

A linear system involving the equation

$$x_1 + x_2 + c = x_0$$

Donald L. Vestal, Jr and Anthony Glackin*

ABSTRACT

Richard Rado's work in Ramsey Theory established conditions under which monochromatic solutions to a linear system must occur. In this paper, we find exact values for a linear system involving the equation $x_1 + x_2 + c = x_0$ and two colors: Let c and k be integers with $-1 \leq c \leq k$. Then the 2-color Rado number for the system

$$\begin{aligned} x_1 + x_2 + c &= x_0, \\ y_1 + y_2 + k &= y_0, \end{aligned}$$

is infinite if c and k have opposite parity, and has a value of $4k+5$ if c and k have the same parity. We also extend this to the continuous result where we color the real numbers.

Keywords: Ramsey theory, rado number, linear system

2020 Mathematics Subject Classification: 05D10.

1. Introduction

In 1916, Issai Schur [21] introduced the combinatorial idea that for any coloring of the natural numbers using t colors, there must be three (not necessarily distinct) integers x, y , and z , all of which are the same color, which satisfy the equation $x+y = z$. (Equivalently, if the natural numbers are partitioned into t disjoint sets, then at least one of the subsets will contain a solution to the equation $x + y = z$.) Such a solution, where the three integers are assigned the same color, is called a monochromatic solution. Since such a monochromatic solution will eventually exist, there must be a smallest integer, which we'll denote $S(t)$, such that any coloring of the integers from 1 to $S(t)$ using t colors must admit

* Corresponding author.

Received 11 Feb 2025; Revised 01 Oct 2025; Accepted 01 Nov 2025; Published Online 21 Nov 2025.

DOI: [10.61091/cn236-07](https://doi.org/10.61091/cn236-07)

© 2025 The Author(s). Published by Combinatorial Press. This is an open access article under the CC BY license (<https://creativecommons.org/licenses/by/4.0/>).

a monochromatic solution. The first few (and only known) values are $S(1) = 2, S(2) = 5, S(3) = 14$, and $S(4) = 161$ (OEIS sequence A030126 [16]).

Following this, Richard Rado (a student of Isai Schur) furthered the work of his mentor to find similar results [17] for linear equations and, more generally, systems of linear equations. Consequently, we have the idea of a t -color Rado number for a system \mathcal{E} of equations: this will be the smallest natural number $S = S(\mathcal{E}, t)$ (if it exists) such that for any coloring of the integers from 1 to S using t colors, there must be a monochromatic solution to the system \mathcal{E} .

While the work of Schur and Rado concerned the existence of monochromatic solutions, in 1982, Beutelspacher and Brestovansky [1] generalized Schur's result by finding the exact values of some of these Rado numbers.

Beutelspacher and Brestovansky For any positive integer k , the 2-color Rado number for the equation $x_1 + x_2 + \cdots + x_k = x_0$ is $k^2 + k - 1$.

There have been many families of equations that have been studied; for example:

- [3] $x + y = kz$,
- [7] $a(x + y) = bz$,
- [19] $x_1 + x_2 + \cdots + x_{m-1} + c = x_m$ where $c \geq 0$,
- [12] $x_1 + x_2 + \cdots + x_{m-1} + c = x_m$ where $c < 0$,
- [20] $x_1 + x_2 + \cdots + x_{m-1} = 2x_m$,
- [6] $a_1x_1 + a_2x_2 + \cdots + a_mx_m = x_0$.

Those equations are all linear, but there have been some results involving nonlinear equations as well:

- [8] $x^2 + y^2 = z^2$,
- [14] $x_1^2 + x_2^2 + \cdots + x_{m-1}^2 = x_m^2$,
- [14] $\frac{1}{x} + \frac{1}{y} = \frac{1}{z}$,
- [5] $x + y^n = z$.

There have also been variations involving multiple equations:

- disjunctive Rado numbers, where we are interested in a solution to one equation or to the other ([4, 10, 13]),
- off-diagonal Rado numbers, where different equations are assigned different colors ([9, 15, 18, 22]).

But while these examples all involve solutions to a single equation, Rado's original result was about linear systems. The examples of disjunctive Rado numbers and off-diagonal Rado numbers involve multiple equations, but not necessarily a simultaneous solution to both (or all) equations. Thus far, we are only aware of one reference that applies to linear systems: in [11], the authors show that the 2-color Rado number for the linear system

$$\begin{aligned} x_1 + x_2 + \cdots + x_n &= z, \\ y_1 + y_2 &= z, \end{aligned}$$

is $n^2 - n + 1$.

For this article, we will consider linear systems involving the equation $x_1 + x_2 + c = x_0$. In an unpublished work, Burr and Loo [3] proved that the 2-color Rado number for this equation, when $c \geq 0$, is $4c + 5$. (This can be trivially expanded to include $c = -1$.) For completeness, we will give the proof in the next section, which will be expanded upon

to prove our main result. The linear system that we will consider will be formed from two equations of this form, but with different variables and possibly different constants. (We'll discuss some cases involving shared variables in the Future Work section as well.) We will also show how the result extends to a continuous version on the real numbers, as in the work of Brady and Haas [2].

2. The discrete version of our linear system

In this section, we prove the following result.

Theorem 2.1. *Let c and k be integers with $-1 \leq c \leq k$. Then the 2-color Rado number for the system*

$$\begin{aligned}x_1 + x_2 + c &= x_0, \\y_1 + y_2 + k &= y_0,\end{aligned}$$

is infinite if c and k have opposite parity, and has a value of $4k + 5$ if c and k have the same parity.

Before giving the proof, we will show the proof of Burr and Loo from [3], as that will help in the proof of Theorem 2.1.

Theorem 2.2 (Burr and Loo). *The 2-color Rado number for the equation $x_1 + x_2 + c$, where c is an integer with $c \geq -1$, is $4c + 5$.*

Proof. When claiming that the 2-color Rado number for this equation is $4c + 5$, we are saying that this is the smallest positive integer r for which any 2-coloring of the integers from 1 to r must have a monochromatic solution to our equation. This means that we must prove two things: there is a coloring of the integers from 1 to $4c + 4$ which does not allow a monochromatic solution (thus establishing the lower bound) and then for any 2-coloring of the integers from 1 to $4c + 5$ there must be a monochromatic solution (thus establishing the upper bound).

For the lower bound, we will prove a stronger result, which will also be used in the continuous version. Namely, the following coloring (using red and blue) of the real numbers in the interval $[1, 4c + 5)$ avoids a monochromatic solution:

$$\begin{aligned}\text{Red:} & \quad [1, c + 2) \cup [3c + 4, 4c + 5), \\ \text{Blue:} & \quad [c + 2, 3c + 4),\end{aligned}$$

Without loss of generality, we may restrict ourselves to considering real solutions with $x_1 \leq x_2$. Clearly there is no blue solution, since $c + 2 \leq x_1 \leq x_2$ would make $3c + 4 \leq x_1 + x_2 + c$, and so x_0 could not be blue.

Suppose x_1 and x_2 were colored red. If we had x_1 and x_2 coming from the first red interval, so $1 \leq x_1 \leq x_2 < c + 2$, then we would get $c + 2 \leq x_1 + x_2 + c < 3c + 4$, which would make x_0 blue. If x_2 came from the second red interval, then we would have $1 \leq x_1$ and $3c + 4 \leq x_2$, which would make $4c + 5 \leq x_1 + x_2 + c$ and so x_0 would be outside the range under consideration.

Since there are no monochromatic real solutions to our equation, there are clearly no monochromatic integer solutions.

For the upper bound, we prove that for any 2-coloring of the integers 1 through $4c+5$, there must be a monochromatic solution to our equation. To show this, we may start by assuming (without loss of generality) that 1 is red, and show that this will eventually force a monochromatic solution. To this end, we will introduce the following notation: assuming that we are trying to avoid a monochromatic solution, a given coloring and a given solution (written as an ordered triple (x_1, x_2, x_0)) may force an extension of our coloring. One example of this:

$$(1, 1, c+2) \rightarrow c+2 \text{ is blue.}$$

Since 1 is red, and $x_1 = 1, x_2 = 1, x_0 = c+2$ is a solution to our equation, then trying to avoid a monochromatic (specifically red in this case) solution will force $c+2$ to be blue. Following this logic, we can force several specific color assignments:

$$\begin{aligned} (1, 1, c+2) &\rightarrow c+2 \text{ is blue,} \\ (c+2, c+2, 3c+4) &\rightarrow 3c+4 \text{ is red,} \\ (1, 3c+4, 4c+5) &\rightarrow 4c+5 \text{ is blue,} \\ (1, 2c+3, 3c+4) &\rightarrow 2c+3 \text{ is blue.} \end{aligned}$$

So far, 1 being red, along with our assumption that there are no monochromatic solutions, has forced 1 and $3c+4$ to be red and $c+2, 2c+3$, and $4c+5$ to be blue. But this then results in the monochromatic (blue) solution:

$$(c+2) + (2c+3) + c = 4c+5.$$

This completes the proof of Burr and Loo's result. □

We now turn to the proof of Theorem 2.1.

Proof of Theorem 2.1. First, we establish the result for the case when c and k have opposite parity. The claim is that the Rado number is infinite, which can be shown by the following coloring: color the odd integers red and the even integers blue.

Note that our equations can be rewritten as $c = x_0 - x_1 - x_2$ and $k = y_0 - y_1 - y_2$. If x_1, x_2, x_0, y_1, y_2 , and y_0 are all red, then they are all odd. This would force both c and k to be odd, which is not the case. Similarly, if x_1, x_2, x_0, y_1, y_2 , and y_0 are all blue, then they are all even, which would force both c and k to be even. Therefore, this coloring avoids a monochromatic solution to our system.

Now suppose c and k have the same parity. To show that the Rado number is at least $4k+5$, we need to give a coloring of the integers 1 through $4k+4$ that avoids a monochromatic solution. But we can use the coloring from the proof of Burr and Loo's result, since that coloring avoids a monochromatic solution to the second equation $y_1 + y_2 + k = y_0$.

Finally, we need to show that any coloring of the integers 1 through $4k+5$ must result in a monochromatic solution to our linear system. From the proof of Burr and Loo, we

know that any 2-coloring integers 1 through $4k+5$ will result in a monochromatic solution to the k -equation and since $c \leq k$, also a monochromatic solution to the c -equation. We want to prove that there is a monochromatic solution to the system, meaning that we need not just monochromatic solutions to both equations, but in the same color.

Suppose, by way of contradiction, that we only have red solutions to the c -equation and only blue solutions to the k -equation. In other words, for our given coloring of the integers from 1 to $4k+5$, there are no red solutions to the k -equation and no blue solutions to the c -equation. We will derive a contradiction based on the coloring of the number 1.

Case 1. 1 is Red.

Using our previous notation (along with subscripts to distinguish which equation our solution corresponds to), we have

$$(1, 1, k+2)_k \rightarrow k+2 \text{ is blue,}$$

to denote the fact that 1 being red and $(1, 1, k+2)_k$ being a solution to the k -equation, along with assumption that there are no red solutions to the k -equation, will force $k+2$ to be blue. Continuing in this way, we have:

$$\begin{aligned} \left(\frac{k-c}{2} + 1, \frac{k-c}{2} + 1, k+2\right)_c &\rightarrow \frac{k-c}{2} + 1 \text{ is red,} \\ \left(\frac{k-c}{2} + 1, \frac{k-c}{2} + 1, 2k-c+2\right)_k &\rightarrow 2k-c+2 \text{ is blue,} \\ (k+2, 2k-c+2, 3k+4)_c &\rightarrow 3k+4 \text{ is red,} \\ (1, 3k+4, 4k+5)_k &\rightarrow 4k+5 \text{ is blue,} \\ (1, 2k+3, 3k+4)_k &\rightarrow 2k+3 \text{ is blue.} \end{aligned}$$

This then forces us into the blue solution $(2k-c+2, 2k+3, 4k+5)_c$ to the c -equation, which is a contradiction.

Case 2 1 is Blue.

Proceeding as before, we have:

$$\begin{aligned} (1, 1, c+2)_c &\rightarrow c+2 \text{ is red,} \\ (c+2, c+2, k+2c+4)_k &\rightarrow k+2c+4 \text{ is blue,} \\ (1, k+2c+4, k+3c+5)_c &\rightarrow k+3c+5 \text{ is red,} \\ (1, k+c+3, k+2c+4)_c &\rightarrow k+c+3 \text{ is red,} \\ (c+2, 2c+3, k+3c+5)_k &\rightarrow 2c+3 \text{ is blue,} \\ (c+2, k+c+3, 2k+2c+5)_k &\rightarrow 2k+2c+5 \text{ is blue,} \\ (1, 2k+c+4, 2k+2c+5)_c &\rightarrow 2k+c+4 \text{ is red,} \\ (k+2, c+2, 2k+c+4)_k &\rightarrow k+2 \text{ is blue,} \\ \left(\frac{k-c}{2} + 1, \frac{k-c}{2} + 1, k+2\right)_c &\rightarrow \frac{k-c}{2} + 1 \text{ is red,} \\ \left(\frac{k-c}{2} + 1, \frac{k-c}{2} + 1, 2k-c+2\right)_k &\rightarrow 2k-c+2 \text{ is blue.} \end{aligned}$$

This then forces us into the blue solution $(2c + 3, 2k - c + 2, 2k + 2c + 5)_c$ to the c -equation, which is a contradiction. \square

Note that both cases here involve the number $\frac{k-c}{2} + 1$; this has to be an integer, which requires c and k to have the same parity.

3. The continuous version of our linear system

In this section, we extend our result to the real numbers, similar to the following result from [2].

Theorem 3.1 (Brady and Haas). *For a given positive real number γ and a positive integer m , suppose that c is a real number which satisfies $-c < \gamma(m - 1)$. Let $R = R(\gamma, m, c)$ denote the least real number such that for any 2-coloring of the real numbers in the interval $[\gamma, R]$, there must be a monochromatic solution to the equation $x_1 + x_2 + \cdots + x_m + c = x_0$. Then $R(\gamma, m, c) = \gamma(m^2 + m - 1) + (m + 2)c$.*

For the continuous version of our linear system, we have the following result.

Theorem 3.2. *Let c and k be real numbers with $-1 \leq c \leq k$. Then the 2-color continuous Rado number for the system*

$$\begin{aligned}x_1 + x_2 + c &= x_0, \\y_1 + y_2 + k &= y_0,\end{aligned}$$

is $4k + 5$.

As in the discrete case, the 2-color continuous Rado number for a system of equations is the smallest real number r with $r \geq 1$ such that for every 2-coloring of the real numbers in the interval $[1, r]$, there must be a monochromatic solution to our system.

To establish the lower bound for this claim, we would need to exhibit a 2-coloring of the interval $[1, 4k + 5)$ that avoid a monochromatic solution. However, we have such a coloring: in the proof of Burr and Loo's result, we showed a coloring of $[1, 4k + 5)$ (replace c with k) that avoids monochromatic solutions to the k -equation.

To establish the upper bound, we need to show that any 2-coloring of $[1, 4k + 5]$ must give a monochromatic solution to our system. But for this, we can use the proof of the upper bound from Theorem 2.1. The numbers used no longer need to be integers, but the algebra still works out the same.

In fact, similar to the result of Brady and Haas, we can further generalize this to start our coloring at any real number γ . (While the value of γ is assumed to be positive in [2], their proof doesn't require this. This result, however, can also follow from the proof of Theorem 3.3 below, with $c = k$.)

Theorem 3.3. *Let c, k , and γ be real numbers with $-\gamma \leq c \leq k$. Then the 2-color*

continuous Rado number for the system

$$\begin{aligned}x_1 + x_2 + c &= x_0, \\y_1 + y_2 + k &= y_0,\end{aligned}$$

where the coloring starts at γ is $4k + 5\gamma$.

Proof. For the lower bound, we can use a variation of the coloring from the proof of Burr and Loo's result; a 2-coloring of the interval $[\gamma, 4k + 5\gamma)$ which avoids a monochromatic solution to the equation $y_1 + y_2 + k$ (and thus a monochromatic solution to our linear system) is:

$$\begin{aligned}\text{Red:} & \quad [\gamma, k + 2\gamma) \cup [3k + 4\gamma, 4k + 5\gamma), \\ \text{Blue:} & \quad [k + 2\gamma, 3k + 4\gamma).\end{aligned}$$

Clearly there is no blue solution, since $k + 2\gamma \leq x_1 \leq x_2$ would make $3k + 4\gamma \leq x_1 + x_2 + k$, and so x_0 could not be blue.

To show that there is no red solution, suppose x_1 and x_2 were colored red. If we had x_1 and x_2 coming from the first red interval, so $\gamma \leq x_1 \leq x_2 < k + 2\gamma$, then we would get $k + 2\gamma \leq x_1 + x_2 + k < 3k + 4\gamma$, which would make x_0 blue. If x_2 came from the second red interval, then we would have $\gamma \leq x_1$ and $3k + 4\gamma \leq x_2$, which would make $4k + 5\gamma \leq x_1 + x_2 + k$ and so x_0 would be outside our range.

For the upper bound, we can also generalize our proof of Theorem 2.1. Suppose we have a 2-coloring of the interval $[\gamma, 4k + 5\gamma]$. As in the proof of Burr and Loo's result, one of the solutions (to the k -equation) listed below must be monochromatic:

$$\begin{aligned}(\gamma, \gamma, k + 2\gamma)_k \\ (k + 2\gamma, k + 2\gamma, 3k + 4\gamma)_k \\ (\gamma, 3k + 4\gamma, 4k + 5\gamma)_k \\ (\gamma, 2k + 3\gamma, 3k + 4\gamma)_k \\ (k + 2\gamma, 2k + 3\gamma, 4k + 5\gamma)_k.\end{aligned}$$

Similarly, replacing k with c , since $c \leq k$, there must also be a monochromatic solution to the c -equation. We need to show that both equations have a monochromatic solution in the same color. Suppose, by way of contradiction, that we have no red solutions to the k -equation and no blue solutions to the c -equation. We will derive a contradiction based on the coloring of the number γ .

Case 1. γ is Red.

Using our previous notation (along with subscripts to distinguish which equation our

solution corresponds to), we have

$$\begin{aligned}
& (\gamma, \gamma, k + 2\gamma)_k \rightarrow k + 2\gamma \text{ is blue,} \\
& \left(\frac{k-c}{2} + \gamma, \frac{k-c}{2} + \gamma, k + 2\gamma \right)_c \rightarrow \frac{k-c}{2} + \gamma \text{ is red,} \\
& \left(\frac{k-c}{2} + \gamma, \frac{k-c}{2} + \gamma, 2k - c + 2\gamma \right)_k \rightarrow 2k - c + 2\gamma \text{ is blue,} \\
& (k + 2\gamma, 2k - c + 2\gamma, 3k + 4\gamma)_c \rightarrow 3k + 4\gamma \text{ is red,} \\
& (\gamma, 3k + 4\gamma, 4k + 5\gamma)_k \rightarrow 4k + 5\gamma \text{ is blue,} \\
& (\gamma, 2k + 3\gamma, 3k + 4\gamma)_k \rightarrow 2k + 3\gamma \text{ is blue.}
\end{aligned}$$

This then forces us into the blue solution $(2k - c + 2\gamma, 2k + 3\gamma, 4k + 5\gamma)_c$ to the c -equation, which is a contradiction.

Case 2. γ is Blue.

Proceeding as before, we have:

$$\begin{aligned}
& (\gamma, \gamma, c + 2\gamma)_c \rightarrow c + 2\gamma \text{ is red,} \\
& (c + 2\gamma, c + 2\gamma, k + 2c + 4\gamma)_k \rightarrow k + 2c + 4\gamma \text{ is blue,} \\
& (\gamma, k + 2c + 4\gamma, k + 3c + 5\gamma)_c \rightarrow k + 3c + 5\gamma \text{ is red,} \\
& (\gamma, k + c + 3\gamma, k + 2c + 4\gamma)_c \rightarrow k + c + 3\gamma \text{ is red,} \\
& (c + 2\gamma, 2c + 3\gamma, k + 3c + 5\gamma)_k \rightarrow 2c + 3\gamma \text{ is blue,} \\
& (c + 2\gamma, k + c + 3\gamma, 2k + 2c + 5\gamma)_k \rightarrow 2k + 2c + 5\gamma \text{ is blue,} \\
& (\gamma, 2k + c + 4\gamma, 2k + 2c + 5\gamma)_c \rightarrow 2k + c + 4\gamma \text{ is red,} \\
& (k + 2\gamma, c + 2\gamma, 2k + c + 4\gamma)_k \rightarrow k + 2\gamma \text{ is blue,} \\
& \left(\frac{k-c}{2} + \gamma, \frac{k-c}{2} + \gamma, k + 2\gamma \right)_c \rightarrow \frac{k-c}{2} + \gamma \text{ is red,} \\
& \left(\frac{k-c}{2} + \gamma, \frac{k-c}{2} + \gamma, 2k - c + 2\gamma \right)_k \rightarrow 2k - c + 2\gamma \text{ is blue.}
\end{aligned}$$

This then forces us into the blue solution $(2c + 3\gamma, 2k - c + 2\gamma, 2k + 2c + 5\gamma)_c$ to the c -equation, which is a contradiction. \square

4. Conclusion and future work

Having established our result for a system where the equations share no variables, it would be natural to consider what happens when some of the variables are shared. Clearly we can't have the exact same variables if c and k are distinct. For $c \neq k$ the system

$$\begin{aligned}
x_1 + x_2 + c &= x_0, \\
x_1 + x_2 + k &= x_0,
\end{aligned}$$

will not have a solution, much less a monochromatic solution.

In a system where two variables are shared, the 2-color Rado number will be infinite. For example, we have the following.

Let c and k be integers with $-1 \leq c < k$. Then the 2-color Rado number for the system

$$\begin{aligned}x_1 + x_2 + c &= y, \\x_1 + x_2 + k &= z,\end{aligned}$$

is infinite.

If there was a solution to this system, then we could write

$$y - c = x_1 + x_2 = z - k,$$

which would mean that $z = y + (k - c)$. Since c and k are distinct in this statement, we could create a 2-coloring that avoids having y and z with the same color: letting $t = k - c$, we can use

$$\begin{aligned}\text{Red: } & \{1, \dots, t\} \cup \{2t + 1, \dots, 3t\} \cup \{4t + 1, \dots, 5t\} \cup \dots, \\ \text{Blue: } & \{t + 1, \dots, 2t\} \cup \{3t + 1, \dots, 4t\} \cup \{5t + 1, \dots, 6t\} \cup \dots\end{aligned}$$

Or, equivalently, for a given integer x , take the value of $q = \lfloor \frac{x}{k-c} \rfloor$, and assign the color red if q is even and blue if q is odd. If two integers differ by $k - c$, then this coloring will assign them different colors, thus making y and z have different colors.

A similar argument shows that the system

$$\begin{aligned}x + y_1 + c &= z, \\x + y_2 + k &= z,\end{aligned}$$

has an infinite 2-color Rado number when c and k are distinct. (In this case, y_1 and y_2 will differ by $k - c$, and the coloring above also avoids a monochromatic solution.)

If we create a system with just one shared variable (as in [11]), then we appear to have the same conclusion as in Theorem 2.1.

Conjecture 4.1. *Let c and k be integers with $-1 \leq c \leq k$. Then the 2-color Rado number for the system*

$$\begin{aligned}x_1 + x_2 + c &= z, \\y_1 + y_2 + k &= z,\end{aligned}$$

is infinite if c and k have opposite parity, and a value of $4k + 5$ if c and k have the same parity.

Clearly the 2-color Rado number for this system is at least as big as $4k + 5$, since any solution here will be a solution to the system described in Theorem 2.1. Computer results suggest that this conjecture is true, along with the following.

Conjecture 4.2. *Let c and k be integers with $-1 \leq c \leq k$. Then the 2-color Rado number for the system*

$$\begin{aligned}x_1 + z + c &= x_0, \\y_1 + z + k &= y_0,\end{aligned}$$

is infinite if c and k have opposite parity, and has a value of $4k + 5$ if c and k have the same parity.

This paper only touches on the Burr-Loo type equation, $x_1 + x_2 + c = x_0$. Future work could look at equations with $c \leq -1$, or with more variables: $x_1 + x_2 + \cdots + x_n + c = x_0$.

Acknowledgment

The authors wish to thank both referees, especially for their excellent comments which led to a generalization for the original form of Theorem 3.3.

References

- [1] A. Beutelspacher and W. Brestovansky. Generalized SCHUR numbers. In *Lecture Notes in Mathematics*. Volume 969, pages 30–38. Springer, 1982. <https://doi.org/10.1007/BFb0062985>.
- [2] C. Brady and R. Haas. Rado numbers for the real line. *Congressus Numerantium*, 177:109–114, 2005.
- [3] S. A. Burr and S. Loo. On rado numbers i. Preprint.
- [4] A. Dileep, J. Moondra, and A. Tripathi. On disjunctive rado numbers for some sets of equations. *Electronic Journal of Combinatorics*, 31(1):P1.69, 2024. <https://doi.org/10.37236/12012>.
- [5] A. Doss, D. Saracino, and D. Vestal. An excursion into nonlinear ramsey theory. *Graphs and Combinatorics*, 29:407–415, 2013. <https://doi.org/10.1007/s00373-012-1140-8>.
- [6] S. Guo and Z. Sun. Determination of the two-color rado number for $a_1x_1 + \cdots + a_mx_m = x_0$. *Journal of Combinatorial Theory, Series A*, 115(2):345–353, 2008. <https://doi.org/10.1016/j.jcta.2007.06.001>.
- [7] H. Harborth and S. Maasberg. Rado number for $a(x + y) = bz$. *Journal of Combinatorial Theory, Series A*, 80(2):356–363, 1997. <https://doi.org/10.1006/jcta.1997.2810>.
- [8] M. J. H. Heule, O. Kullmann, and V. W. Marek. Solving and verifying the boolean pythagorean triples problem via cube-and-conquer. In N. Creignou and D. L. Berre, editors, *Proceedings of SAT*. Volume 9710, Lecture Notes in Computer Science, pages 228–245. Springer, 2016. https://doi.org/10.1007/978-3-319-40970-2_15.
- [9] J. Jing and Y. M. Mei. On some exact formulas for 2-color off-diagonal rado numbers. *Journal of Combinatorial Mathematics and Combinatorial Computing*, 119:75–83, 2024. <https://doi.org/10.61091/jcmcc119-08>.
- [10] B. Johnson and D. Schaal. Disjunctive rado numbers. *Journal of Combinatorial Theory, Series A*, 112(2):263–276, 2005. <https://doi.org/10.1016/j.jcta.2005.02.007>.
- [11] B. M. Kim, W. Hwang, and B. C. Song. 2-color rado number for $x_1 + x_2 + \cdots + x_n = y_1 + y_2 = z$. *Korean Journal of Mathematics*, 28(2):379–389, 2020. <https://doi.org/10.11568/kjm.2020.28.2.379>.
- [12] W. Kosek and D. Schaal. Rado numbers for the equation $\sum_{i=1}^{m-1} x_i + c = x_m$ for negative values of c . *Advances in Applied Mathematics*, 27(4):805–815, 2001. <https://doi.org/10.1006/aama.2001.0762>.

-
- [13] W. Kosek and D. Schaal. A note on disjunctive rado numbers. *Advances in Applied Mathematics*, 31(2):433–439, 2003. [https://doi.org/10.1016/S0196-8858\(03\)00020-4](https://doi.org/10.1016/S0196-8858(03)00020-4).
- [14] K. Myers and J. Parrish. Some nonlinear rado numbers. *INTEGERS: Electronic Journal of Combinatorial Number Theory*, 18B:A6, 2018.
- [15] K. Myers and A. Robertson. Two color off-diagonal rado-type numbers. *Electronic Journal of Combinatorics*, 14(1):R53, 2007. <https://doi.org/10.37236/971>.
- [16] OEIS Foundation Inc. Schur's numbers, entry a030126 in the on-line encyclopedia of integer sequences. <http://oeis.org/A030126>, 2023.
- [17] R. Rado. Studien zur kombinatorik. *Mathematische Zeitschrift*, 36:424–480, 1933.
- [18] A. Robertson and D. Schaal. Off-diagonal generalized SCHUR numbers. *Advances in Applied Mathematics*, 26(3):252–257, 2001. <https://doi.org/10.1006/aama.2000.0718>.
- [19] D. Sabo, D. Schaal, and J. Tokaz. Disjunctive rado numbers for $x_1 + x_2 + c = x_3$. *INTEGERS: Electronic Journal of Combinatorial Number Theory*, 7:A2, 2007.
- [20] D. Schaal and D. Vestal. Rado numbers for $x_1 + x_2 + \cdots + x_{m-1} = 2x_m$. *Congressus Numerantium*, 191:105–116, 2008.
- [21] I. Schur. Über die kongruenz $x^m + y^m = z^m \pmod{p}$. *Jahresbericht der Deutschen Mathematiker-Vereinigung*, 25:114–116, 1916.
- [22] O. X. M. Yao and E. X. W. Xia. Two formulas of 2-color off-diagonal rado numbers. *Graphs and Combinatorics*, 31:299–307, 2015. <https://doi.org/10.1007/s00373-013-1378-9>.

Donald L. Vestal, Jr

Department of Mathematics & Statistics, South Dakota State University

Brookings, South Dakota 57007

E-mail donald.vestal@sdstate.edu

Anthony Glackin

Department of Mathematics & Statistics, South Dakota State University

Brookings, South Dakota 57007

E-mail anthony.glackin@jacks.sdstate.edu