

A note on valency dependance invariants of $L(G(k))$ graph

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ABSTRACT

Let $L(G(k))$ be a line graph of a k -subdivided graph $G(k)$ of any connected, simple and undirected graph G . In this paper, we fixed the valency dependence invariants of $L(G(k))$ for $k \geq 2$. Our results are the generalization of the results proved in [1, 3, 8].

Keywords: valency based invariants, line graph, k -subdivided graph

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1. Introduction

Let G be a simple and connected graph with vertex set $V(G)$ and edge set $E(G)$. The number of elements in the vertex set and edge set are called the order and size of the graph G respectively. The degree of a vertex ' v ' in a simple graph G is denoted as $deg(v)$ and defined as the number of edges incidents to vertex v .

In chemical graph theory, we apply the concepts of graph theory to describe the mathematical model of a variety of chemical structures. The atoms of the molecules correspond to the vertices and the chemical bond is reflected by edges. Topological indices are numerical parameters of chemical graphs associated with quantitative structure property relationship (QSPR) and quantitative structure activity relationship (QSAR) [2, 9]. The major of topological indices is distance based, degree based, and spectrum based. Among these classes, degree based topological indices are of great importance and are helpful tools for chemists. The concept of topological index came from the work done by Wiener, when he was working on boiling point of paraffin [17].

One of the well-known degree based topological indices is the atom-bond connectivity (ABC) index of a graph, proposed by Estrada et al. and defined as [4]:

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$$ABC(G) = \sum_{uv \in E(G)} \sqrt{\frac{d_u + d_v - 2}{d_u d_v}}. \quad (1)$$

Vukičević and Furtula defined the geometric arithmetic index as [16]:

$$GA(G) = \sum_{uv \in E(G)} \frac{2\sqrt{d_u d_v}}{d_u + d_v}. \quad (2)$$

The fourth member of the class of ABC index was introduced by Ghorbani and Hoseinzadeh [5]:

$$ABC_4(G) = \sum_{uv \in E(G)} \sqrt{\frac{s_u + s_v - 2}{s_u s_v}}. \quad (3)$$

The fifth GA index was introduced by Graovac et al. [6]:

$$GA_5(G) = \sum_{uv \in E(G)} \frac{2\sqrt{s_u s_v}}{s_u + s_v}, \quad (4)$$

where s_v is the summation of degrees of all neighbors of vertex v in G .

2. Discussion and main results

In 2011, Ranjini et al. [10, 11] studied the Zagreb indices of the line graphs of the subdivision of the tadpole, wheel, steerage, and stepping stool charts. Sardar et al. [13] computed the topological files of the line diagrams of Banana tree and Firecracker diagrams. Nadeem et al., saddiqui et al. and Soleimani et al. [9, 12, 14] studied the topological properties of nanostructures. In 2015, Su and Xu [15] figured the general aggregate availability records and co-lists of the line graph of the tadpole and haggie charts with subdivision. Nadeem et al. [9, 16] calculated ABC_4 and GA_5 of the line graphs of the tadpole, wheel, stepping stool, $2D$ -lattice, nanotube, and nanotorus of $TUC_4C_8[p, q]$. Recently, Imran et al. [7] found the first and second Zagreb index, the hyper Zagreb index, multiple Zagreb indices and Zagreb polynomials of the line graph of wheel and ladder graphs by taking $k = 2$.

For accurate and optimal structure design it is important to use topological and connectivity indices as structural descriptors. Let G be a finite, simple, undirected and connected graph with order p and size q . A k -subdivided graph $G(k)$ of G is obtained by replacing every edge of graph G by a path P_{k+2} . A line graph $L(G)$ of graph G is a graph having q vertices and two vertices are adjacent in $L(G)$ if and only if their corresponding edges are adjacent in G . It is not easy to describe the structure of line graph of any graph in general. However, the line graph of $G(k)$ has following properties.

- Every vertex of degree d becomes a complete graph K_d and every path P_{k+2} becomes a path P_{k+1} .

- The graph $L(G(k))$ has p number of independent complete graphs K_{d_i} for $1 \leq i \leq p$ as subgraphs.
- The graph $L(G(k))$ has q number of independent paths P_{k+1} .
- Let (d_i, d_j) be a type of edge in G then this edge with respect to degree will become a path $(d_i, \underbrace{2, 2, \dots, 2}_{k-time}, d_j)$ in $G(k)$ and $(d_i, \underbrace{2, 2, \dots, 2}_{(k-1)-time}, d_j)$ in $L(G(k))$.
- Let (d_i, d_j) be a type of edge in G then this edge with respect to neighborhood degree will become a path $(2d_i, \underbrace{2, 2, \dots, 2}_{k-time}, 2d_j)$ in $G(k)$ and $(d_i(d_i-1)+2, \underbrace{2, 2, \dots, 2}_{(k-1)-time}, d_j(d_j-1)+2)$ in $L(G(k))$.

In [3, 8, 7] the authors have calculated the neighborhood degree based topological indices for the line graph of wheel, ladder and tadpole graph only for $k = 1, 2$. They have found the topological indices by using elementary counting. In this article, we study the structure of line graph of k -subdivided graph. Then we define degree base and neighborhood degree base topological indices for the line graph of k -subdivided graphs in general.

3. Degree based topological indices of $L(G(k))$

In this section, we elaborate this concept by using a lemma as follows:

Lemma 3.1. *Consider graph G on p vertices and q edges with degree sequences $3 \leq d_1 \leq d_2 \leq d_3 \dots \leq d_p$. Let $e_i = (d_{\alpha_i}, d_{\beta_i})$ be all s -types of edges with count ζ_i , $i = 1, 2, \dots, s$ and μ_i and λ_i are the number of vertices of degree d_{α_i} and d_{β_i} respectively. Then line graph $L(G(k))$ of k -subdivided graph of G , will have the following degree base edges and their counts.*

Edge Type	$(d_{\alpha_i}, d_{\alpha_i})$	$(d_{\alpha_i}, 2)$	$(2, 2)$	$(2, d_{\beta_i})$	$(d_{\beta_i}, d_{\beta_i})$
Count	$\mu_i \binom{d_{\alpha_i}}{2}$	ζ_i	$(k - 2)q$	ζ_i	$\lambda_i \binom{d_{\beta_i}}{2}$

Proof.

We assume that $k \geq 2$. Since the line graph of complete bipartite graph $K_{1,d}$ is complete graph K_d and line graph of path P_{b+1} is a path P_b . Therefore every vertex of degree $d_i \geq 3$ in G becomes a complete graph K_{d_i} and every path P_{k+2} in G becomes a path P_{k+1} in $L(G(k))$. Thus graph $L(G(k))$ has p number of independent complete graphs K_{d_i} for $1 \leq i \leq p$ as subgraphs. Each path P_{k+2} between the pair of vertices (d_m, d_n) becomes a path P_{k+1} in $L(G(k))$ for $1 \leq m, n \leq q$ and vertices have degrees (from left to right) as $d_m, \underbrace{2, 2, \dots, 2}_{k-time}, d_n$ in $L(G(k))$. Therefore, we will have q total number of paths in graph $L(G(k))$. By using these properties, we conclude the proof. □

Theorem 3.2. Let $H \cong G(k)$ (for $k > 1$), ABC index of line graph of H is

$$ABC(L(H)) = \mu_i \frac{(d_{\alpha_i} - 1)^{3/2}}{\sqrt{2}} + \lambda_i \frac{(d_{\beta_i} - 1)^{3/2}}{\sqrt{2}} + \frac{(k-2)q}{\sqrt{2}} + \xi_i \frac{2}{\sqrt{2}}.$$

Proof. From Eq. (1),

$$\begin{aligned} ABC(L(H)) &= \sum_{uv \in E(L(H))} \sqrt{\frac{d_u + d_v - 2}{d_u d_v}} \\ &= \sum_{(d_{\alpha_i}, d_{\alpha_i})} \sqrt{\frac{d_{\alpha_i} + d_{\alpha_i} - 2}{d_{\alpha_i} d_{\alpha_i}}} + \sum_{(d_{\alpha_i}, 2)} \sqrt{\frac{d_{\alpha_i} + 2 - 2}{2 d_{\alpha_i}}} \\ &\quad + \sum_{(2, 2)} \sqrt{\frac{2 + 2 - 2}{2 \times 2}} + \sum_{(2, d_{\beta_i})} \sqrt{\frac{d_{\beta_i} + 2 - 2}{2 d_{\beta_i}}} + \sum_{(d_{\beta_i}, d_{\beta_i})} \sqrt{\frac{d_{\beta_i} + d_{\beta_i} - 2}{d_{\beta_i} d_{\beta_i}}} \\ &= \mu_i \binom{d_{\alpha_i}}{2} \sqrt{\frac{2d_{\alpha_i} - 2}{d_{\alpha_i}^2}} + \lambda_i \binom{d_{\beta_i}}{2} \sqrt{\frac{2d_{\beta_i} - 2}{d_{\beta_i}^2}} + \xi_i \left[\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \right] + \frac{(k-2)q}{\sqrt{2}} \\ &= \mu_i \frac{(d_{\alpha_i} - 1)^{3/2}}{\sqrt{2}} + \lambda_i \frac{(d_{\beta_i} - 1)^{3/2}}{\sqrt{2}} + \frac{(k-2)q}{\sqrt{2}} + \xi_i \frac{2}{\sqrt{2}}. \end{aligned}$$

□

Theorem 3.3. Let $H \cong G(k)$ (for $k > 1$), GA index of line graph of H is

$$GA(L(H)) = \mu_i \frac{d_{\alpha_i}(d_{\alpha_i} - 1)}{2} + \lambda_i \frac{d_{\beta_i}(d_{\beta_i} - 1)}{2} + (k-2)q + 2\sqrt{2}\xi_i \left[\frac{(\sqrt{d_{\alpha_i}} + \sqrt{d_{\beta_i}})(d_{\alpha_i}d_{\beta_i} + 2)}{(d_{\alpha_i} + 2)(d_{\beta_i} + 2)} \right].$$

Proof. From Eq. (2),

$$\begin{aligned} GA(L(H)) &= \sum_{uv \in E(L(H))} \frac{2\sqrt{d_u d_v}}{d_u + d_v} \\ &= \sum_{(d_{\alpha_i}, d_{\alpha_i})} \frac{2\sqrt{d_{\alpha_i}^2}}{2d_{\alpha_i}} + \sum_{(d_{\alpha_i}, 2)} \frac{2\sqrt{2d_{\alpha_i}}}{2 + d_{\alpha_i}} + \sum_{(2, 2)} \frac{2\sqrt{4}}{4} + \sum_{(2, d_{\beta_i})} \frac{2\sqrt{2d_{\beta_i}}}{2 + d_{\beta_i}} + \sum_{(d_{\beta_i}, d_{\beta_i})} \frac{2\sqrt{d_{\beta_i}^2}}{2d_{\beta_i}} \\ &= \mu_i \binom{d_{\alpha_i}}{2} + \lambda_i \binom{d_{\beta_i}}{2} + (k-2)q + \xi_i \left[\frac{2\sqrt{2d_{\alpha_i}}}{2 + d_{\alpha_i}} + \frac{2\sqrt{2d_{\beta_i}}}{2 + d_{\beta_i}} \right] \\ &= \mu_i \frac{d_{\alpha_i}(d_{\alpha_i} - 1)}{2} + \lambda_i \frac{d_{\beta_i}(d_{\beta_i} - 1)}{2} + (k-2)q \\ &\quad + 2\sqrt{2}\xi_i \left[\frac{\sqrt{d_{\alpha_i}}(d_{\beta_i} + 2) + \sqrt{d_{\beta_i}}(d_{\alpha_i} + 2)}{(d_{\alpha_i} + 2)(d_{\beta_i} + 2)} \right] \\ &= \mu_i \frac{d_{\alpha_i}(d_{\alpha_i} - 1)}{2} + \lambda_i \frac{d_{\beta_i}(d_{\beta_i} - 1)}{2} + (k-2)q \\ &\quad + 2\sqrt{2}\xi_i \left[\frac{(\sqrt{d_{\alpha_i}} + \sqrt{d_{\beta_i}})(d_{\alpha_i}d_{\beta_i} + 2)}{(d_{\alpha_i} + 2)(d_{\beta_i} + 2)} \right]. \end{aligned}$$

□

4. Neighborhood degree based topological indices of $L(G(k))$

Firstly, We introduce this concept in the form of a lemma as follows.

Lemma 4.1. *Let G be any finite and simple graph with order p and size q , having degree sequence $3 \leq d_1 \leq d_2 \leq \dots \leq d_{p-1} \leq d_p$. Let $e_i = (d_{\alpha_i}, d_{\beta_i})$ be all type of edges with each has count ζ_i and $i = 1, 2, \dots, s$ in graph G . For $k \geq 2$, the the line graph $L(G(k))$ of k -subdivided graph of G , will have the following neighborhood degree base edges:*

1. For $k = 2$, we will have two type of neighborhood degree base edges: first type of edges depend on the degrees of vertices of G are $(d_\gamma(d_\gamma - 1) + 2, d_\gamma(d_\gamma - 1) + 2)$; with count $\frac{d_\gamma(d_\gamma - 1)}{2}$. The second type of edges depend on the type of edges of G are $(d_{\alpha_i}(d_{\alpha_i} - 1) + 2, d_{\alpha_i} + d_{\beta_i})$; $(d_{\alpha_i} + d_{\beta_i}, d_{\beta_i}(d_{\beta_i} - 1) + 2)$; $\zeta_i; \zeta_i$ respectively with $1 \leq \gamma \leq p$ and $1 \leq i \leq s$.
2. For $k = 3$, neighborhood degree base edges are $(d_\gamma(d_\gamma - 1) + 2, d_\gamma(d_\gamma - 1) + 2)$; $(d_{\alpha_i}(d_{\alpha_i} - 1) + 2, d_{\alpha_i} + 2)$; $(d_{\alpha_i} + 2, d_{\beta_i} + 2)$; $(d_{\beta_i} + 2, d_{\beta_i}(d_{\beta_i} - 1) + 2)$; with count $\frac{d_\gamma(d_\gamma - 1)}{2}$; $\zeta_i; \zeta_i; \zeta_i$ respectively with $1 \leq \gamma \leq p$ and $1 \leq i \leq s$.
3. For $k = 4$, neighborhood degree base edges are $(d_\gamma(d_\gamma - 1) + 2, d_\gamma(d_\gamma - 1) + 2)$; $(d_{\alpha_i}(d_{\alpha_i} - 1) + 2, d_{\alpha_i} + 2)$; $(d_{\alpha_i} + 2, 4)$; $(4, d_{\beta_i} + 2)$; $(d_{\beta_i} + 2, d_{\beta_i}(d_{\beta_i} - 1) + 2)$; with count $\frac{d_\gamma(d_\gamma - 1)}{2}$; $\zeta_i; \zeta_i; \zeta_i; \zeta_i$ respectively with $1 \leq \gamma \leq p$ and $1 \leq i \leq s$.
4. For $k \geq 5$, neighborhood degree base edges are $(d_{\alpha_i}(d_{\alpha_i} - 1) + 2, d_{\alpha_i} + 2)$; $(d_{\alpha_i} + 2, 4)$; $(4, d_{\beta_i} + 2)$; $(d_{\beta_i} + 2, d_{\beta_i}(d_{\beta_i} - 1) + 2)$; with each have count ζ_i ; and $(d_\gamma(d_\gamma - 1) + 2, d_\gamma(d_\gamma - 1) + 2)$; $(4, 4)$; with count $\frac{d_\gamma(d_\gamma - 1)}{2}$; $(k - 4)q$; respectively with $1 \leq \gamma \leq p$ and $1 \leq i \leq s$.

Proof. We assume that $k \geq 5$. Since the line graph of complete bipartite graph $K_{1,d}$ is complete graph K_d . and line graph of path P_{b+1} is a path P_b . Therefore, every vertex of degree $d_i \geq 3$ in G becomes a complete graph K_{d_i} and every path P_{k+2} in G becomes a path P_{k+1} in $L(G(k))$. Thus graph $L(G(k))$ has p number of independent complete graphs K_{d_i} for $1 \leq i \leq p$ as subgraphs. Each path P_{k+2} between the pair of vertices (d_m, d_n) becomes a path P_{k+1} in $L(G(k))$ for $1 \leq m, n \leq q$ and vertices have neighborhood degrees as $d_i(d_i - 1) + 2, d_i + 2, \underbrace{4, 4, \dots, 4}_{(k-2)\text{-time}}, d_j + 2, d_j(d_j - 1) + 2$ in $L(G(k))$. By using these properties, we conclude the proof for $k \geq 5$ and all other cases for $k = 2, 3, 4$ also follow the same lines. □

Remark 4.2. It may be noted that the count of edge type (d_m, d_n) is same as that of edge type (d_n, d_m) .

Theorem 4.3. *Let $H \cong G(k)$ (for $k \geq 4$), The generalized $ABC_4(H)$ index is*

$$ABC_4(L(G)) = \binom{d_\gamma}{2} \left[\frac{\sqrt{2}}{\sqrt{d_\gamma^2 - d_\gamma + 2}} \right] + (k - 4) \frac{\sqrt{6}}{4}$$

$$\begin{aligned}
& + \xi_i \left[\sqrt{\frac{d_{\alpha_i}^2 + 2}{(d_{\alpha_i}^2 - d_{\alpha_i} + 2)(d_{\alpha_i} + 2)}} + \frac{1}{2} \sqrt{\frac{d_{\alpha_i} + 4}{d_{\alpha_i} + 2}} \right. \\
& \left. + \frac{1}{2} \sqrt{\frac{d_{\beta_i} + 4}{d_{\beta_i} + 2}} + \sqrt{\frac{d_{\beta_i}^2 + 2}{(d_{\beta_i} + 2)(d_{\beta_i}^2 - d_{\beta_i} + 2)}} \right].
\end{aligned}$$

Proof. From Eq. (3),

$$\begin{aligned}
ABC_4(H) &= \sum_{uv \in E(H)} \sqrt{\frac{s_u + s_v - 2}{s_u s_v}} \\
&= \sum_{(d_\gamma(d_\gamma-1)+2, d_\gamma(d_\gamma-1)+2)} \sqrt{\frac{d_\gamma(d_\gamma-1) + 2 + d_\gamma(d_\gamma-1) + 2 - 2}{(d_\gamma(d_\gamma-1) + 2)(d_\gamma(d_\gamma-1) + 2)}} \\
&+ \sum_{(d_{\alpha_i}(d_{\alpha_i}-1)+2, d_{\alpha_i}+2)} \sqrt{\frac{d_{\alpha_i}(d_{\alpha_i}-1) + 2 + d_{\alpha_i} + 2 - 2}{(d_{\alpha_i}(d_{\alpha_i}-1) + 2)(d_{\alpha_i} + 2)}} + \sum_{(4,4)} \sqrt{\frac{4+4-2}{4 \times 4}} \\
&+ \sum_{(d_{\alpha_i}+2, 4)} \sqrt{\frac{d_{\alpha_i} + 2 + 4 - 2}{4(d_{\alpha_i} + 2)}} + \sum_{(4, d_{\beta_i}+2)} \sqrt{\frac{d_{\beta_i} + 2 + 4 - 2}{4(d_{\beta_i} + 2)}} \\
&+ \sum_{(d_{\beta_i}+2, d_{\beta_i}(d_{\beta_i}-1)+2)} \sqrt{\frac{d_{\beta_i}(d_{\beta_i}-1) + 2 + d_{\beta_i} + 2 - 2}{(d_{\beta_i}(d_{\beta_i}-1) + 2)(d_{\beta_i} + 2)}} \\
&= \binom{d_\gamma}{2} \frac{\sqrt{2}}{\sqrt{d_\gamma^2 - d_\gamma + 2}} + (k-4)q \frac{\sqrt{6}}{4} \\
&+ \xi_i \left[\sqrt{\frac{d_{\alpha_i}^2 + 2}{(d_{\alpha_i}^2 - d_{\alpha_i} + 2)(d_{\alpha_i} + 2)}} + \frac{1}{2} \sqrt{\frac{d_{\alpha_i} + 4}{d_{\alpha_i} + 2}} \right. \\
& \left. + \frac{1}{2} \sqrt{\frac{d_{\beta_i} + 4}{d_{\beta_i} + 2}} + \sqrt{\frac{d_{\beta_i}^2 + 2}{(d_{\beta_i} + 2)(d_{\beta_i}^2 - d_{\beta_i} + 2)}} \right].
\end{aligned}$$

□

Theorem 4.4. Let $H \cong G(k)$ (for $k \geq 4$), The generalized $GA_5(H)$ index is

$$\begin{aligned}
GA_5(L(H)) &= \binom{d_\gamma}{2} + (k-4)q \\
&+ \xi_i \left[\frac{2\sqrt{d_{\alpha_i}^3 + d_{\alpha_i}^2 + 4}}{d_{\alpha_i} + 4} + \frac{4\sqrt{d_{\alpha_i} + 2}}{d_{\alpha_i} + 6} + \frac{2\sqrt{d_{\beta_i}^3 + d_{\beta_i}^2 + 4}}{d_{\beta_i} + 4} + \frac{4\sqrt{d_{\alpha_i} + 2}}{d_{\alpha_i} + 6} \right].
\end{aligned}$$

Proof. From Eq. (4),

$$GA_5(L(H)) = \sum_{uv \in E(L(H))} \frac{2\sqrt{s_u s_v}}{s_u + s_v}$$

$$\begin{aligned}
&= \sum_{(d_\gamma(d_\gamma-1)+2, d_\gamma(d_\gamma-1)+2)} \frac{2\sqrt{(d_\gamma(d_\gamma-1)+2)^2}}{2(d_\gamma(d_\gamma-1)+2)} \\
&+ \sum_{(d_{\alpha_i}(d_{\alpha_i}-1)+2, d_{\alpha_i}+2)} \frac{2\sqrt{(d_{\alpha_i}+2)(d_{\alpha_i}(d_{\alpha_i}-1)+2)}}{d_{\alpha_i}(d_{\alpha_i}-1)+2+d_{\alpha_i}+2} \\
&+ \sum_{(d_{\alpha_i}+2, 4)} \frac{2\sqrt{4(d_{\alpha_i}+2)}}{d_{\alpha_i}+2+4} + \sum_{(4,4)} \frac{2\sqrt{4 \times 4}}{4+4} + \sum_{(4, d_{\beta_i}+2)} \frac{2\sqrt{4(d_{\beta_i}+2)}}{d_{\beta_i}+2+4} \\
&+ \sum_{(d_{\beta_i}+2, d_{\beta_i}(d_{\beta_i}-1)+2)} \frac{2\sqrt{(d_{\beta_i}+2)(d_{\beta_i}(d_{\beta_i}-1)+2)}}{d_{\beta_i}+2+d_{\beta_i}(d_{\beta_i}-1)+2} \\
&= \binom{d_\gamma}{2} + (k-4)q \\
&+ \xi_i \left[\frac{2\sqrt{d_{\alpha_i}^3 + d_{\alpha_i}^2 + 4}}{d_{\alpha_i} + 4} + \frac{4\sqrt{d_{\alpha_i} + 2}}{d_{\alpha_i} + 6} + \frac{2\sqrt{d_{\beta_i}^3 + d_{\beta_i}^2 + 4}}{d_{\beta_i} + 4} + \frac{4\sqrt{d_{\alpha_i} + 2}}{d_{\alpha_i} + 6} \right].
\end{aligned}$$

□

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