COMMON FIXED POINTS IN CONE METRIC SPACES FOR MK-PAIRS AND L-PAIRS

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ABSTRACT. In this paper we introduce some contractive conditions of Meir-Keeler type for a pair of mappings, called *MK-pair* and *L-pair*, in the framework of cone metric spaces and we prove theorems which assure existence and uniqueness of common fixed points for *MK-pairs* and *L-pairs*. As an application we obtain a result of common fixed point of a p-MK-pair, a mapping and a multifunction, in complete cone metric spaces. These results extend and generalize well-known comparable results in the literature.

1. INTRODUCTION AND PRELIMINARIES.

The study of common fixed points of mappings satisfying certain contractive conditions has been at the centre of strong research activity, being the applications of fixed point very important in several areas of mathematics. Common fixed point results have been obtained for commuting mappings by Jungck [9], for weakly commuting mappings by Sessa [20], for compatible mappings by Jungck [10], for R-weakly commuting mappings by Pant [17]. These results require the continuity of one of the two maps involved. Recently Jungck [11, 13] defined a pair of self mappings to be weakly compatible if they commute at their coincidence points. In recent years, several authors have obtained coincidence point results for various classes of mappings on a metric space, using these concepts. By using the notion of weakly uniformly strict p-contraction Cardinali and Rubbioni [4] extended the Meir-Keeler fixed point theorem to multifunctions.

Huang and Zhang [7] generalized the concept of a metric space, replacing the set of real numbers by an ordered Banach space and obtained some fixed point theorems for mappings satisfying different contractive conditions. Vetro [21], in the framework of cone metric spaces, introduced a generalized contractive condition and proved a common fixed point theorem for a pair of weakly compatible mappings. This theorem generalizes

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some results of Huang and Zhang [7]. Further to this, Abbas and Jungck [1] proved common fixed point theorems for a pair of weakly compatible mappings. Recently, Rezapour and Hamlbarani [19] proved that there are no normal cones with normal constant $c < 1$ and for each $k > 1$ there are cones with normal constant $c > k$. Then, omitting the assumption of normality they obtained generalizations of some results of [7]. Following these results, recently a lot of papers have been dedicated to show that results of fixed point or common fixed point known in the setting of metric spaces hold in the framework of cone metric spaces. For a survey of fixed point theory and related results in cone metric spaces, we refer to [2, 3, 5, 6, 8, 12, 13, 14, 18, 22] and the references therein.

The purpose of this paper is to present common fixed points for mappings which satisfy contractive conditions of Meir-Keeler type. We obtain also a coincidence point result.

We recall the definition of cone metric spaces and some of their properties [7].

Let B be a real Banach space and P be a subset of B. By θ we denote the zero element of B and by Int$P$ the interior of P. The subset P is called an *order cone* if and only if:

(i) P is closed, nonempty, and $P \neq \{\theta\}$;

(ii) $a, b \in \mathbb{R}$, $a, b \geq 0$, $x, y \in P \Rightarrow ax + by \in P$;

(iii) $x, -x \in P \Rightarrow x = \theta$.

On this basis, we define a partial ordering $\leq$ with respect to P by $x \leq y$ if and only if $y - x \in P$. We write $x < y$ to indicate that $x \leq y$ but $x \neq y$, while $x \ll y$ if and only if $y - x \in \text{Int}P$. For the symbol $\ll$, the following hold:

(j) $\theta < \varepsilon$ and $\theta \ll \delta$ imply $\theta \ll \varepsilon + \delta$;

(jj) if $\theta \ll \varepsilon$ and $\theta \ll \delta$, then there exists a positive real number $\lambda < 1$ such that $\lambda\delta \ll \varepsilon$.

The order cone P is called *normal* if there is a number $c > 0$ such that for all $x, y \in B$, $\theta \leq x \leq y$ implies $\|x\| \leq c\|y\|$. The least positive number satisfying the above inequality is called the normal constant of P. The order cone P is called *regular* if every nondecreasing sequence which is order bounded from above is convergent, that is, if $\{x_n\} \subset B$ is a sequence such that

$$x_1 \leq x_2 \leq \cdots \leq x_n \leq \cdots \leq y$$

for some $y \in B$, then there is an $x \in B$ such that $\|x_n - x\| \to 0$ $(n \to +\infty)$. We observe that a regular order cone is a normal order cone (Lemma 1.1 of [19]).

In the following we always suppose that B is a Banach space, P is an order cone in B with Int$P \neq \emptyset$ and $\leq$ is the partial ordering with respect to P.

Definition 1. Let X be a nonempty set. Suppose the mapping $d : X \times X \to B$ satisfies

(i) $\theta < d(x,y)$ for all $x, y \in X$ with $x \neq y$ and $d(x,y) = \theta$ if $x = y$;

(ii) $d(x,y) = d(y,x)$ for all $x, y \in X$;

(iii) $d(x,y) \leq d(x,z) + d(y,z)$ for all $x, y, z \in X$.

Then d is called a cone metric on X and (X,d) is called a cone metric space.

It is obvious that cone metric spaces generalize metric spaces.

Example 1. Let $B = \mathbb{R}^2$, $P = \{(x,y) \in B : x, y \geq 0\} \subset \mathbb{R}^2$, $X = \mathbb{R}$ and $d : X \times X \to B$ such that $d(x,y) = (|x-y|, \alpha|x-y|)$, where $\alpha \geq 0$ is a constant. Then (X,d) is a cone metric space.

Definition 2. Let (X,d) be a cone metric space and $\{x_n\}$ be a sequence in X. We say that:

(i) $\{x_n\}$ converges to $x \in X$ if for every $c \in B$ with $\theta \ll c$ there is an N such that for all $n \geq N$, $d(x_n, x) \ll c$. We denote this by $\lim_{n \to +\infty} x_n = x$ or $x_n \to x$ $(n \to +\infty)$;

(ii) if for any $c \in B$ with $\theta \ll c$, there is an N such that for all $n, m \geq N$, $d(x_n, x_m) \ll c$, then $\{x_n\}$ is called a Cauchy sequence in X.

Let (X,d) be a cone metric space, if every Cauchy sequence is convergent in X, then X is called a complete cone metric space.

Lemma 1. ([7], Lemmas 1 and 4). *Let (X,d) be a cone metric space, P be a normal order cone. Let $\{x_n\}$ be a sequence in X. Then*

(i) $\{x_n\}$ converges to x if and only if $d(x_n, x) \to \theta$ $(n \to +\infty)$;

(ii) $\{x_n\}$ is a Cauchy sequence if and only if $d(x_n, x_m) \to \theta$ $(n, m \to +\infty)$.

Definition 3. The mappings $f, g : X \to X$ are an *MK-pair* if for every $\varepsilon > \theta$, there exists $\delta \gg \theta$ such that

$$d(g(x), g(y)) < \varepsilon + \delta \quad \text{implies} \quad d(f(x), f(y)) < \varepsilon$$

for all $x, y \in X$.

Definition 4. Let P be an order cone. A map $\varphi : P \to P$ is called an *L-map* if $\varphi(\theta) = \theta$, $\theta < \varphi(\omega)$ for $\omega \in P \setminus \{\theta\}$, and for every $\omega \in P \setminus \{\theta\}$ there exists $\delta \gg \theta$ such that $\varphi(t) \leq \omega$ for all $t \in [\omega, \omega + \delta]$, where $[\omega, \omega + \delta] = \{t \in P : \omega \leq t \leq \omega + \delta\}$.

The notion of L-map is an extension of L-function [15] for cone metric spaces.

Definition 5. The mappings $f, g : X \to X$ are an *L-pair* if there exists an L-map φ such that for all $x, y \in X$

(i) $\theta < d(g(x), g(y))$ implies $d(f(x), f(y)) < \varphi(d(g(x), g(y)))$;
(ii) $g(x) = g(y)$ implies $f(x) = f(y)$.

Remark 1. If the mappings $f, g : X \to X$ are an *L-pair*, then they are also an *MK-pair*.

2. Common fixed point theorems for MK-pairs and L-pairs.

In this section we prove some results on common fixed points for *MK-pairs* and *L-pairs* in cone metric spaces. We also obtain a coincidence point result. Let (X, d) be a cone metric space and let P be an order cone. Let $f, g : X \to X$ be mappings with $f(X) \subset g(X)$. Let $x_0 \in X$ be arbitrary. Choose $x_1 \in X$ such that $f(x_0) = g(x_1)$. This can be done, since $f(X) \subset g(X)$. Continuing this process, having chosen $x_n \in X$, we choose $x_{n+1} \in X$ such that $g(x_{n+1}) = f(x_n)$ for all $n \in \mathbb{N}$. $\{f(x_n)\}$ is called an f-g-sequence with initial point x_0.

Definition 6. The mappings $f, g : X \to X$ are weakly compatible if f and g commute at their coincidence point (i.e., $f(g(x)) = g(f(x))$ whenever $f(x) = g(x)$). A point $y \in X$ is called point of coincidence of two self-mappings f and g on X if there exists a point $x \in X$ such that $y = g(x) = f(x)$.

Theorem 1. *Let (X, d) be a cone metric space, P be a regular order cone and let $f, g : X \to X$ be an MK-pair such that $f(X) \subset g(X)$. Suppose that f and g are weakly compatible, and that $f(X)$ or $g(X)$ is complete. Then the mappings f and g have a unique common fixed point in X. Moreover for any $x_0 \in X$, the f-g-sequence $\{f(x_n)\}$ with initial point x_0 converges to the common fixed point.*

Proof. We note that the hypothesis that f, g are an *MK-pair* implies:
(j) $d(f(x), f(y)) \leq d(g(x), g(y))$ for all $x, y \in X$;
(jj) $d(f(x), f(y)) < d(g(x), g(y))$ for every $x, y \in X$ such that $\theta < d(g(x), g(y))$.

We fix $x_0 \in X$ and we prove that every f-g-sequence $\{f(x_n)\}$ of initial point x_0 is a Cauchy sequence in $f(X)$. If $f(x_n) = f(x_{n-1})$ for some $n \in \mathbb{N}$, then $f(x_m) = f(x_n)$ for all $m \in \mathbb{N}$ with $m > n$ and so $\{f(x_n)\}$ is a Cauchy sequence. We suppose that $f(x_n) \neq f(x_{n-1})$ for all $n \in \mathbb{N}$. By (jj), we deduce

$$d(f(x_{n+1}), f(x_n)) < d(g(x_{n+1}), g(x_n)) = d(f(x_n), f(x_{n-1})),$$

and so the sequence $\{d(f(x_{n+1}), f(x_n))\}$ is decreasing.

Consequently, there exists $\omega \in P$ such that

$$\omega = \lim_{n \to +\infty} d(f(x_{n+1}), f(x_n)).$$

We show that $\omega = \theta$. Assume $\omega > \theta$ and let $\delta \gg \theta$ as in Definition 3. We choose $N \in \mathbb{N}$ such that for all $n \geq N$, $d(f(x_n), f(x_{n-1})) \ll \omega + \delta$. If $n \geq N$, from $d(g(x_n), g(x_{n+1})) = d(f(x_n), f(x_{n-1})) \ll \omega + \delta$, we obtain

$$\omega \leq d(f(x_n), f(x_{n+1})) < \omega,$$

which gives $\omega = \theta$.

Fix $\theta \ll \varepsilon$ and let $\theta \ll \delta$ be as in Definition 3, it is not restrictive to suppose that $\delta \ll \varepsilon$. Now we choose N such that for every $m \geq N$ we have $d(f(x_m), f(x_{m+1})) \ll \delta$. Fix $m \geq N$. We prove that

$$d(f(x_m), f(x_{n+1})) \ll \varepsilon + \delta \ll 2\varepsilon \tag{1}$$

for all $n \geq m$. We note that (1) holds when $n = m$. We assume that (1) holds for some $n \geq m$. Thus, we have

$$\begin{aligned} d(g(x_{m+1}), g(x_{n+2})) &= d(f(x_m), f(x_{n+1})) \\ &\ll \varepsilon + \delta, \end{aligned}$$

which implies $d(f(x_{m+1}), f(x_{n+2})) < \varepsilon$.

We deduce

$$\begin{aligned} d(f(x_m), f(x_{n+2})) &\leq d(f(x_m), f(x_{m+1})) + d(f(x_{m+1}), f(x_{n+2})) \\ &\ll \varepsilon + \delta. \end{aligned}$$

Therefore, (1) holds when $n := n + 1$. By induction, we deduce (1) holds for all $n \geq m$. Hence $\{f(x_n)\}$ is a Cauchy sequence. Suppose that $f(X)$ is a complete subspace of X, then there exists $y \in f(X) \subset g(X)$ such that $f(x_n) \to y$ and also $g(x_n) \to y$. (This holds also if $g(X)$ is complete with $y \in g(X)$.) Let $z \in X$ be such that $g(z) = y$. We show that $f(z) = g(z)$. From

$$\theta \leq d(f(x_n), f(z)) \leq d(g(x_n), g(z)),$$

by Lemma 5 of [7], we obtain

$$\begin{aligned} \theta \leq d(y, f(z)) &= \lim_{n\to+\infty} d(f(x_n), f(z)) \\ &\leq \lim_{n\to+\infty} d(g(x_n), g(z)) \\ &= d(y, g(z)) = \theta \end{aligned}$$

which implies $y = f(z) = g(z)$, and so y is a point of coincidence of f and g.

From $f(z) = g(z)$, being the mappings f and g weakly compatible, it follows that

$$f(y) = f(g(z)) = g(g(z)) = g(y).$$

We show that $f(y) = g(y) = y$. If $g(y) \neq y$, in virtue of (jj), we obtain

$$\begin{aligned} d(f(y), f(z)) &< d(g(y), g(z)) \\ &= d(f(y), f(z)), \end{aligned}$$

which gives $f(y) = y = g(y)$. Then y is a common fixed point for the mappings f and g. The uniqueness follows from the hypothesis that f and g are an MK-pair. □

If $B = \mathbb{R}$, $P = \mathbb{R}_+$ and $g = I_X$ the identity mapping on X, from Theorem 1 we obtain the following fixed point theorem of Meir and Keeler.

Theorem 2 (Meir and Keeler [16]). *Let (X, d) be a complete metric space and let $f : X \to X$ be a Meir-Keeler contraction, i.e., for every $\varepsilon > 0$, there exists $\delta > 0$ such that $d(x, y) < \varepsilon + \delta$ implies $d(f(x), f(y)) < \varepsilon$ for all $x, y \in X$. Then f has a unique fixed point.*

From Theorem 1 and Remark 1, we obtain the following theorem.

Theorem 3. *Let (X, d) be a cone metric space, P be a regular order cone and let $f, g : X \to X$ be an L-pair such that $f(X) \subset g(X)$. Suppose that f and g are weakly compatible. If $f(X)$ or $g(X)$ is a complete subspace of X, then the mappings f and g have a unique common fixed point in X. Moreover for any $x_0 \in X$, the f-g-sequence $\{f(x_n)\}$ of initial point x_0 converges to the common fixed point.*

In the proof of Theorems 1, we have proved also the following coincidence point result.

Theorem 4. *Let (X, d) be a cone metric space, P be a regular order cone and let $f, g : X \to X$ be an MK-pair or an L-pair such that $f(X) \subset g(X)$. Suppose $f(X)$ or $g(X)$ is a complete subspace of X, then the mappings f and g have a unique point of coincidence in X.*

From Theorems 1 and 3, if we choose $g = I_X$ the identity mapping on X, we obtain the following corollaries.

Corollary 1. *Let (X, d) be a cone metric space, P be a regular cone and let $f : X \to X$ be a mapping. Suppose that, for every $\varepsilon > \theta$, there exists $\delta \gg \theta$ such that $d(x, y) < \varepsilon + \delta$ implies $d(f(x), f(y)) < \varepsilon$ for all $x, y \in X$. If $f(X)$ or X is complete, then the mapping f has a unique fixed point in X. Moreover for any $x_0 \in X$, the sequence $\{f(x_n)\}$ of initial point x_0 converges to the fixed point.*

Corollary 1 is the extension of the Meir and Keeler fixed point theorem for regular cone metric spaces.

Corollary 2. *Let (X, d) be a cone metric space, P be a regular cone and let $f : X \to X$ be a mapping. Suppose that there exists an L-map φ such that*

$$\theta < d(x, y) \quad \textit{implies} \quad d(f(x), f(y)) < \varphi(d(x, y))$$

for all $x, y \in X$. If $f(X)$ or X is complete, then the mapping f has a unique fixed point in X. Moreover for any $x_0 \in X$, the sequence $\{f(x_n)\}$ of initial point x_0 converges to the fixed point.

Definition 7. Let (X,d) be a cone metric space, $f : X \to X$ be a mapping and $F : X \to 2^X$ be a multifunction, where 2^X is the family of nonempty subsets of X. The mapping f is a selection of F if $f(x) \in F(x)$ for every $x \in X$.

Definition 8. Let (X,d) be a cone metric space, $g : X \to X$ be a mapping and $F : X \to 2^X$ be a multifunction. F and g are called a p-MK-pair if the following property holds:

(i) there exists $p \in \mathbb{N}$ such that for every $\theta < \varepsilon$ there exists $\theta \ll \delta$ such that for each $x, y \in X$ admitting the representation:

$$\exists\, x_0, \ldots, x_{p-1} \in X \text{ with } x \in F(x_{p-1}), x_{p-1} \in F(x_{p-2}) \ldots, x_1 \in F(x_0)$$

and

$$\exists\, y_0, \ldots, y_{p-1} \in X \text{ with } y \in F(y_{p-1}), y_{p-1} \in F(y_{p-2}) \ldots, y_1 \in F(y_0),$$

then $d(x,y) < \varepsilon$ whenever $d(g(x_0), g(y_0)) < \varepsilon + \delta$.

Definition 9. Let (X,d) be a cone metric space, $g : X \to X$ be a mapping and $F : X \to 2^X$ be a multifunction. F and g are called p-weakly compatible if for every selection f of F the mappings f^p and g are weakly compatible.

Theorem 5. *Let (X,d) be a cone metric space, P be a regular order cone, $g : X \to X$ and $F : X \to 2^X$. Assume that F and g are a p-MK-pair with $\bigcup_{x \in X} F(x) \subset g(X)$ and that F and g are p-weakly compatible. If F has a selection that commute with g at each fixed point of g and if $g(X)$ is a complete subspace of X, then the mappings F and g have a common fixed point in X.*

Proof. Let $f : X \to X$ be a selection of the multifunction F, that commute with g at each fixed point of g. We prove that f^p and g are an MK-pair. Let $\theta < \varepsilon$ and let $\theta \ll \delta$ be provided by Definition 8. We fix $x_0, y_0 \in X$ such that $d(g(x_0), g(y_0)) < \varepsilon + \delta$ and define

$$x_1 = f(x_0), \ldots, x_p = f(x_{p-1}), \quad y_1 = f(y_0), \ldots, y_p = f(y_{p-1}).$$

Since f is a selection of F, we deduce that

$$x_1 \in F(x_0), \ldots, x_p \in F(x_{p-1}), \quad y_1 \in F(y_0), \ldots, y_p \in F(y_{p-1}).$$

Being F and g a p-MK-pair, this implies

$$d(f^p(x_0), f^p(y_0)) = d(x_p, y_p) < \varepsilon.$$

Consequently f^p and g are an MK-pair. By Theorem 1 the mappings f^p and g have a unique fixed point, say y. From $f^p(f(y)) = f(y)$ and $f(y) = f(g(y)) = g(f(y))$, we deduce that $y = g(y) = f(y) \in F(y)$, that is y is a common fixed point of F and g. □

If in Theorem 5 we choose $g = I_X$ the identity mapping on X, we obtain the following corollary.

Corollary 3. *Let (X, d) be a complete cone metric space, P be a regular order cone and $F : X \to 2^X$. If F and I_X are a p-MK-pair, then the mapping F has a fixed point in X.*

Corollary 3 coincides with Theorem 3.1 of [4], if (X, d) is a complete metric space.

Example 2. Let $X = \{0\} \cup \{1/n : n \in \mathbb{N}\}$, $B = \mathbb{R}^2$ be a Banach space with the euclidean norm and let $P = \{(x, y) \in E : x, y \geq 0\}$. Let $d : X \times X \to E$ be defined by

$$d(x, y) = (|x - y|, \alpha |x - y|) \text{ for all } x, y \in X, \text{ where } \alpha \geq 0.$$

Define $F : X \to 2^X$ by

$$F(x) = \begin{cases} \{0\} \cup \{\frac{1}{2n}\} & x = \frac{1}{2n-1} \\ \{0\} & \text{otherwise.} \end{cases}$$

It is easy to verify that F and I_X are a 2-MK-pair and that F and I_X are 2-weakly compatible. Consequently, by Corollary 3, F has a fixed point.

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