The Smarandachely Adjacent Vertex Total Coloring of a kind of 3-regular Graph*

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Abstract: A proper total coloring of a graph G is called smarandachely adjacent vertex total coloring of graph if for any two adjacent and distinct vertices u and v in G, the set of colors assigned to the vertices and the edges incident to u doesn't contain the set of colors assigned to the vertices and the edges incident to v, vice versa. The minimal number of colors required for a smarandachely adjacent vertex total coloring of graph is called the smarandachely adjacent vertex total chromatic number of graph. In this paper,we define a kind of 3-regular Multilayer Cycle $Re(n, m)$ and obtain the Smarandachely adjacent vertex total chromatic number of it.

Keywords: 3-regular graph; Smarandachely adjacent vertex total coloring; Smarandachely adjacent vertex total chromatic number.
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1. Introduction

A series of new coloring educed by computer science, information science, light transmission and so on, which are very difficult problem, such as the vertex distinguishing edge coloring[1], the adjacent vertex distinguishing edge coloring[2,3,4], the adjacent vertex distinguishing total coloring[5−8], $D(\beta)$-vertex-distinguishing total colorings of graphs[9],The adjacent-vertex-strongly-distinguishing total coloring of graphs[10], Vertex-distinguishing total coloring of graphs[11] and the relation of total chromatic number with adjacent strong edge chromatic number of regular graph[12]. The counter adjacent vertex-distinguishing edge coloring of graphs is educed by light transmission.Basing on those, Zhang et-al.further proposed The Smarandachely Adjacent Vertex Total Coloring of graphs[11]. In this paper, we define three special subgraphs and a kind of Generalized three-regular ring graph, using the three special subgraphs we obtain the Smarandachely-adjacent-vertex total chromatic number of the kinds Generalized three-regular ring graph.

Definition 1.1.[10] A *proper* $k - total - coloring$ of G is a mapping from $V(G) \cup E(G)$ to $\{1, 2, \cdots, k\}$ such that any two adjacent or incident elements of $V(G) \cup E(G)$ are assigned different colors.If $\forall uv \in E(G)$, then $C(u) \neq C(v)$,we say that f is a *adjacent − vertex − distinguishing total coloring* of G, or a $k - AVDTC$ of G for short. The minimum number such that G has $k - AVDTC$ is denoted by $\chi_{at}(G)$, and it is called *adjacent − vertex − distinguishing total chromatic* number of G,where $C(u) = \{f(u)\} \bigcup \{f(uv)|uv \in E(G)\}$.

Conjecture 1.1.[10] For a simple graph G,then $\chi_{at}(G) \leq \Delta(G) + 3$.

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Definition 1.2.[14] A Smarandachely-adjacent-vertex total coloring of graph G(V,E) is a *proper* k*−total−coloring* of G and satisfying: for $uv \in E(G)$, we have $|C(u)\backslash C(v)| \geq 1$ and $|C(v)\backslash C(u)| \geq 1$, it is denoted simply by $k-SAVTC$. The minimum number such that G has $k-SAVTC$ is denoted by $\chi_{sat}(G)$ and is called *smarandachely total chromatic* number of G .Where $C(u) = \{f(u)\} \bigcup \{f(uv)|uv \in E(G)\}$.

Conjecture 1.2.[14] For connected simple graph G with order at least 2 then

$\Delta(G) + 2 \leq \chi_{sat}(G) \leq \Delta(G) + 3$. Where $\Delta(G)$ is the maximum degree of graph G.

Conjecture 1.3.[14] For a r−regular graph, then $\chi_{sat}(G) = \Delta(G)+2$, where graph

G is not odd complete graph.

Definition 1.3. Let $G(V,E)$ be a simple graph, If$V(G) = \{V(C_n^1), V(C_{2n}^2), V(C_{2n}^3),$

$\cdots, V(C_{2n}^{m-1}), V(C_n^m)\}$, where$V(C_n^1) = \{v_{1,1}, v_{1,2}, \cdots, v_{1,n}\}, V(C_{2n}^i) = \{v_{i,1}, v_{i,2},$ $\cdots v_{i,n}, v_{i,n+1}, v_{i,n+2}, \cdots, v_{i,2n}\}, i = 2,3,\cdots,m-1, V(C_n^m) = \{v_{m,1}, v_{m,2}, \cdots, v_{m,n}\}$;

$E(G) = \{\{v_{i,j}v_{i,j+1}\} \bigcup \{v_{i,n}v_{i,1}\}|i = 1, m, j = 1,2,\cdots,n.\ j+1(\ mod\, n)\} \bigcup \{\{v_{i,j}v_{i,j+1}\} \bigcup \{v_{i,2n}v_{i,1}\}|i = 2,3,\cdots,m-1, j = 1,2,\cdots,2n.\ j+1(\ mod\ 2n)\} \bigcup \{v_{1,j}v_{2,k}|k = 2j-1, j = 1,2,\cdots,n.\} \bigcup \{v_{i,j}v_{i+1,j}|i = 3,5,\cdots,m-2\ (\ m \equiv 0\ (mod\ 2))\ or\ i = 3,5,\cdots,m-3\ (\ m \equiv 1\ (mod\ 2)), j = 1,3,5,\cdots,2n-1.\} \bigcup \{v_{i,j}v_{i+1,j}|i = 2,4,6,\cdots,m-3\ (\ m \equiv 0\ (mod\ 2))\ or\ i = 2,4,6,\cdots,m-2\ (\ m \equiv 1\ (mod\ 2)), j = 2,4,6,\cdots,2n.\} \bigcup \{u_{m-1,j}u_{m,k}|j = 1,3,5,\cdots,2n-1\ (\ m \equiv 0\ (mod\ 2))\ or\ i = 2,4,6,\cdots,2n.\ (\ m \equiv 1\ (mod\ 2)), k = 1,2,3,\cdots,n\}$; The graph is called a 3-regular multi-cycle, denoted by $Re(m,n)$.

For other terminologies and notations we can refer to [15-17].

Definitions not given here may be found in [10,11,12,13].

2. Main result

Lemma 2.1.[14] Let G be a simple graph, then $\chi_{sat}(G) \geq \Delta + 2$.

Theorem 2.1. For a 3-regular graph $Re(n,2)(n \geq 3)$, then $\chi_{sat}(Re(n,2)) = 5$.

Proof. $\chi_{sat}(Re(n,m)) = \chi_{sat}(P_2 \times C_n) = 5$.

Theorem 2.2. For a kind of 3-regular graph $Re(n,m)(n \geq 3, m \geq 3)$, then

$\chi_{sat}(Re(n,m)) = 5$.

Proof. Case 1. When $n \equiv 0(\ mod\ 2)$,

$$f(u_{1,j}) = \begin{cases} 1, & j \equiv 1(\ mod\ 2); \\ 2, & j \equiv 0(\ mod\ 2). \end{cases} \quad j = 1,2,\cdots,n.$$

$$f(u_{1,j}u_{1,j+1}) = \begin{cases} 3, & j \equiv 1(\ mod\ 2); \\ 4, & j \equiv 0(\ mod\ 2). \end{cases} \quad j = 1,2,\cdots,n.(\text{ if } j+1 > n, j+1(\ mod\ n))$$

Case 1.1. When $m \equiv 1(\ mod\ 2)$,

$$f(u_{i,j}) = \begin{cases} 1 & j \equiv 3(\ mod\ 4); \\ 2, & j \equiv 1(\ mod\ 4); \\ 3, & j \equiv 0(\ mod\ 4) \text{ or } j \equiv 2(\ mod\ 4). \end{cases} \quad i = 2,4,\cdots,m-1, j = 1,2,\cdots,2n.$$

$$f(u_{i,j}) = \begin{cases} 1, & j \equiv 0(\ mod\ 4); \\ 2, & j \equiv 2(\ mod\ 4); \\ 3, & j \equiv 1(\ mod\ 4) \text{ or } j \equiv 3(\ mod\ 4). \end{cases} \quad i = 3,5,\cdots,m-2, j = 1,2,\cdots,2n.$$

$$f(u_{i,j}u_{i,j+1}) = \begin{cases} 1, & j \equiv 0(\ mod\ 4); \\ 2, & j \equiv 2(\ mod\ 4); \\ 4, & j \equiv 1(\ mod\ 4) \text{ or } j \equiv 3(\ mod\ 4). \end{cases} \quad i = 2,4,\cdots,m-1, j =$$

$1,2,\cdots,2n.(\text{ if } j+1 > 2n,\ j+1(\ mod\ 2n))$

$$f(u_{i,j}u_{i,j+1}) = \begin{cases} 1, & j \equiv 1(\ mod\ 4); \\ 2, & j \equiv 3(\ mod\ 4); \\ 4, & j \equiv 0(\ mod\ 4) \text{ or } j \equiv 2(\ mod\ 4). \end{cases} \quad i = 3,5,\cdots,m-2, j =$$

$1,2,\cdots,2n.$(if $j+1>2n, j+1(\ mod\ 2n)$)

$$f(u_{m,j})=\begin{cases}1, & j\equiv 0(\ mod\ 2);\\ 2, & j\equiv 1(\ mod\ 2).\end{cases}\quad j=1,2,\cdots,n.$$

$$f(u_{i,j}u_{m,j+1})=\begin{cases}3, & j\equiv 0(\ mod\ 2);\\ 4, & j\equiv 1(\ mod\ 2).\end{cases}\quad j=1,2,\cdots,n.(\text{ if } j+1>n, j+1(\ mod\ n))$$

All other edges are colored by 5, where

$$C(u_{1,j})=C(u_{m,j})=\begin{cases}\{1,3,4,5\}, & j\equiv 1(\ mod\ 2);\\ \{2,3,4,5\}, & j\equiv 0(\ mod\ 2).\end{cases}\quad j=1,2,\cdots,n.$$

$$C(u_{i,j})=\begin{cases}\{1,3,4,5\}, & j\equiv 0(\ mod\ 4);\\ \{2,3,4,5\}, & j\equiv 2(\ mod\ 4);\\ \{1,2,4,5\}, & j\equiv 1(\ mod\ 4)\text{ or } j\equiv 3(\ mod\ 4).\end{cases}\quad i=2,4,\cdots,m-1, j=$$

$1,2,\cdots,2n.$

$$C(u_{i,j})=\begin{cases}\{1,3,4,5\}, & j\equiv 1(\ mod\ 4);\\ \{2,3,4,5\}, & j\equiv 3(\ mod\ 4);\\ \{1,2,4,5\}, & j\equiv 0(\ mod\ 4)\text{ or } j\equiv 2(\ mod\ 4).\end{cases}\quad i=3,5,\cdots,m-2, j=$$

$1,2,\cdots,2n.$

Case 1.2. When $m\equiv 0(\ mod\ 2)$.

$$f(u_{i,j})=\begin{cases}1, & j\equiv 3(\ mod\ 4);\\ 2, & j\equiv 1(\ mod\ 4);\\ 3, & j\equiv 0(\ mod\ 4)\text{ or } j\equiv 2(\ mod\ 4).\end{cases}\quad i=2,4,\cdots,m-2, j=1,2,\cdots,2n.$$

$$f(u_{i,j})=\begin{cases}1, & j\equiv 0(\ mod\ 4);\\ 2, & j\equiv 2(\ mod\ 4);\\ 3, & j\equiv 1(\ mod\ 4)\text{ or } j\equiv 3(\ mod\ 4).\end{cases}\quad i=3,5,\cdots,m-1, j=1,2,\cdots,2n.$$

$$f(u_{i,j}u_{i,j+1})=\begin{cases}1, & j\equiv 0(\ mod\ 4);\\ 2, & j\equiv 2(\ mod\ 4);\\ 4, & j\equiv 1(\ mod\ 4)\text{ or } j\equiv 3(\ mod\ 4).\end{cases}$$

$i=2,4,\cdots,m-1, j=1,2,\cdots,2n.$(if $j+1>2n, j+1(\ mod\ 2n)$)

$$f(u_{i,j}u_{i,j+1})=\begin{cases}1, & j\equiv 1(\ mod\ 4);\\ 2, & j\equiv 3(\ mod\ 4);\\ 4, & j\equiv 0(\ mod\ 4)\text{ or } j\equiv 2(\ mod\ 4).\end{cases}$$

$i=3,5,\cdots,m-1, j=1,2,\cdots,2n.$(if $j+1>2n, j+1(\ mod\ 2n)$)

$$f(u_{m,j})=\begin{cases}1, & j\equiv 0(\ mod\ 2);\\ 2, & j\equiv 1(\ mod\ 2).\end{cases}\quad j=1,2,\cdots,n.$$

$$f(u_{m,j}u_{m,j+1})=\begin{cases}3, & j\equiv 1(\ mod\ 2);\\ 4, & j\equiv 0(\ mod\ 2).\end{cases}\quad j=1,2,\cdots,n.(\text{ if } j+1>n, j+1(\ mod\ n))$$

All other edges are colored by 5, where,

$$C(u_{1,j})=\begin{cases}\{1,3,4,5\}, & j\equiv 1(\ mod\ 2);\\ \{2,3,4,5\}, & j\equiv 0(\ mod\ 2).\end{cases}\quad j=1,2,\cdots,n.$$

$$C(u_{i,j})=\begin{cases}\{1,3,4,5\}, & j\equiv 0(\ mod\ 4);\\ \{2,3,4,5\}, & j\equiv 2(\ mod\ 4);\\ \{1,2,4,5\}, & j\equiv 1(\ mod\ 4)\text{ or } j\equiv 3(\ mod\ 4).\end{cases}\quad i=2,4,\cdots,m-2, j=$$

$1,2,\cdots,2n.$

$$C(u_{i,j})=\begin{cases}\{1,3,4,5\}, & j\equiv 1(\ mod\ 4);\\ \{2,3,4,5\}, & j\equiv 3(\ mod\ 4);\\ \{1,2,4,5\}, & j\equiv 0(\ mod\ 4)\text{ or } j\equiv 2(\ mod\ 4).\end{cases}\quad i=3,5,\cdots,m-1, j=$$

$1,2,\cdots,2n.$

$$C(u_{m,j})=\begin{cases}\{1,3,4,5\}, & j\equiv 0(\ mod\ 2);\\ \{2,3,4,5\}, & j\equiv 1(\ mod\ 2).\end{cases}\quad j=1,2,\cdots,n.$$

Case 2. When $n\equiv 1(\ mod\ 2)$. Case 2.1 When $n\equiv 0(\ mod\ 3)$.

$$f(u_{1,j})=\begin{cases}1, & j\equiv 1(\ mod\ 3);\\ 2, & j\equiv 2(\ mod\ 3);\\ 3, & j\equiv 0(\ mod\ 3).\end{cases}\quad j=1,2,\cdots,n.$$

$$f(u_{1,j}u_{1,j+1}) = \begin{cases} 3, & j \equiv 1(\ mod\ 3); \\ 4, & j \equiv 2(\ mod\ 3); \\ 5, & j \equiv 0(\ mod\ 3). \end{cases} \quad j = 1, 2, \cdots, n.(\text{ if } j+1 > n, j+1(\ mod\ n))$$

$$f(u_{1,j}u_{2,k}) = \begin{cases} 1, & j \equiv 2(\ mod\ 3); \\ 2, & j \equiv 0(\ mod\ 3); \\ 4, & j \equiv 1(\ mod\ 3). \end{cases} \quad k = 2j - 1, j = 1, 2, \cdots, n.$$

$$f(u_{2,j}) = \begin{cases} 1, & j \equiv 5(\ mod\ 6); \\ 2, & j \equiv 4(\ mod\ 6); \\ 3, & j \equiv 2(\ mod\ 6) \text{ or } j \equiv 0(\ mod\ 6); \\ 5, & j \equiv 1(\ mod\ 6) \text{ or } j \equiv 3(\ mod\ 6). \end{cases} \quad j = 1, 2, \cdots, 2n.$$

$$f(u_{2,j}u_{2,j+1}) = \begin{cases} 1, & j \equiv 1(\ mod\ 6); \\ 3, & j \equiv 4(\ mod\ 6); \\ 2, & j \equiv 2(\ mod\ 6) \text{ or } j \equiv 0(\ mod\ 6); \\ 4, & j \equiv 3(\ mod\ 6) \text{ or } j \equiv 5(\ mod\ 6). \end{cases}$$

$j = 1, 2, \cdots, 2n.(\text{ if } j + 1 > 2n, j + 1(\ mod\ 2n))$

Case 2.1.1. When $m \equiv 0(\ mod\ 2)$.

$$f(u_{i,j}) = \begin{cases} 1, & j \equiv 0(\ mod\ 2); \\ 2, & j \equiv 1(\ mod\ 2). \end{cases} \quad i = 3, 5, \cdots, m - 3, j = 1, 2, \cdots, 2n.$$

$$f(u_{i,j}) = \begin{cases} 1, & j \equiv 1(\ mod\ 2); \\ 2, & j \equiv 0(\ mod\ 2). \end{cases} \quad i = 2, 4, cdots, m - 2, j = 1, 2, \cdots, 2n.$$

$$f(u_{i,j}u_{i,j+1}) = \begin{cases} 3, & j \equiv 1(\ mod\ 2); \\ 4, & j \equiv 0(\ mod\ 2). \end{cases}$$

$i = 3, 5, \cdots, m - 2, j = 1, 2, \cdots, 2n.(\text{ if } j + 1 > 2n, j + 1(\ mod\ 2n))$

$$f(u_{i,j}u_{i,j+1}) = \begin{cases} 3, & j \equiv 0(\ mod\ 2); \\ 4, & j \equiv 1(\ mod\ 2). \end{cases}$$

$i = 4, 6, \cdots, m - 2, j = 1, 2, \cdots, 2n.(\text{ if } j + 1 > 2n, j + 1(\ mod\ 2n))$

$$f(u_{m-1,j}) = \begin{cases} 1, & j \equiv 4(\ mod\ 6); \\ 4, & j \equiv 5(\ mod\ 6); \\ 3, & j \equiv 2(\ mod\ 6) \text{ or } j \equiv 0(\ mod\ 6); \\ 5, & j \equiv 1(\ mod\ 6) \text{ or } j \equiv 3(\ mod\ 6). \end{cases} \quad j = 1, 2, \cdots, 2n.$$

$$f(u_{m-1,j}u_{m-1,j+1}) = \begin{cases} 3, & j \equiv 4(\ mod\ 6); \\ 4, & j \equiv 3(\ mod\ 6); \\ 1, & j \equiv 1(\ mod\ 6) \text{ or } j \equiv 5(\ mod\ 6); \\ 2, & j \equiv 2(\ mod\ 6) \text{ or } j \equiv 0(\ mod\ 6). \end{cases}$$

$j = 1, 2, \cdots, 2n.(\text{ if } j + 1 > 2n, j + 1(\ mod\ 2n))$

$$f(u_{m,j}) = \begin{cases} 1, & j \equiv 1(\ mod\ 3); \\ 2, & j \equiv 2(\ mod\ 3); \\ 3, & j \equiv 0(\ mod\ 3). \end{cases} \quad j = 1, 2, 3, \cdots, n.$$

$$f(u_{m,j}u_{m,j+1}) = \begin{cases} 3, & j \equiv 1(\ mod\ 3); \\ 4, & j \equiv 2(\ mod\ 3); \\ 5, & j \equiv 0(\ mod\ 3). \end{cases} \quad j = 1, 2, \cdots, n.(\text{ if } j+1 > n, j+1(\ mod\ n))$$

$$f(u_{m,j}u_{m-1,k}) = \begin{cases} 1, & j \equiv 2(\ mod\ 3); \\ 2, & j \equiv 0(\ mod\ 3); \\ 4, & j \equiv 1(\ mod\ 3). \end{cases} \quad k = 2j - 1, j = 1, 2, \cdots, 2n.$$

All other edges are colored by 5, where,

$$C(u_{1,j}) = \begin{cases} \{1, 3, 4, 5\}, & j \equiv 1(\ mod\ 3); \\ \{1, 2, 3, 4\}, & j \equiv 2(\ mod\ 3); \\ \{2, 3, 4, 5\}, & j \equiv 0(\ mod\ 3). \end{cases} \quad j = 1, 2, \cdots, n.$$

$$C(u_{2,j}) = \begin{cases} \{1, 2, 3, 5\}, & j \equiv 2(\ mod\ 6); \\ \{1, 2, 3, 4\}, & j \equiv 5(\ mod\ 6); \\ \{1, 2, 4, 5\}, & j \equiv 1(\ mod\ 6) \text{ or } j \equiv 3(\ mod\ 6); \\ \{2, 3, 4, 5\}, & j \equiv 4(\ mod\ 6) \text{ or } j \equiv 0(\ mod\ 6) \end{cases} \quad j = 1, 2, \cdots, 2n.$$

$$C(u_{i,j}) = \begin{cases} \{1,3,4,5\}, & j \equiv 1(\ mod\ 2); \\ \{2,3,4,5\}, & j \equiv 0(\ mod\ 2). \end{cases} \quad i = 3,5,\cdots,m-3, j = 1,2,\cdots,2n.$$

$$C(u_{i,j}) = \begin{cases} \{1,3,4,5\}, & j \equiv 0(\ mod\ 2); \\ \{2,3,4,5\}, & j \equiv 1(\ mod\ 2). \end{cases} \quad i = 4,6,\cdots,m-2, j = 1,2,\cdots,2n.$$

$$C(u_{m-1,j}) = \begin{cases} \{1,3,4,5\}, & j \equiv 4(\ mod\ 6); \\ \{1,2,3,4\}, & j \equiv 5(\ mod\ 6); \\ \{1,2,4,5\}, & j \equiv 1(\ mod\ 6) \text{ or } j \equiv 3(\ mod\ 6); \\ \{1,2,3,5\}, & j \equiv 2(\ mod\ 6) \text{ or } j \equiv 0(\ mod\ 6) \end{cases}$$

$j = 1,2,\cdots,2n.$

$$C(u_{m,j}) = \begin{cases} \{1,3,4,5\}, & j \equiv 1(\ mod\ 3); \\ \{1,2,3,4\}, & j \equiv 2(\ mod\ 3); \\ \{2,3,4,5\}, & j \equiv 0(\ mod\ 3). \end{cases} \quad j = 1,2,\cdots,n.$$

Case 2.1.2. When $m \equiv 1(\ mod\ 2)$.

$$f(u_{i,j}) = \begin{cases} 1, & j \equiv 0(\ mod\ 2); \\ 2, & j \equiv 1(\ mod\ 2). \end{cases} \quad i = 3,5,\cdots,m-2, j = 1,2,\cdots,2n.$$

$$f(u_{i,j}) = \begin{cases} 1, & j \equiv 1(\ mod\ 2); \\ 2, & j \equiv 0(\ mod\ 2). \end{cases} \quad i = 4,6,\cdots,m-3, j = 1,2,\cdots,2n.$$

$$f(u_{i,j}u_{i,j+1}) = \begin{cases} 3, & j \equiv 1(\ mod\ 2); \\ 4, & j \equiv 0(\ mod\ 2). \end{cases}$$

$i = 3,5,\cdots,m-2, j = 1,2,\cdots,2n.(\text{ if } j+1 > 2n, j+1(\ mod\ 2n))$

$$f(u_{i,j}u_{i,j+1}) = \begin{cases} 3, & j \equiv 0(\ mod\ 2); \\ 4, & j \equiv 1(\ mod\ 2). \end{cases}$$

$i = 4,6,\cdots,m-3, j = 1,2,\cdots,2n.(\text{ if } j+1 > 2n, j+1(\ mod\ 2n))$

$$f(u_{m-1,j}) = \begin{cases} 5, & j \equiv 0(\ mod\ 6); \\ 2, & j \equiv 2(\ mod\ 6) \text{ or } j \equiv 4(\ mod\ 6); \\ 1, & j \equiv 1(\ mod\ 6) \text{ or } j \equiv 3(\ mod\ 6) \text{ or } j \equiv 5(\ mod\ 6). \end{cases}$$

$j = 1,2,\cdots,2n.$

$$f(u_{m,j}) = \begin{cases} 1, & j \equiv 0(\ mod\ 3); \\ 3, & j \equiv 1(\ mod\ 3); \\ 4, & j \equiv 2(\ mod\ 3). \end{cases} \quad j = 1,2,\cdots,n.$$

$$f(u_{m-1,j}u_{m-1,j+1}) = \begin{cases} 2, & j \equiv 5(\ mod\ 6); \\ 4, & j \equiv 3(\ mod\ 6) \text{ or } j \equiv 1(\ mod\ 6); \\ 3, & j \equiv 2(\ mod\ 6) \text{ or } j \equiv 4(\ mod\ 6) \\ & \text{or } j \equiv 0(\ mod\ 6). \end{cases}$$

$j = 1,2,\cdots,2n.(\text{ if } j+1 > 2n, j+1(\ mod\ 2n))$

$$f(u_{m,j}u_{m,j+1}) = \begin{cases} 1, & j \equiv 1(\ mod\ 3); \\ 2, & j \equiv 0(\ mod\ 3); \\ 3, & j \equiv 2(\ mod\ 3). \end{cases} \quad j = 1,2,\cdots,n.(\text{ if } j+1 > n, j+1(\ mod\ n))$$

$$f(u_{m,j}u_{m,k}) = \begin{cases} 4, & j \equiv 0(\ mod\ 3); \\ 5, & j \equiv 1(\ mod\ 3) \text{ or } j \equiv 2(\ mod\ 3). \end{cases} \quad k = 2j, j = 1,2,\cdots,n.$$

All other edges are colored by 5, where,

$$C(u_{1,j}) = \begin{cases} \{1,3,4,5\}, & j \equiv 1(\ mod\ 3); \\ \{1,2,3,4\}, & j \equiv 2(\ mod\ 3); \\ \{2,3,4,5\}, & j \equiv 0(\ mod\ 3). \end{cases} \quad j = 1,2,\cdots,n.$$

$$C(u_{2,j}) = \begin{cases} \{1,2,3,5\}, & j \equiv 2(\ mod\ 6); \\ \{1,2,3,4\}, & j \equiv 5(\ mod\ 6); \\ \{1,2,4,5\}, & j \equiv 1(\ mod\ 6) \text{ or } j \equiv 3(\ mod\ 6); \\ \{2,3,4,5\}, & j \equiv 4(\ mod\ 6) \text{ or } j \equiv 0(\ mod\ 6) \end{cases} \quad j = 1,2,\cdots,2n.$$

$$C(u_{i,j}) = \begin{cases} \{1,3,4,5\}, & j \equiv 1(\ mod\ 2); \\ \{2,3,4,5\}, & j \equiv 0(\ mod\ 2). \end{cases} \quad i = 3,5,\cdots,m-2, j = 1,2,\cdots,2n.$$

$$C(u_{i,j}) = \begin{cases} \{1,3,4,5\}, & j \equiv 0(\ mod\ 2); \\ \{2,3,4,5\}, & j \equiv 1(\ mod\ 2). \end{cases}$$

$i = 4,6,\cdots,m-2, j = 1,2,\cdots,2n.$

$$C(u_{m-1,j}) = \begin{cases} \{1,2,3,5\}, & j \equiv 5(\ mod\ 6); \\ \{1,3,4,5\}, & j \equiv 1(\ mod\ 6) \text{ or } j \equiv 3(\ mod\ 6); \\ \{2,3,4,5\}, & j \equiv 2(\ mod\ 6) \\ & \text{or } j \equiv 4(\ mod\ 6) \text{ or } j \equiv 0(\ mod\ 6). \end{cases} \quad j = 1,2,\cdots,2n.$$

$$C(u_{m,j}) = \begin{cases} \{1,2,3,5\}, & j \equiv 1(\ mod\ 3); \\ \{1,3,4,5\}, & j \equiv 2(\ mod\ 3); \\ \{1,2,3,4\}, & j \equiv 0(\ mod\ 3). \end{cases} \quad j = 1,2,\cdots,n.$$

Case 2.2. When $n \equiv 1(\ mod\ 2)$.

$$f(u_{1,j}) = \begin{cases} 1, & j \equiv 1(\ mod\ 2); \\ 2, & j \equiv 0(\ mod\ 2). \end{cases} \quad j = 1,2,\cdots,n-1.$$

$$f(u_{2,j}) = \begin{cases} 3, & j \equiv 1(\ mod\ 2); \\ 4, & j \equiv 0(\ mod\ 2). \end{cases} \quad j = 1,2,\cdots,2n-2.$$

$f(u_n u_1) = 5, f(u_{2,1}) = 5, f(u_{2,2n}) = 2;$

$$f(u_{1,j}u_{1,j+1}) = \begin{cases} 3, & j \equiv 1(\ mod\ 2); \\ 4, & j \equiv 0(\ mod\ 2). \end{cases} \quad j = 1,2,\cdots,n-1.$$

$$f(u_{2,j}u_{2,j+1}) = \begin{cases} 1, & j \equiv 1(\ mod\ 2); \\ 2, & j \equiv 0(\ mod\ 2). \end{cases} \quad j = 1,2,\cdots,2n-3.$$

$f(u_{2,2m}u_{2,1}) = 3,\ f(u_{1,1}u_{2,1}) = 4,\ f(u_{1,n}u_{2,2n-1}) = 2;$

Case 2.2.1. When $m \equiv 1(\ mod\ 2)$.

$$f(u_{i,j}) = \begin{cases} 3, & j \equiv 1(\ mod\ 2); \\ 4, & j \equiv 0(\ mod\ 2). \end{cases} \quad i = 4,6,\cdots,m-1, j = 1,2,\cdots,2n$$

$$f(u_{i,j}u_{i,j+1}) = \begin{cases} 1, & j \equiv 1(\ mod\ 2); \\ 2, & j \equiv 0(\ mod\ 2). \end{cases}$$

$i = 3,4,\cdots,m-1, j = 1,2,\cdots,2n.(\text{ if } j+1 > 2n, j+1(\ mod\ 2n))$

$$f(u_{m,j}) = \begin{cases} 1, & j \equiv 1(\ mod\ 2); \\ 2, & j \equiv 0(\ mod\ 2). \end{cases} \quad j = 1,2,\cdots,n-1. f(u_{m,n}) = 3.$$

$$f(u_{m,j}u_{m,j+1}) = \begin{cases} 3, & j \equiv 1(\ mod\ 2); \\ 4, & j \equiv 0(\ mod\ 2). \end{cases}$$

$j = 1,2,\cdots,n-2.\ f(u_{m,n-1}u_{m,n}) = 2. f(u_{m,n}u_{m,1}) = 4;$

All other edges are colored by 5, where,

$$C(u_{1,j}) = \begin{cases} \{2,3,4,5\}, & j \equiv 0(\ mod\ 2); \\ \{1,3,4,5\}, & j \equiv 1(\ mod\ 2). \end{cases} \quad j = 2,3,\cdots,n-1.$$

$C(u_{1,1}) = \{1,2,3,4\}, C(u_{1,n}) = \{1,2,4,5\};$

$$C(u_{2,j}) = \begin{cases} \{1,2,4,5\}, & j \equiv 0(\ mod\ 2); \\ \{1,2,3,5\}, & j \equiv 1(\ mod\ 2). \end{cases} \quad j = 2,3,\cdots,2n-2.$$

$C(u_{2,1}) = \{1,3,4,5\}, C(u_{2,2n-1}) = \{1,2,3,4\}, C(u_{2,2n}) = \{2,3,4,5\};$

$$C(u_{1,j}) = \begin{cases} \{1,2,4,5\}, & j \equiv 1(\ mod\ 2); \\ \{1,2,3,5\}, & j \equiv 0(\ mod\ 2). \end{cases} \quad i = 3,5,\cdots,m-2, j = 1,2,\cdots,2n.$$

$$C(u_{i,j}) = \begin{cases} \{1,2,3,5\}, & j \equiv 1(\ mod\ 2); \\ \{1,2,4,5\}, & j \equiv 0(\ mod\ 2). \end{cases}$$

$i = 4,6,\cdots,m-1, j = 1,2,\cdots,2n.$

$$C(u_{m,j}) = \begin{cases} \{1,3,4,5\}, & j \equiv 1(\ mod\ 2); \\ \{2,3,4,5\}, & j \equiv 0(\ mod\ 2). \end{cases} \quad j = 1,2,\cdots,n-2.$$

$C(u_{m,m-1}) = \{1,2,3,5\}, C(u_{m,n}) = \{2,3,4,5\};$

Case 2.2.2. When $m \equiv 0(\ mod\ 2)$.

$$f(u_{i,j}) = \begin{cases} 3, & j \equiv 1(\ mod\ 2); \\ 4, & j \equiv 0(\ mod\ 2). \end{cases} \quad i = 4,6,\cdots,m-2, j = 1,2,\cdots,2n.$$

$$f(u_{i,j}) = \begin{cases} 3, & j \equiv 0(\ mod\ 2); \\ 4, & j \equiv 1(\ mod\ 2). \end{cases} \quad i = 3,5,\cdots,m-1, j = 1,2,\cdots,2n.$$

$$f(u_{i,j}u_{i,j+1}) = \begin{cases} 1, & j \equiv 1(\ mod\ 2); \\ 2, & j \equiv 0(\ mod\ 2). \end{cases}$$

$i = 3,4,\cdots,m-1, j = 1,2,\cdots,2n.(\text{ if } j+1 > 2n, j+1(\ mod\ 2n))$

$$f(u_{m,j}) = \begin{cases} 1, & j \equiv 1(\ mod\ 2); \\ 2, & j \equiv 0(\ mod\ 2). \end{cases}$$

$j = 1, 2, \cdots, 2n. f(u_{m,n}) = 5, f(u_{m-1,2n-1}u_{m,n}) = 3.$

$$f(u_{m,j}u_{m,j+1}) = \begin{cases} 3, & j \equiv 1(\ mod\ 2); \\ 4, & j \equiv 0(\ mod\ 2). \end{cases} \quad j = 1, 2, \cdots, 2n.$$

$f(u_{m,n-1}u_{mn}) = 2, f(u_{m,n}u_{m,1}) = 4.$

All other edges are colored by 5, where,

$$C(u_{i,j}) = \begin{cases} \{2, 3, 4, 5\}, & j \equiv 0(\ mod\ 2); \\ \{1, 3, 4, 5\}, & j \equiv 1(\ mod\ 2). \end{cases} \quad j = 2, 3, 4, \cdots, n-1.$$

$C(u_{1,1}) = \{1, 2, 3, 4\}, C(u_{1,n}) = \{1, 2, 4, 5\};$

$$C(u_{2,j}) = \begin{cases} \{1, 3, 4, 5\}, & j \equiv 0(\ mod\ 2); \\ \{1, 2, 3, 5\}, & j \equiv 1(\ mod\ 2). \end{cases} \quad j = 2, 3, \cdots, 2n-2.$$

$C(u_{2,1}) = \{1, 3, 4, 5\}, C(u_{2,2n-1}) = \{1, 2, 3, 4\}, C(u_{2,2n}) = \{2, 3, 4, 5\};$

$$C(u_{i,j}) = \begin{cases} \{1, 2, 4, 5\}, & j \equiv 1(\ mod\ 2); \\ \{1, 2, 3, 5\}, & j \equiv 0(\ mod\ 2). \end{cases}$$

$i = 3, 5, \cdots, m-1, j = 1, 2, \cdots, 2n.$

$$C(u_{i,j}) = \begin{cases} \{1, 2, 3, 5\}, & j \equiv 1(\ mod\ 2); \\ \{1, 2, 4, 5\}, & j \equiv 0(\ mod\ 2). \end{cases}$$

$i = 4, 6, \cdots, m-2, j = 1, 2, \cdots, 2n.$

$C(u_{m,1}) = \{1, 2, 3, 5\};$

$$C(u_{m,j}) = \begin{cases} \{1, 3, 4, 5\}, & j \equiv 1(\ mod\ 2); \\ \{2, 3, 4, 5\}, & j \equiv 0(\ mod\ 2). \end{cases} \quad j = 2, 3, \cdots, n.$$

Case 2.3. When $n \equiv 2(\ mod\ 2)$.

$$f(u_{1,j}) = \begin{cases} 1, & j \equiv 1(\ mod\ 2); \\ 2, & j \equiv 0(\ mod\ 2). \end{cases} \quad j = 1, 2, \cdots, n-1.$$

$f(u_{1,n}) = 3, f(u_{1,n}u_{1,1}) = 2, f(u_{1,n}u_{2,2n-1}) = 1;$

$$f(u_{2,j}) = \begin{cases} 3, & j \equiv 1(\ mod\ 2); \\ 4, & j \equiv 0(\ mod\ 2). \end{cases} \quad j = 1, 2, \cdots, 2n-1.$$

$$f(u_{2,j}u_{2,j+1}) = \begin{cases} 1, & j \equiv 1(\ mod\ 2); \\ 2, & j \equiv 0(\ mod\ 2). \end{cases} \quad j = 1, 2, \cdots, 2n-4.$$

$f(u_{2,2n}u_{2,1}) = 4, f(u_{2,2n-1}u_{2,2n-1}) = 2, f(u_{2,2n-2}u_{2,2n-1}) = 3,$
$f(u_{2,2n-3}u_{2,2n-2}) = 1, f(u_{2,2n-1}) = 5, f(u_{2,2n}) = 1;$

Case 2.3.1. When $m \equiv 0(\ mod\ 2)$.

$$f(u_{i,j}) = \begin{cases} 3, & j \equiv 0(\ mod\ 2); \\ 4, & j \equiv 1(\ mod\ 2). \end{cases} \quad i = 3, 5, \cdots, m-3, j = 1, 2, \cdots, 2n.$$

$$f(u_{i,j}) = \begin{cases} 3, & j \equiv 1(\ mod\ 2); \\ 4, & j \equiv 0(\ mod\ 2). \end{cases} \quad i = 4, 6, \cdots, m-2, j = 1, 2, \cdots, 2n.$$

$$f(u_{i,j}u_{i,j+1}) = \begin{cases} 1, & j \equiv 1(\ mod\ 2); \\ 2, & j \equiv 0(\ mod\ 2). \end{cases}$$

$i = 3, 4, \cdots, m-2, j = 1, 2, \cdots, 2n.($ if $j+1 > 2n, j+1(\ mod\ 2n))$

$$f(u_{m-1,j}) = \begin{cases} 3, & j \equiv 0(\ mod\ 2); \\ 4, & j \equiv 1(\ mod\ 2). \end{cases} \quad j = 1, 2, \cdots, 2n-2.$$

$$f(u_{m,j}) = \begin{cases} 1, & j \equiv 1(\ mod\ 2); \\ 2, & j \equiv 0(\ mod\ 2). \end{cases} \quad j = 1, 2, \cdots, n-1.$$

$$f(u_{m-1,j}u_{m-1,j+1}) = \begin{cases} 1, & j \equiv 1(\ mod\ 2); \\ 2, & j \equiv 0(\ mod\ 2). \end{cases} \quad j = 1, 2, \cdots, 2n-2.$$

$f(u_{m-1,2n-1}u_{m-1,2n}) = 4, f(u_{m-1,2n}u_{m-1,1}) = 2;$

$$f(u_{m,j}) = \begin{cases} 1, & j \equiv 1(\ mod\ 2); \\ 2, & j \equiv 0(\ mod\ 2). \end{cases} \quad j = 1, 2, \cdots, n-1.$$

$$f(u_{m,j}u_{m,j+1}) = \begin{cases} 3, & j \equiv 1(\ mod\ 2); \\ 4, & j \equiv 0(\ mod\ 2). \end{cases} \quad j = 1, 2, \cdots, n-1.$$

$f(u_{m-1,2n-1}u_{m,n}) = 1, f(u_{m,n}u_{m,1}) = 2;$

All other edges are colored by 5, where,

$$C(u_{1,j}) = \begin{cases} \{2,3,4,5\}, & j \equiv 0(\ mod\ 2); \\ \{1,3,4,5\}, & j \equiv 1(\ mod\ 2). \end{cases} \quad j = 2,3,\cdots,n-1.$$

$C(u_{1,1}) = \{1,2,3,5\}, C(u_{1,n}) = \{1,2,3,4\}$;

$$C(u_{2,j}) = \begin{cases} \{1,2,4,5\}, & j \equiv 0(\ mod\ 2); \\ \{1,2,3,5\}, & j \equiv 1(\ mod\ 2). \end{cases} \quad j = 2,3,\cdots,2n-3.$$

$C(u_{2,1}) = \{1,3,4,5\}, C(u_{2,2n-2}) = \{1,3,4,5\}$,
$C(u_{2,2n-1}) = \{1,2,3,5\}, C(u_{2,n}) = \{1,2,4,5\}$;

$$C(u_{i,j}) = \begin{cases} \{1,2,4,5\}, & j \equiv 1(\ mod\ 2); \\ \{1,2,3,5\}, & j \equiv 0(\ mod\ 2). \end{cases} \quad i = 3,5,\cdots,m-3, j = 1,2,\cdots,2n.$$

$$C(u_{i,j}) = \begin{cases} \{1,2,4,5\}, & j \equiv 0(\ mod\ 2); \\ \{1,2,3,5\}, & j \equiv 1(\ mod\ 2). \end{cases} \quad i = 4,6,\cdots,m-2, j = 1,2,\cdots,2n.$$

$$C(u_{m-1,j}) = \begin{cases} \{1,2,4,5\}, & j \equiv 1(\ mod\ 2); \\ \{1,2,3,5\}, & j \equiv 0(\ mod\ 2). \end{cases} \quad j = 1,2,\cdots,2n-1.$$

$C(u_{m-1,n}) = \{2,3,4,5\}$;

$$C(u_{m,j}) = \begin{cases} \{2,3,4,5\}, & j \equiv 0(\ mod\ 2); \\ \{1,3,4,5\}, & j \equiv 1(\ mod\ 2). \end{cases} \quad j = 2,3,\cdots,n-1.$$

$C(u_{m,1}) = \{1,2,3,5\}, C(u_{m,n}) = \{1,2,3,4\}$;

Case 2.3.2. When $m \equiv 1(\ mod\ 2)$.

$$f(u_{i,j}) = \begin{cases} 3, & j \equiv 0(\ mod\ 2); \\ 4, & j \equiv 1(\ mod\ 2). \end{cases} \quad i = 3,5,\cdots,m-2, j = 1,2,\cdots,2n.$$

$$f(u_{i,j}) = \begin{cases} 3, & j \equiv 1(\ mod\ 2); \\ 4, & j \equiv 0(\ mod\ 2). \end{cases} \quad i = 4,6,\cdots,m-3, j = 1,2,\cdots,2n.$$

$$f(u_{i,j}u_{i,j+1}) = \begin{cases} 1, & j \equiv 1(\ mod\ 2); \\ 2, & j \equiv 0(\ mod\ 2). \end{cases} \quad i = 3,4,\cdots,m-2, j = 1,2,\cdots,2n.$$

$$f(u_{m-1,j}) = \begin{cases} 3, & j \equiv 1(\ mod\ 2); \\ 4, & j \equiv 0(\ mod\ 2). \end{cases} \quad j = 1,2,\cdots,2n-2.$$

$f(u_{m-1,2n-1}) = 3, f(u_{m-1,2n}) = 2$;

$$f(u_{m-1,j}u_{m-1,j+1}) = \begin{cases} 1, & j \equiv 1(\ mod\ 2); \\ 2, & j \equiv 0(\ mod\ 2). \end{cases} \quad j = 1,2,\cdots,2n-2.$$

$f(u_{m-1,2n-1}u_{m-1,2n}) = 1, f(u_{m-1,2n}u_{m-1,1}) = 4$;

$$f(u_{m,j}) = \begin{cases} 2, & j \equiv 0(\ mod\ 2); \\ 3, & j \equiv 1(\ mod\ 2). \end{cases} \quad j = 1,2,\cdots,n-3.$$

$f(u_{m,n-1}) = 3, f(u_{m,n}) = 5, f(u_{m,n-2}) = 1$;

$$f(u_{m,j}u_{m,j+1}) = \begin{cases} 3, & j \equiv 2(\ mod\ 3); \\ 4, & j \equiv 1(\ mod\ 3) \text{ or } j \equiv 0(\ mod\ 3). \end{cases} \quad j = 1,2,\cdots,n-2.$$

$f(u_{m,n-1}u_{m,n}) = 2, f(u_{m,n}u_{m,1}) = 1$,
$f(u_{m-1,2n}u_{m,n}) = 3, f(u_{m,n-1}u_{m,n}) = 4$;

All other edges are colored by 5, where,

$$C(u_{1,j}) = \begin{cases} \{2,3,4,5\}, & j \equiv 0(\ mod\ 2); \\ \{1,3,4,5\}, & j \equiv 1(\ mod\ 2). \end{cases} \quad j = 2,3,\cdots,n-1.$$

$C(u_{1,1}) = \{1,2,3,5\}, C(u_{1,n}) = \{1,2,3,4\}$;

$$C(u_{2,j}) = \begin{cases} \{1,2,4,5\}, & j \equiv 0(\ mod\ 2); \\ \{1,2,3,5\}, & j \equiv 1(\ mod\ 2). \end{cases} \quad j = 2,3,\cdots,2n-3.$$

$C(u_{2,2n-2}) = \{1,3,4,5\}, C(u_{2,2n-1}) = \{1,2,3,5\}, C(u_{2,n}) = \{1,2,4,5\}$;

$$C(u_{i,j}) = \begin{cases} \{1,2,4,5\}, & j \equiv 1(\ mod\ 2); \\ \{1,2,3,5\}, & j \equiv 0(\ mod\ 2). \end{cases} \quad i = 3,5,\cdots,m-2, j = 1,2,\cdots,2n.$$

$$C(u_{i,j}) = \begin{cases} \{1,2,4,5\}, & j \equiv 0(\ mod\ 2); \\ \{1,2,3,5\}, & j \equiv 1(\ mod\ 2). \end{cases} \quad i = 4,6,\cdots,m-3, j = 1,2,\cdots,2n.$$

$$C(u_{m-1,j}) = \begin{cases} \{1,2,4,5\}, & j \equiv 0(\ mod\ 2); \\ \{1,2,3,5\}, & j \equiv 1(\ mod\ 2). \end{cases} \quad j = 2,3,\cdots,n-1.$$

$C(u_{m-1,n}) = \{1,2,3,4\}, C(u_{m-1,1}) = \{1,3,4,5\}$;

$$C(u_{m,j}) = \begin{cases} \{1,3,4,5\}, & j \equiv 1(\ mod\ 2); \\ \{2,3,4,5\}, & j \equiv 0(\ mod\ 2). \end{cases} \quad j = 1, 2, \cdots, n-3.$$

$C(u_{m,n-2}) = \{1,3,4,5\}, C(u_{m,n-1}) = \{2,3,4,5\}, C(u_{m,n}) = \{1,2,3,5\}.$

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