2$, then it is easy to get the following by the proof of Lemma 5.

Corollary 7. *Let G be graph with $\lambda(G) = 1$ and $\alpha'(G) \leq 2$. If G contains no spanning eulerian trail, then G is either obtained from $K_{2,t}$ by adding several pendant edges on the vertices of degree t, or obtained from $K_{2,t}^*$ by adding at least two pendant edges on a vertex of degree t.*

3 Supereulerian graphs in terms of $\lambda(G) \geq \alpha(G) - 1$

In this section, we shall weaken the condition $\kappa(G) \geq \alpha(G) - 1$ to $\lambda(G) \geq \alpha(G) - 1$ and obtain a characterization similar to that of Theorem 2 for non-supereulerian graphs. We assume that G' is the reduction of G. By the definition, one can see $\alpha(G) \geq \alpha(G')$, and $\lambda(G') \geq \lambda(G)$ if G' is not trivial.

Recall that $\kappa(G) \leq \lambda(G)$. By Theorem 3, we may assume $\lambda(G) \leq 3$, and then $\alpha(G) - 1 \leq 3$. By the characterization of Theorem 2, if $\lambda(G) = 3$, then either G' is K_1 or G is the Petersen graph. The non-supereulerian graphs are also characterized by Theorem 2 for the case $\lambda(G) = \kappa(G) = 2$ and $\alpha(G) \leq 3$. Moreover, if $\lambda(G) = 1$ and $\alpha(G) \leq 2$, then G is not supereulerian. The rest is the case for $2 = \lambda(G) > \kappa(G) = 1$ and $\alpha(G) \leq 3$.

Define $\mathcal{F}_1 = \{K_{2,3}, K_{2,3}(1), K_{2,3}(2)\}$. Let $K_{2,3}^*$ and $K_{2,3}^*(1)$ be the graphs obtained from $K_{2,3}$ and $K_{2,3}(1)$ by replacing a vertex whose neighbors are both vertices of degree 3 in $K_{2,3}$ and $K_{2,3}(1)$ with a complete graph of order at least three, respectively. It is easy to see that the reduction of $K_{2,3}'$, $K_{2,3}^*$ and $K_{2,3}^*(1)$ are the graph $K_{2,3}$.

Lemma 8. *Assume $\lambda(G) = 2 > \kappa(G)$. If G is not supereulerian, then G is either the graph obtained from $K_{2,3}$ by joining a complete graph on a vertex of degree 2, or the graph obtained from $K_{2,3}(1)$ by joining a complete graph on a vertex of degree 2 whose neighbors in G are both vertices of degree 3.*

Proof. Assume G is not supereulerian. Let G' be the reduction of G. Then G' is not K_1 and $\lambda(G') = 2$. If $\kappa(G') = 2$, then by Theorem 2 that $G' \in \mathcal{F}_1$. Since $\kappa(G) = 1$, there is one vertex v_1 of G' that is not trivial and it is a cut vertex of G. Assume that v_1 is obtained by contracting the collapsible subgraph H_1 of G. Note that $|V(H_1)| \geq 3$. If $d_{G'}(v_1) = 3$, then one can find an independent set of size 4. Thus, v_1 is a vertex of degree 2. If v_1 has an neighbor in G' of degree 2, then it is easy to find an independent set of size 4. So the neighbors of v_1 in G' are both vertices of degree 3. Since $\alpha(G) \leq 3$, H_1 must be a complete subgraph of G. It is not difficult to see that the vertices of degree 3 in G' are trivial and there is exactly one vertex of degree 2 that is not trivial. Thus, G is either the graph obtained from $K_{2,3}$ by joining a complete graph on a vertex of degree 2, or the graph obtained from $K_{2,3}(1)$ by joining a complete graph on a vertex of degree 2 whose neighbors in G' are both vertices of degree 3.

Now, we may assume $\kappa(G') = 1$. Let u be a cut vertex of G'. Note that $\lambda(G') \geq 2$ and G' is triangle free. Then u lies on at least two 4-cycles which have exactly one common vertex u. These 4-cycles imply an independent set of size at least 4. This is impossible. We complete the proof. □

Combining Theorem 2 and Lemma 8, we give a similar statement of Theorem 2.

Theorem 9. *Let G be a 2-edge connected simple graph with $\lambda(G) \geq \alpha(G) - 1$. If G is not supereulerian, then exactly one of the following holds.*

(a) G is in {the Petersen graph, $K_{2,3}, K_{2,3}(1), K_{2,3}(2), K'_{2,3}$}, or

(b) G is one of the two 2-connected graphs obtained from $K_{2,3}$ and $K_{2,3}(1)$ by replacing a vertex whose neighbors are both vertices of degree 3 in $K_{2,3}$ and $K_{2,3}(1)$ with a complete graph of order at least three.

(c) G is either the graph obtained from $K_{2,3}$ by joining a complete graph on a vertex of degree 2, or the graph obtained from $K_{2,3}(1)$ by joining a complete graph on a vertex of degree 2 whose neighbors in G are both vertices of degree 3.

In the next section we consider the dominating closed trail in graphs with $\lambda(G) \geq \alpha'(G) - 2$.

4 The dominating closed trail in graphs with $\lambda(G) \geq \alpha'(G) - 2$

In this section we consider the dominating closed trail in the graphs satisfying $\lambda(G) \geq \alpha'(G) - 2$. We assume the graph has minimum degree at least 2.

Theorem 10 (Chen [8]). *Let G be a reduced graph with $\delta(G) \geq 3$. Then $\alpha'(G) \geq \min\{\frac{n-1}{2}, \frac{n+4}{3}\}$.*

The theorem above implies that if a reduced graph has $\alpha'(G) \leq 5$, then $n \leq 11$.

Theorem 11 (Chen [6]). *A 3-edge-connected graph with at most 13 vertices either is supereulerian, or its reduction is the Petersen graph.*

Theorem 12 (Catlin and Lai [5]). *Let G' be the reduction of a graph G. If every edge of G' lies on a cycle of length at most 7, then G contains a dominating closed trail.*

Thus, if $\lambda(G) \geq 3$, then either G is supereulerian or its reduction is the Petersen graph. It is easy to see that if G satisfying $\lambda(G) = 3 \geq \alpha'(G) - 2$ and its reduction is the Petersen graph, then G is the Petersen graph. Thus, we have the following.

Corollary 13. *If a 3-edge connected graph G satisfying $\lambda(G) \geq \alpha'(G) - 2$, then it is either a supereulerian graph, or the Petersen graph. Moreover, G contains a dominating closed trail.*

Next, we assume $\lambda(G) \leq 2$ and $\alpha'(G) \leq \lambda(G) - 2$. By Corollary 6, one can see that if $\alpha'(G) \leq 3$, then G has a dominating closed trail. If $\lambda(G) = 1$, then $\alpha'(G) \leq 3$.

Lemma 14. *Let G' be the reduction of G with $\delta(G) \geq 2$ and $\alpha'(G) \leq \lambda(G) + 2$. If $\lambda(G) = 1$, then G contains no dominating closed trail, and G' either is K_2 or the graph C_4' obtained by adding a pendant edge on a 4-cycle.*

Proof. Note that $\lambda(G) = 1$ and $\delta(G) \geq 2$. Then G has no dominating closed trail.

Let e be a cut edge of G', and let G_1', G_2' be the two components of $G' - e$. If G_1' and G_2' are both trivial components, then $G' \cong K_2$. Note G' is triangle free. If neither G_1' nor G_2' is a trivial component, then G' contains a matching of size at least 4. This is impossible. Assume G_1' is trivial and obtained by contracting the subgraph G_1 (it is of at least 3 vertices) of G. Then, G_1 is a triangle and G_2' is a 4-cycle. We complete the proof. □

Lemma 15. *Let G be a graph with $\delta(G) \geq 3$ and $\lambda(G) = 2$. If $|V(G)| \leq 8$, then G is collapsible.*

Proof. It is easy to see that G is collapsible if $|V(G)| \leq 5$. Assume $|V(G)| \geq 6$. Let $\{e_1, e_2\}$ be an edge cut of G and G_1, G_2 be the two components of $G - \{e_1, e_2\}$. If $|V(G)| \leq 7$, then one of G_1, G_2 is of order 3. And if G_1 (G_2) is of order 3, then G_1 is a triangle. Thus $|V(G/G_1)| \leq 5$ if $|V(G)| \leq 7$, and then G is collapsible. So we may assume $|V(G)| \geq 8$ and G_i be of order 4. Since $\delta(G) \geq 3$, G_i is collapsible. So G is collapsible. □

Lemma 16. *Let G' be the reduction of graph G with $\delta(G) \geq 2$ and $\alpha'(G) \leq \lambda(G) + 2$. If $\lambda(G) = 2$, then G contains a dominating closed trail.*

Proof. It can be seen that each of the graphs in $\mathcal{S}$ and $\mathcal{C}$ (in Lemma 5) contains a dominating closed trail. Moreover, it is not difficult to see that if $|V(G)| \leq 7$, then G contains a dominating closed trail. So we may assume $\alpha'(G) = 4$ and $\lambda(G) = 2$. If $\delta(G) \geq 3$, then by Theorem 10 we have $n \leq 8$. By the lemma above, G contains a dominating closed trail. So we may assume $\delta(G) = 2$.

Note that if $\delta(G') \geq 3$ and $\alpha'(G') \leq 4$, then by Theorem 10 we have $n \leq 8$. Thus, G' is collapsible and then G contains a dominating closed trail. We may assume $\delta(G') = 2$ from now on. Let the length of a longest cycle of G' is l.

By Theorem 12, we may assume $l \geq 8$. Note that $\alpha'(G') \leq 4$. Then $G - V(C)$ contains no edges and no non-trivial vertices. Thus C implies a dominating closed trail of G. □

Combining the lemmas above, we state the main result of this section as follows.

Theorem 17. *If a graph G satisfies $\delta(G) \geq 2$ and $\alpha'(G) \leq \lambda(G) + 2$, then either G contains a dominating closed trail, or its reduction is in $\{K_2, C_4'\}$.*

In the end, we pose a problem on the 3-edge connected graph.

Problem 18. *What is the minimum integer t such that a 3-edge connected graph G satisfying $\lambda(G) \geq \alpha'(G) - t$ is neither supereulerian nor the Petersen graph ? Furthermore, what is the minimum integer t such that a 3-edge connected graph G satisfying $\lambda(G) \geq \alpha'(G) - t$ contains no dominating closed trail ?*

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