Some results on specific graphs

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Abstract. The aim of this paper is to show that the corona $P_n \bigodot P_m$ between two paths P_n and P_m is cordial for all $n \geq 1$ and $m \geq 1$. Also, we prove that except for n and m are congruent to $2(mod4)$, the corona $C_n \bigodot C_m$ between two cycles C_n and C_m is cordial. Furthermore, we show that if $n \equiv 2(mod4)$ and m is odd, then $C_n \bigodot C_m$ is not cordial.

1. Introduction.

Graph theory has applications in many other fields, including physics, chemistry, biology, communication, economics, engineering, operator research, and especially computer science.

Graphs labelings are currently the subject of much study. A graph labeling is an assignment of integers to the vertices or edges, or both, subject to certain conditions. They first introduced in the late 1960s. Two of the most important types of labelings are called graceful and harmonious. Graceful labelings were introduced independently by Rosa [12] in 1966 and Golomb [10] in 1972, while harmonious labelings were first studied by Graham and Sloane [11] in 1980. A third important type of labeling, which interests us and contains aspects of both of the other two, is called cordial and was introduced by Cahit [1] in 1990. Whereas the label of an edge vw for graceful and harmonious labeling are given respectively by $f(v) - f(w)$ and $f(v) + f(w)$ (modulo the number of edges), cordial labelings use only labels 0 and 1 and the induced label $(f(v) + f(w))(mod2)$, which of course equals $f(v) - f(w)$. Because arithmetic modulo 2 is an integral part of computer science, cordial labelings have close connections with that field. A highly recommended reference on this subject is the survey by Gallian [9].

Cordial graphs are defined as follows: Let $G = (V, E)$ be a graph. A binary vertex labeling is a mapping $f : V \rightarrow \{0,1\}$. For each edge $e = uv \in E$, the induced edge labeling $f^* : E \rightarrow \{0,1\}$ defined by the formula $f^*(vw) = f(v) + f(w)(mod2)$. (Thus, for any edge e, $f^*(e) = 0$ if its two vertices have the same label and $f^*(e) = 1$ if they have different labels). Let v_0 and v_1 be the numbers of vertices labeled by 0 and 1 respectively, and let e_0 and e_1 be the corresponding numbers of edge. Such a labeling is called cordial if both $|v_0 - v_1| \leq 1$ and $|e_0 - e_1| \leq 1$. A graph is called cordial if it has a cordial labeling [2-8].

The corona $G_1 \bigodot G_2$ of two graphs G_1(with n_1 vertices and m_1 edges) and G_2 (with n_2 vertices and m_2 edges) is defined as the graph obtained by taking one copy of G_1 and n_1 copies of G_2, and then joining the $i-^{th}$ vertex of G_1 with an edge to every vertex in the $i-^{th}$ copy of G_2 [9]. It follows from the definition of the corona that $G_1 \odot G_2$ has $n_1 + n_1.n_2$ vertices and $m_1 + n_1.m_2 + n_1.n_2$ edges. It is easy to see that $G_1 \odot G_2$ is not isomorphic to $G_2 \odot G_1$.

The main object of this paper is to show that the corona $P_n \odot P_m$ between two paths P_n and P_m is cordial for all $n \geq 1$ and $m \geq 1$. Also, we show that except for n and m are congruent to $2(mod4)$, the corona $C_n \odot C_m$ between two cycles C_n and C_m is cordial. Finally, we prove that if $n \equiv 2(mod4)$ and m is odd, then $C_n \odot C_m$ is not cordial.

It is well known that $P_1 \odot P_m$ is isomorphic to $P_1 + P_m$ and consequently is cordial for all $m \geq 1$ (see [2,3,13]). So, in the sequel, we need only to prove the corona $P_n \odot P_m$ between two paths P_n and P_m is cordial for all $n > 1$ and all $m \geq 1$.

Throughout, all graphs in our work are finite, connected and simple.

2. Terminology and some notations.

We introduce some notations and terminology for a graph with $4r$ vertices, we let L_{4r} denote the labeling 0011 0011...0011 (repeated r-times). In most cases, we then modify this by adding symbols at one end or the other (or both); thus $010L_{4r}$ denotes the labeling 010 0011 0011...0011 of the path P_{4r+3} or the cycle C_{4r+3} when $r \geq 1$ and 010 when $r = 0$. Also, we let M_r (zero- one repeated r-times) denote the labelling 0101...01 if r is an even number and 010...010 if r is an odd number, i.e., for example, $M_7 = 0101010$ and $M_6 = 010101$. Moreover, we let 0_r denotes the labelling 0000...0000 (zero repeated r- times) and 1_r denotes the labelling 111...1111 (one repeated r- times), i.e., 0_5 is 00000, 1_5 is 11111(see[2-8]).

Throughout, the following additional notations shall be used in our new technique which is different from the ones used in previous works [2-8]. Thus our notations may be helpful in the sequel. For a given labeling of the corona $G_1 \odot G_2$, we denote v_i and e_i (for $i = 0, 1$) to represent the numbers of vertices and edges, respectively, labeled by i. Let us denote x_i and a_i to be the numbers of vertices and edges labeled by i for the graph G_1. Also, we let y_i and b_i be those for G_2, which have connected to the vertices label 0 of G_1. Likewise, $y_i^{/}$ and $b_i^{/}$ be those for G_2, which have connected to the vertices label 1 of G_1. It is easily to verify that $v_0 = x_0 + x_0.y_0 + x_1.y_0^{/}$, $v_1 = x_1 + x_0.y_1 + x_1.y_1^{/}$,

$e_0 = a_0 + x_0.b_0 + x_1.b_0' + x_0.y_0 + x_1.y_1'$ and $e_1 = a_1 + x_0.b_1 + x_1.b_1' + x_0.(x_0.y_1) + x_1.y_0'$. Thus $v_0 - v_1 = (x_0 - x_1) + x_0.(y_0 - y_1) + x_1.(y_0' - y_1')$ and $e_0 - e_1 = (a_0 - a_1) + x_0.(b_0 - b_1) + x_1.(b_0' - b_1') + x_0.(y_0 - y_1) - x_1.(y_0' - y_1')$. In particular, if we have only one labeling for copies of G_2, .i.e., $y_i = y_i'$ and $b_i = b_i'$, then $v_0 = x_0 + n_1.y_0$, $v_1 = x_1 + n_1.y_1$, $e_0 = a_0 + n_1.b_0 + x_0.y_0 + x_1.y_1$ and $e_1 = a_1 + n_1.b_1 + x_0.y_1 + x_1.y_0$. Thus $v_0 - v_1 = (x_0 - x_1) + n_1.(y_0 - y_1)$ and $e_0 - e_1 = (a_0 - a_1) + n_1.(b_0 - b_1) + (x_0 - x_1).(y_0 - y_1)$, where n_1 is the order of G_1. When it comes to the proof, we need only to show that, for each specified combination of labelings, $|v_0 - v_1| \leq 1$ and $|e_0 - e_1| \leq 1$.

Finally, we use the specific labeling $[A : B_1, B_2, B_3, ..., B_n]$ of the corona $G_1 \odot G_2$, where A is the labeling of the vertices of G_1 of order n and B_i, $1 \leq i \leq n$ is the labeling of the vertices of the copy of G_2, which is connected to the $i-^{th}$ vertex of G_1.

3. The corona between two paths.

In this section, we show that the corona $P_n \odot P_m$ between two paths P_n and P_m is cordial for all $n \geq 1$ and $m \geq 1$.

Lemma 3.1. If $m \equiv 0(mod4)$, then the corona $P_n \odot P_m$ between two paths P_n and P_m is cordial for all $n \geq 1$.

Proof. Let $m = 4s$, where $s \geq 1$, then we label the vertices of all n copies of P_m as $B_0 = L_{4s}$, i.e., $y_0 = 2s$, $y_1 = 2s$, $b_0 = 2s$ and $b_1 = 2s - 1$. Let $n = 4r + i$, where $r \geq 0$ and $i = 0, 1, 2, 3$, then for given values of i with $0 \leq i \leq 3$, we may use the labeling A_i for P_n as shown in Table 3.1. Using Table 3.1 and the fact that $v_0 - v_1 = (x_0 - x_1) + n.(y_0 - y_1)$ and $e_0 - e_1 = (a_0 - a_1) + n.(b_0 - b_1) + (x_0 - x_1).(y_0 - y_1)$, we can compute the values appeared in the last two columns of Table 3.2. Since all of these are 0 or 1, the lemma follows.

$n = 4r + i$, i= 0,1,2,3	Labeling of P_n	x_0	x_1	a_0	a_1
$i = 0$	$A_0 = M_{4r}$, r≥ 1	$2r$	$2r$	0	$4r - 1$
$i = 1$	$A_1 = M_{4r}0, r \geq 0$	$2r + 1$	$2r$	0	$4r$
$i = 2$	$A_2 = M_{4r}01, r \geq 0$	$2r + 1$	$2r + 1$	0	$4r + 1$
$i = 3$	$A_3 = M_{4r}010, r \geq 0$	$2r + 2$	$2r + 1$	0	$4r + 2$

Table 3.1. Labelings of P_n

$n=4r+i$, $i=0,1,2,3$	$m=4s+j$, $j=0$	P_n	P_m	v_0-v_1	e_0-e_1
0	0	A_0	B_0	0	1
1	0	A_1	B_0	1	1
2	0	A_2	B_0	0	1
3	0	A_3	B_0	1	1

Table 3.2.Combinations of labelings.

Lemma 3.2. If m is not congruent to $0(mod4)$, then the corona $P_n \odot P_m$ between two paths P_n and P_m is cordial for all $n \geq 1$ and $m > 4$.

Proof. Let $n = 4r+i$ (for $i = 0,1,2,3$ and $r \geq 0$) and $m = 4s+j$ (for $j = 1,2,3$ and $s > 1$), then for given values of i with $0 \leq i \leq 3$, we use the labeling A_i for P_n as given in Table 3.3. For given values of j with $1 \leq j \leq 3$, we may use the following labelings B_j and B_j' for all the n copies of P_m, where B_i is the labelings of all copies of P_m connected to the vertices of P_n, which are labeled 0 in A_i, and B_i' is the labelings of all copies of P_m connected to the vertices of P_n, which are labeled 1 in A_i as given in Table 3.3. Using Table 3.3 and the fact that $v_0 - v_1 = (x_0 - x_1) + x_0.(y_0 - y_1) + x_1.(y_0' - y_1')$ and $e_0 - e_1 = (a_0 - a_1) + x_0.(b_0 - b_1) + x_1.(b_0' - b_1') + x_0.(y_0 - y_1) - x_1.(y_0' - y_1')$, we can compute the values shown in the last two columns of Table 3.4. Since all of these are -1, 0 or 1, the lemma follows.

$n=4r+i$, i= 0,1,2,3	Labeling of P_n	x_0	x_1	a_0	a_1
$i=0$	$A_0 = L_{4r}$, r≥ 1	$2r$	$2r$	$2r$	$2r-1$
$i=1$	$A_1 = L_{4r}0, r \geq 0$	$2r+1$	$2r$	$2r$	$2r$
$i=2$	$A_2 = L_{4r}01, r \geq 0$	$2r+1$	$2r+1$	$2r$	$2r+1$
$i=3$	$A_3 = L_{4r}001, r \geq 0$	$2r+2$	$2r+1$	$2r+1$	$2r+1$

$m=4s+j$, j= 1,2,3	Labeling of P_m	y_0	y_1	b_0	b_1
$j=1$	$B_1 = L_{4s}1$	$2s$	$2s+1$	$2s+1$	$2s-1$
$j=2$	$B_2 = L_{4s}10$	$2s+1$	$2s+1$	$2s+1$	$2s$
$j=3$	$B_3 = L_{4s}110$	$2s+1$	$2s+2$	$2s+2$	$2s$

$m = 4s + j$, j= 1,2,3	Labeling of P_m	y'_0	y'_1	b'_0	b'_1
$j = 1$	$B'_1 = L_{4s}0$	$2s+1$	$2s$	$2s$	$2s$
$j = 2$	$B'_2 = L_{4s}01$	$2s+1$	$2s+1$	$2s$	$2s+1$
$j = 3$	$B'_3 = L_{4s}001$	$2s+2$	$2s+1$	$2s+1$	$2s+1$

Table 3.3. Labelings of P_n and P_m.

$n = 4r + i$, $i = 0, 1, 2, 3$	$m = 4s + j$, $j = 1, 2, 3$	P_n	P_m	$v_0 - v_1$	$e_0 - e_1$
0	1	A_0	B_1, B'_1	0	1
0	2	A_0	B_2, B'_2	0	1
0	3	A_0	B_3, B'_3	0	1
1	1	A_1	B_1, B'_1	0	1
1	2	A_1	B_2, B'_2	1	1
1	3	A_1	B_3, B'_3	0	1
2	1	A_2	B_1, B'_1	0	1
2	2	A_2	B_2, B'_2	0	-1
2	3	A_2	B_3, B'_3	0	-1
3	1	A_3	B_1, B'_1	0	1
3	2	A_3	B_3, B'_3	1	1
3	3	A_3	B_3, B'_3	0	1

Table 3.4.Combinations of labelings.

Lemma 3.3. The corona $P_n \odot P_m$ between two paths P_n and P_m is cordial for all $n \geq 1$ and $m \leq 3$.

Proof. We have two cases for n:

Case 1. $1 \leq n \leq 3$. For $n = 1$, the lemma is obviously true. Now for $2 \leq n \leq 3$. Appropriate labelings are the following:

$P_2 \odot P_1 : [00 : 1, 1]$,
$P_2 \odot P_2 : [00 : 01, 11]$,
$P_2 \odot P_3 : [00 : 011, 011]$,
$P_3 \odot P_1 : [000 : 1, 1, 1]$,
$P_3 \odot P_2 : [000 : 01, 11, 11]$ and
$P_3 \odot P_3 : [000 : 011, 011, 011]$.

Case 2. $n \geq 4$. Suppose that $n = 4r + i$, where $i = 0, 1, 2, 3$ and $r \geq 1$. We have three cases for m.

Subcase 2.1. $m = 1$. The following labelings suffice:

$P_{4r} \odot P_1 : [L_{4r} : 0, 0, 0, ..., 0, 1]$,
$P_{4r+1} \odot P_1 : [L_{4r}0 : 1, 0, 1, 0, ..., 1, 0]$,
$P_{4r+2} \odot P_1 : [L_{4r}10 : 0, 1, 0, ..., 1, 0, 1]$ and
$P_{4r+3} \odot P_1 : [L_{4r}001 : 1, 0, 1, 0, ..., 1, 0, 1]$.

<u>**Subcase 2.2.**</u> $m = 2$. The following labelings suffice:
$P_{4r} \bigodot P_2 : [M_{4r} : 00, 11, 00, ...(2r - times), ...$
$00, 11, 01, 01, 01, ...(2r - times)..., 01, 01, 01]$,
$P_{4r+1} \bigodot P_2 : [M_{4r}0 : 00, 11, 00, ...(2r - times), ...00, 11, 01, 01, 01, ...((2r + 1) - times)..., 01, 01, 01]$,
$P_{4r+2} \bigodot P_2 : [L_{4r}10 : 00, 11, 00, ...(2r - times), ...00, 11, 01, 01, 01, ...((2r + 2) - times)..., 01, 01, 01]$ and
$P_{4r+3} \bigodot P_2 : [L_{4r}100 : 00, 11, 00, ...(2r - times), ...$
$00, 11, 01, 01, 01, ...((2r + 3) - times)..., 01, 01, 01]$.
<u>**Subcase 2.3.**</u> $m = 3$. The following labelings suffice:
$P_{4r} \bigodot P_3 : [M_{4r} : 001, 110, 001, ...(4r - times), ..., 011, 110]$,
$P_{4r+1} \bigodot P_3 : [M_{4r}0 : 001, 110, 001, ...(4r - times), ..., 110, 110]$,
$P_{4r+2} \bigodot P_3 : [L_{4r}10 : 001, 110, 001, ...(4r - times), ..., 110, 001, 110]$ and
$P_{4r+3} \bigodot P_3 : [L_{4r}100 : 001, 110, 001, ...(4r - times), ..., 110, 001, 110, 110]$,
the lemma thus proved.

The following theorem can now be established.
Theorem 3.1. The corona $P_n \bigodot P_m$ between two paths P_n and P_m is cordial for all $n \geq 1$ and $m \leq 1$.
Proof. The proof follows directly from the forgoing Lemmas 3.1 – 3.3.

4. The corona between two cycles.

In this section we show that except for n and m are congruent to $2(mod4)$, the corona $C_n \bigodot C_m$ between two cycles C_n and C_m is cordial. Moreover if $n \equiv 2(mod4)$ and m is odd, then $C_n \bigodot C_m$ is not cordial. Let us start to prove the following:
Lemma 4.1. If $m \equiv 0(mod4)$, then the corona $C_n \bigodot C_m$ between two cycles C_n and C_m is cordial for all $n \geq 3$ except for $n \equiv 2(mod4)$.
Proof. Let $m = 4s$, where $s \geq 1$, then we label the vertices of all n copies of C_m as $B_0 = L_{4s}$, i.e., $y_0 = 2s$, $y_1 = 2s$, $b_0 = 2s$ and $b_1 = 2s$. Let $n = 4r + i$, where $r \geq 0$ and $i = 0, 1, 3$, then for given values of i, we may use the labeling A_i for C_n as given in Table 4.1. Using Table 4.1 and the fact that $v_0 - v_1 = (x_0 - x_1) + n.(y_0 - y_1)$ and $e_0 - e_1 = (a_0 - a_1) + n.(b_0 - b_1) + (x_0 - x_1).(y_0 - y_1)$, we can compute the values shown in the last two columns of Table 4.2. Since all of these are -1, 0 or 1, the lemma follows.

$n = 4r + i$, i= 0,1,3	Labeling of C_n	x_0	x_1	a_0	a_1
$i = 0$	$A_0 = L_{4r}$, r≥ 1	$2r$	$2r$	$2r$	$2r$
$i = 1$	$A_1 = L_{4r}0, r \geq 1$	$2r + 1$	$2r$	$2r + 1$	$2r$
$i = 3$	$A_3 = L_{4r}001, r \geq 0$	$2r + 2$	$2r + 1$	$2r + 1$	$2r + 2$

Table 4.1. Labelings of C_n

$n = 4r + i$, $i = 0, 1, 3$	$m = 4s + j$, $j = 0$	C_n	C_m	$v_0 - v_1$	$e_0 - e_1$
0	0	A_0	B_0	0	0
1	0	A_1	B_0	1	1
3	0	A_3	B_0	1	-1

Table 4.2.Combinations of labelings.

Lemma 4.2. If m is not congruent to $0(mod4)$, then the corona $C_n \bigodot C_m$ between two cycles C_n and C_m is cordial for all $m > 3$ and $n \geq 3$ except for $n \equiv 2(mod4)$.

Proof. Let $n = 4r + i$ (for $i = 0, 1, 3$ and $r \geq 1$) and $m = 4s + j$ (for $j = 1, 2, 3$ and $s \geq 1$), then for given values of i with $i = 0, 1, 3$, we may use the labeling A_i, or A_i' for C_n as given in Table 4.3. For given values of j with $1 \leq j \leq 3$, we use the following labelings B_j and B_j' for all the n copies of C_m, where B_i is the labelings of all copies of C_m connected to the vertices of C_n, which are labeled 0 in A_i or A_i', and B_i' is the labelings of all copies of C_m connected to the vertices of C_n, which are labeled 1 in A_i or A_i' as given in Table 4.3. Using Table 4.3 and the fact that $v_0 - v_1 = (x_0 - x_1) + x_0.(y_0 - y_1) + x_1.(y_0' - y_1')$ and $e_0 - e_1 = (a_0 - a_1) + x_0.(b_0 - b_1) + x_1.(b_0' - b_1') + x_0.(y_0 - y_1) - x_1.(y_0' - y_1')$, we can compute the values shown in the last two columns of Table 4.4. Since all these are all -1, 0 or 1, the lemma follows.

$n = 4r + i$, i= 0,1,3	Labeling of C_n	x_0	x_1	a_0	a_1
$i = 0$	$A_0 = L_{4r}$, r≥ 1	$2r$	$2r$	$2r$	$2r$
$i = 1$	$A_1 = L_{4r}0, r \geq 1$	$2r + 1$	$2r$	$2r + 1$	$2r$
	$A_1' = L_{4r-4}M_5$, r≥ 1	$2r + 1$	$2r$	$2r - 1$	$2r + 2$
$i = 3$	$A_3 = L_{4r}001$,r≥ 0	$2r + 2$	$2r + 1$	$2r + 1$	$2r + 2$

$m = 4s+j$, j= 1,2,3	Labeling of C_m	y_0	y_1	b_0	b_1
$j=1$	$B_1 = L_{4s}1$, s≥ 1	$2s$	$2s+1$	$2s+1$	$2s$
$j=2$	$B_2 = L_{4s}10, s \geq 1$	$2s+1$	$2s+1$	$2s+2$	$2s$
$j=3$	$B_3 = L_{4s}110, s \geq 1$	$2s+1$	$2s+2$	$2s+3$	$2s$

$m = 4s+j$, j= 1,2,3	Labeling of C_m	y'_0	y'_1	b'_0	b'_1
$j=1$	$B'_1 = L_{4s}0$, s≥ 1	$2s+1$	$2s$	$2s+1$	$2s$
$j=2$	$B'_2 = L_{4s}01, s \geq 1$	$2s+1$	$2s+1$	$2s$	$2s+2$
$j=3$	$B'_3 = L_{4s}001, s \geq 1$	$2s+2$	$2s+1$	$2s+1$	$2s+2$

Table 4.3. Labelings of C_n and C_m.

$n = 4r+i$, $i = 0,1,2,3$	$m = 4s+j$, $j = 1,2,3$	C_n	C_m	$v_0 - v_1$	$e_0 - e_1$
0	1	A_0	B_1, B'_1	0	0
0	2	A_0	B_2, B'_2	0	0
0	3	A_0	B_3, B'_3	0	0
1	1	A_1	B_1, B'_1	0	1
1	2	A'_1	B_2, B'_2	1	-1
1	3	A'_1	B_3, B'_3	0	-1
3	1	A_3	B_1, B'_1	0	-1
3	2	A_3	B_3, B'_3	1	1
3	3	A_3	B_3, B'_3	0	1

Table 4.4.Combinations of labelings.

Lemma 4.3. Except for $n \equiv 2(mod4)$, the corona $C_n \odot C_3$ between two cycles C_n and C_3 is cordial for all $n \geq 3$.

Proof. We have two cases for n.

Case 1. $n = 3$. The following labelings is suffice.

$C_3 \odot C_3 : [001 : 111; 110; 100]$.

Case 2. $n > 3$. Suppose that $n = 4r + i$, where $i = 0, 1, 3$ and $r \geq 1$. Then the following labelings are the following.

$C_{4r} \odot C_3 : [L_{4r} : 001, 001, 110, 110, ...(4r - times)..., 001, 001, 110, 110]$,
$C_{4r+1} \odot C_3 : [L_{4r}0 : 001, 001, 110, 110, ...(4r{-}times)..., 001, 001, 110, 110, 110]$
and $C_{4r+3} \odot C_3 : [L_{4r}100 : 001, 001, 110, 110, ...(4r - times)..., 001, 001,$ $110, 110, 110, 001, 110]$, the lemma follows.

Now we can establish the following:

Theorem 4.1. Except for $n \equiv 2(mod4)$, the corona $C_n \odot C_m$ between

two cycles C_n and C_m is cordial for all $n \geq 3$ and $m \geq 3$.
Proof. The proof follows from Lemma 4.1, Lemma 4.2 and Lemma 4.3.

In this part, we discuss the cordiality of the corona $C_n \odot C_m$ between two cycles C_n and C_m when $n \equiv 2(mod4)$. Seoud and Abdel Maqusoud [14] proved that if G is a graph with n vertices and m edges, and every vertex has odd degree then G is not cordial when $n+m \equiv 2(mod4)$. So, we can establish the following lemma.

Lemma 4.4. If $n \equiv 2(mod4)$ and m is odd, then the corona $C_n \odot C_m$ between two cycles C_n and C_m is not cordial.

Proof. Suppose that $n = 4r+2$, where $r \geq 1$ and $m = 2s+1$. Then it is easy to verify that every vertex of the corona $C_n \odot C_m$ has odd degree, the order of the corona $C_n \odot C_m$ is $8rs + 8r + 4s + 4$ and its size is $16rs + 12r = 8s + 6$. Therefor according to Seoud and Abdel Maqusoud theorem mentioned above we conclude that the corona $C_n \odot C_m$ between two cycles C_n and C_m is not cordial, the lemma follows.

Lemma 4.5. If $n \equiv 2(mod4)$ and $m \equiv 0(mod4)$, then the corona $C_n \odot C_m$ between two cycles C_n and C_m is cordial.

Proof. Let $n = 4r+2$, where $r \geq 1$ and $m = 4s$, where $s \geq 1$. Then we label the vertices of C_n as $L_{4r}11$, i.e., $x_0 = 2r$, $x_1 = 2r+2$, $a_0 = 2r+2$ and $a_1 = 2r$. Now, we label the n copies of C_m as the following. We label the vertices of the first copy of C_m which connected to the first vertex of C_n which are labeled zero as $0_{2s+1}1_{2s-1}$, i.e., $y_0 = 2s+1$, $y_1 = 2s-1$, $b_0 = 4s-2$ and $b_1 = 2$, we label the vertices of the second copy of C_m which connected to the second vertex of C_n which are labeled zero as M_{4s}, i.e., $y_0^/ = 2s$, $y_1^/ = 2s$, $b_0^/ = 0$ and $b_1^/ = 4s$, and we label the vertices of the other $n-2$ copies of C_m which connected to the other $4r$ vertices of C_n as L_{4s}, i.e., $y_0^{//} = 2s$, $y_1^{//} = 2s$, $b_0^{//} = 2s$ and $b_1^{//} = 2s$. Therefore $v_0 = x_0 + 1.y_0 + 1.y_0^/ + 4r.y_0^{//} = 8rs + 4s + 2r + 1$, $v_1 = x_1 + 1.y_1 + 1.y_1^/ + 4r.y_1^{//} = 8rs + 4s + 2r + 1$, $e_0 = a_0 + 1.b_0 + 1.b_0^/ + 4r.b_0^{//} + 1.y_0 + 1.y_0^/ + (x_0 - 2).y_0^{//} + x_1.y_1^{//} = 16rs + 2r + 8s + 1$ and $e_1 = a_1 + 1.b_1 + 1.b_1^/ + 4r.b_1^{//} + 1.y_1 + 1.y_1^/ + (x_0 - 2).y_1^{//} + x_1.y_0^{//} = 16rs + 2r + 8s + 1$. Then $v_0 = v_1$ and $e_0 = e_1$, the lemma follows.

From the above results, we can establish the following:

Theorem 4.2. Except for n and m are congruent to $2(mod4)$, the corona $C_n \odot C_m$ between two cycles C_n and C_m is cordial. Moreover if $n \equiv 2(mod4)$ and m is odd, then $C_n \odot C_m$ is not cordial.

Proof. The proof follows from Theorem 4.1, Lemma 4.4 and Lemma 4.5.

References

[1] I. Cahit, On cordial and 3-equitable labelings of graphs, Utilities Math., 37 (1990).

[2] Adel T. Diab and E.A. Elsakhawi, Some Results on Cordial Graphs, Proc. Math. Phys. Soc. Egypt, No.77, pp.67-87(2002).

[3] Adel T. Diab, Study of Some Problems of Cordial Graphs, ARS Combinatoria 92, pp.255-261(2009).

[4] Adel T. Diab, On Cordial Labelings of the Second Power of Paths with Other Graphs, ARS Combinatoria 97A, pp.327-343(2010).

[5] Adel T. Diab, Generalization of Some Results on Cordial Graphs, ARS Combinatoria 99, pp.161-173(2009).

[6] Adel T. Diab, On Cordial Labelings of Wheels with Other Graphs, ARS Combinatoria 100, pp.265-279 (2011).

[7] Adel T. Diab and Sayed Anwer Elsaid Mohammed, On Cordial Labelings of Fans with Other Graphs, accepted for publication in ARS Combinatoria.

[8] Adel T. Diab, On Cordial Labelings of the Second Power of Cycles with Other Graphs, accepted for publication in Utilitas Mathematica.

[9] J. A. Gallian, A dynamic survey of graph labeling, The Electronic Journal of Combinatorics,17, (November 13, 2010)DS6.

[10] S. W. Golomb, How to number a graph in Graph Theory and Computing,R.C. Read, ed., Academic Press, New York (1972) 23-37.

[11] R. L. Graham and N.J.A. Sloane, On additive bases and harmonious graphs, SIAM J. Alg. Discrete Math.,1(1980) 382-404.

[12] A. Rosa, On certain valuations of the vertices of a graph, Theory of Graphs(Internat. Symposium, Rome, July 1966),Gordon and Breach, N.Y.and Dunod Paris (1967) 349-355.

[13] M. A. Seoud, Adel T. Diab and E.A. Elsakhawi, On Strongly C-Harmonious, Relatively Prime, Odd Graceful and Cordial Graphs, Proc.Math.Phys.Soc.Egypt, No.73, pp.33-55(1998).

[14] M. A. Seoud and Abdel Maqusoud, J. Egypt. Math. Soc.,Vol.7(1), pp.127-135 (1999).

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