Journal of Combinatorial Mathematics and Combinatorial Computing, 117: 55–64
DOI:10.61091/jcmcc117-06
http://www.combinatorialpress.com/jcmcc
Received 12 April 2019, Accepted 27 December 2019, Published 14 November 2023

Article

On Regular Distance Magic Graphs of Odd Order

Petr Kovář[1,2*] **Adam Silber**[1] **Pavla Kabelíková–Hrušková**[1] **and Michal Kravčenko**[1,2]

[1] VŠB – Technical University of Ostrava, Department of Applied Mathematics, Czech Republic
[2] IT4Innovations, VŠB – Technical University of Ostrava, Ostrava-Poruba, Czech Republic

* **Correspondence:** petr.kovar@vsb.cz

Abstract: Let $G = (V, E)$ be a graph with n vertices. A bijection $f : V \to \{1, 2, \dots, n\}$ is called a *distance magic labeling* of G if there exists an integer k such that $\sum_{u \in N(v)} f(u) = k$ for all $v \in V$, where $N(v)$ is the set of all vertices adjacent to v. Any graph which admits a distance magic labeling is a *distance magic graph*. The existence of regular distance magic graphs of even order was solved completely in a paper by Fronček, Kovář, and Kovářová. In two recent papers, the existence of 4-regular and of $(n-3)$-regular distance magic graphs of odd order was also settled completely. In this paper, we provide a similar classification of all feasible odd orders of r-regular distance magic graphs when $r = 6, 8, 10, 12$. Even though some nonexistence proofs for small orders are done by brute force enumeration, all the existence proofs are constructive.

Keywords: Graph labeling, distance magic, regular graphs
Mathematics Subject Classification: 05C70 , 05C78

---

1. Introduction and known results

Magic labelings were introduced in the 1960s. The concept became popular around 2000, and nowadays hundreds of papers on various magic-type labelings can be found. For a comprehensive survey we recommend [1].

In this paper we focus on distance magic labeling. Let $G = (V, E)$ be a simple undirected graph of order n. The *distance magic labeling* of G is a bijection $f : V \to \{1, 2, \dots, n\}$ for which such an integer k exists that $\sum_{u \in N(v)} f(u) = k$ for all $v \in V$, where $N(v)$ is the set of all vertices adjacent to v. The sum $\sum_{u \in N(v)} f(u)$ is the *weight* $w_f(v)$ of the vertex v and the constant k is the *magic constant* of f. If a graph admits a distance magic labeling, it is called a *distance magic graph*. For convenience we label vertices by their labels.

The same concept was also introduced independently by other authors and it was called *1-vertex magic vertex* labeling [2] or *sigma* labeling (Σ-labeling). Nowadays, the terminology seems to have settled on "distance magic labeling".

An interesting motivation for distance magic labelings of regular graphs is the scheduling of fair incomplete tournaments. In a complete tournament of n teams we can assign each team its *strength* from 1 to n according its rank. The best team has a rank of 1 and strength n, a rank 2 team has strength $n - 1$, etc.

In a complete tournament each team meets all $n - 1$ opponents, so the total strength of opponents of the best team is the lowest, while the last team plays $n - 1$ teams with the highest total strength. In an incomplete tournament each team plays just $r < n-1$ games and there naturally arises the problem of scheduling an *equalized* incomplete tournament in which the total strength of all opponents is the same for every team. This corresponds precisely to finding an r-regular distance magic graph with n vertices. At the same time the complement of such a graph represents the games of a *fair* incomplete tournament in which the total strength of all opponents of each team mimics the difficulty in a complete tournament. Both problems are directly related and the solution of one is also the solution to its complementary problem. For details, we refer to [2] or to survey [3]. In the former, the question of existence of regular distance magic graphs with an even number of vertices was settled completely:

Proposition 1 ([2]). *An r-regular distance magic graph of even order n exists if and only if $2 \le r \le n-2$, $r \equiv 0 \pmod 2$ and either $n \equiv 0 \pmod 4$ or $n \equiv r+2 \equiv 2 \pmod 4$.*

The existence of regular distance magic graphs of large odd orders was addressed in [4]. The constructions cover cases where regularity r is an odd number and the number of vertices is sufficiently large. For certain values of n and r, an r-regular distance magic graph can be constructed by compositions of regular graphs, see [5, 6]. The constructions in [4, 5] and [6] leave only a finite number of unsolved cases for each particular regularity r. For $(n-3)$-regular graphs, the following was shown in [6]:

Proposition 2 ([2]). *An $(n-3)$-regular distance magic graph G with n vertices exists iff $n \equiv 3 \pmod 6$. G is isomorphic to $K_{n/3}[\overline{K_3}]$.*

In [7], the existence of 4-regular distance magic graphs of odd order was also settled completely. In this paper, we give a complete list of all orders for which an r-regular distance magic graph exists when $r = 6, 8, 10, 12$. Not only do we close the gap of missing orders, but we also offer an alternative approach which gives a constructive proof for all feasible odd orders n. Moreover, for a fixed regularity we use a single construction. Contrary to the constructions in [4] the resulting graphs are always connected. Some nonexistence proofs for small orders are based on brute force (exhaustive computer search).

2. On 6-regular graphs

In this section, we show all orders for which a 6-regular distance magic graph exists. In this and all subsequent sections, f and f' always denote some distance magic labelings. First we prove the following.

Lemma 1. *Let G be a 6-regular graph of order n with a subgraph $K_{4,2}$ with partite sets $\{u_1, u_2, u_3, u_4\}$ and $\{v_1, v_2\}$. If G allows a distance magic labeling f such that the sums of vertex labels $f(u_1)+f(u_2) = f(u_3) + f(u_4) = f(v_1) + f(v_2) = n + 1$, then there exists a 6-regular distance magic graph of order $n + 4t$ for all non-negative integers t.*

Proof. It is enough to show that given a 6-regular distance magic graph with n vertices with a subgraph $K_{4,2}$ with partite sets $\{u_1, u_2, u_3, u_4\}$ and $\{v_1, v_2\}$ such that the sums of labels $f(u_1) + f(u_2) = f(u_3) + f(u_4) = f(v_1) + f(v_2) = n + 1$, we can alway construct a 6-regular distance magic graph with $n + 4$ vertices labeled by a distance magic labeling f' which contains another subgraph $K_{4,2}$ with some partite sets $\{u'_1, u'_2, u'_3, u'_4\}$ and $\{v'_1, v'_2\}$ such that the sums of labels $f'(u'_1) + f'(u'_2) = f'(u'_3) + f'(u'_4) = f'(v'_1) + f'(v'_2) = n + 5$ in f'.

Suppose G is a 6-regular distance magic graph of order n with distance magic labeling f. By H we denote the $K_{4,2}$ subgraph in G. In Figure 1, the edges of H are highlighted in red.

We show that there exists a distance magic labeling f' of G' with $n + 4$ vertices, where G' arises from G by removing the eight edges of H, adding four new vertices x_1, x_2, y_1, y_2 and eight edges of a $K_{4,2}$ with partite sets $\{u_1, u_2, v_1, v_2\}$ and $\{x_1, x_2\}$, another eight edges of a $K_{4,2}$ with partite sets

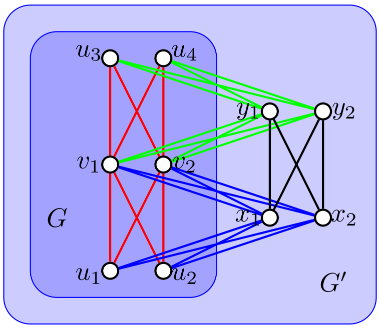

Figure 1. Adding four vertices to a 6-regular distance magic graph.

$\{v_1, v_2, u_3, u_4\}$ and $\{y_1, y_2\}$, and four edges of a $K_{2,2}$ with partite sets $\{x_1, x_2\}$ and $\{y_1, y_2\}$ respectively. In Figure 1, the edges are drawn in blue, green, and black. It is easy to observe that G' is connected and 6-regular with $n+4$ vertices. Consider the following labeling f' of G'.

$$f'(v) = \begin{cases} f(v)+2 & \text{for } v \in V(G), \\ 1 & \text{for } v = x_1, \\ n+4 & \text{for } v = x_2, \\ 2 & \text{for } v = y_1, \\ n+3 & \text{for } v = y_2. \end{cases}$$

Clearly, f' is a bijection $V(G') \to \{1, 2, \ldots, n+4\}$. Now, we show that the weight $w(v)$ of every vertex v in G' is $3(n+5)$. For x_1 and x_2 we have

$$\begin{aligned} w_{f'}(x_1) = w_{f'}(x_2) &= \sum_{i=1}^{2} (f'(u_i) + f'(v_i)) + f'(y_1) + f'(y_2) \\ &= 2(n+1) + 8 + 2 + (n+3) = 3(n+5). \end{aligned}$$

For y_1 and y_2 the computation is analogous. For u_1, u_2, u_3, u_4 two neighbors with the sum $n+1$ are replaced by two neighbors with the sum $n+5$, and the labels of each of the four remaining neighbors are increased by 2.

$$\begin{aligned} w_{f'}(u_1) &= w_{f'}(u_2) = w_{f'}(u_3) = w_{f'}(u_4) \\ &= w_f(u_1) - (n+1) + (n+5) + 8 = 3(n+1) + 12 = 3(n+5) \end{aligned}$$

For v_1, v_2 two pairs of neighbors each with the sum $n+1$ are replaced by two pairs of vertices with the sum $n+5$, and the labels of each of the two remaining neighbors are increased by 2.

$$\begin{aligned} w_{f'}(v_1) &= w_{f'}(v_2) \\ &= w_f(v_1) - 2(n+1) + 2(n+5) + 4 \\ &= w_f(v_1) + 12 = 3(n+1) + 12 = 3(n+5) \end{aligned}$$

Finally, for all remaining vertices v in G' we increase the weight by $6 \cdot 2 = 12$, so $w_{f'}(v) = w_f(v) + 12 = 3(n+1) + 12 = 3(n+5)$. Thus, f' is a distance magic labeling of G'. Also, all the green (as well as blue) edges form a $K_{4,2}$ subgraph with the sum of corresponding pairs of vertices $(n+4)+1$, and the claim follows by induction.

□

Theorem 1. *There exists a 6-regular distance magic graph of odd order n if and only if $n = 9$ or $n \geq 13$.*

Proof. For odd $n < 9$ clearly no such graph exists. It was shown by brute force that for odd $n = 11$ no 6-regular distance magic graph exists. Regular distance magic graphs for $n = 9$ and $n = 15$ are shown in Figures 2 and 3, respectively. The required subgraph $K_{4,2}$ is always highlighted in red. We check that $f(u_1) + f(u_2) = f(u_3) + f(u_4) = f(v_1) + f(v_2) = 2 + 8 = 3 + 7 = 1 + 9 = 10$ in $C_3[\overline{K}_3]$ and $f(u_1) + f(u_2) = f(u_3) + f(u_4) = f(v_1) + f(v_2) = 1 + 15 = 3 + 13 = 2 + 14 = 16$ in $C_5[\overline{K}_3]$. For all remaining odd $n \geq 13$ the result follows by Lemma 1.

□

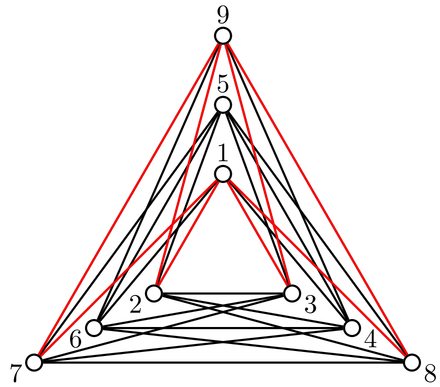

Figure 2. A 6-regular distance magic graph $C_3[\overline{K}_3]$ with 9 vertices and $k = 30$.

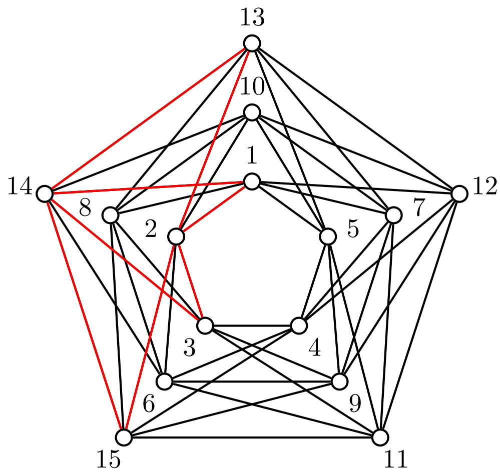

Figure 3. A 6-regular distance magic graph $C_5[\overline{K}_3]$ with 15 vertices and $k = 48$.

3. On 8-regular graphs

A similar classification of orders can be done for 8-regular distance magic graphs.

Lemma 2. *Let G be an 8-regular graph of order n with a subgraph $K_{2,2,2}$. If G allows a distance magic labeling f such that the sum of vertex labels in each partite set of the $K_{2,2,2}$ subgraph is $n + 1$, then there exists an 8-regular distance magic graph with $n + 4t$ vertices for all non-negative integers t.*

Proof. We show that given an 8-regular distance magic graph of order n with the $K_{2,2,2}$ subgraph with partite sets $\{u_1, u_2\}$, $\{v_1, v_2\}$ and $\{w_1, w_2\}$ in which the sums of labels $f(u_1) + f(u_2) = f(v_1) + f(v_2) = f(w_1) + f(w_2) = n + 1$, we can alway construct an 8-regular distance magic graph of order $n + 4$ which contains another $K_{2,2,2}$ such that the sum in each partite set is $n + 5$.

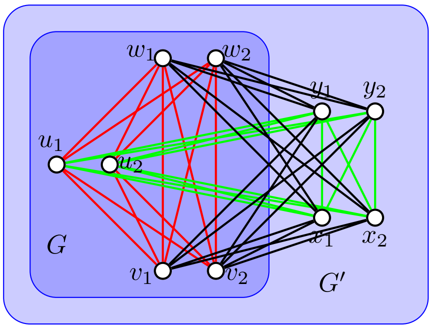

Figure 4. Adding four vertices to an 8-regular distance magic graph.

Suppose G is an 8-regular distance magic graph of order n with distance magic labeling f and the specified subgraph $K_{2,2,2}$. In Figure 4, the edges of $K_{2,2,2}$ are highlighted in red.

There exists a distance magic labeling of G' with $n+4$ vertices, where G' arises from G by removing the 12 edges of the $K_{2,2,2}$ subgraph, by adding 4 new vertices x_1, x_2, y_1, y_2, by adding 24 edges of a $K_{6,4}$ with partite sets $\{u_1, u_2, v_1, v_2, w_1, w_2\}$ and $\{x_1, x_2, y_1, y_2\}$, and by adding 4 edges of a $K_{2,2}$ with partite sets $\{x_1, x_2\}$ and $\{y_1, y_2\}$. In Figure 4, the edges are drawn in black and in green. It is easy to observe that G' is a connected 8-regular graph with $n + 4$ vertices. Consider the following labeling f' of G'.

$$f'(v) = \begin{cases} f(v) + 2 & \text{for } v \in V(G), \\ 1 & \text{for } v = x_1, \\ n + 4 & \text{for } v = x_2, \\ 2 & \text{for } v = y_1, \\ n + 3 & \text{for } v = y_2. \end{cases}$$

Using similar steps as in the proof of Lemma 1 it is easy to verify that f' is a distance magic labeling of G'. Moreover, all the green edges form a $K_{2,2,2}$ subgraph with the sum of vertex labels $(n + 4) + 1$ in each partite set, and the claim follows by induction.

□

Theorem 2. *There exists an 8-regular distance magic graph of odd order n if and only if $n \geq 15$.*

Proof. For odd $n < 11$ clearly no such graph exists. For $n = 11$ it does not exist according to Proposition 2. The case $n = 13$ was excluded by a brute force search.

Regular distance magic graphs for $n = 15$ and $n = 17$ are shown in Figures 5 and 6, respectively. The subgraphs $K_{2,2,2}$ required in Lemma 2 are highlighted in red. Clearly, the sums are $f(u_1)+f(u_2) = f(v_1) + f(v_2) = f(w_1) + f(w_2) = 1 + 15 = 4 + 12 = 5 + 11 = 16$ in Figure 5 and $f(u_1) + f(u_2) = f(v_1) + f(v_2) = f(w_1) + f(w_2) = 1 + 17 = 2 + 16 = 5 + 13 = 18$ in Figure 6. For all remaining odd $n \geq 19$ the result follows by Lemma 2.

□

4. On 10-regular graphs

In the same way we continue for 10-regular graphs.

Lemma 3. *Let G be a 10-regular graph of order n with a subgraph $C_4[\overline{K}_2]$. If G allows a distance magic labeling such that the sum of vertex labels is $n + 1$ for each pair of vertices corresponding to each $\overline{K}_2$ in the composition, then there exists a 10-regular distance magic graph with $n + 4t$ vertices for all non-negative integers t.*

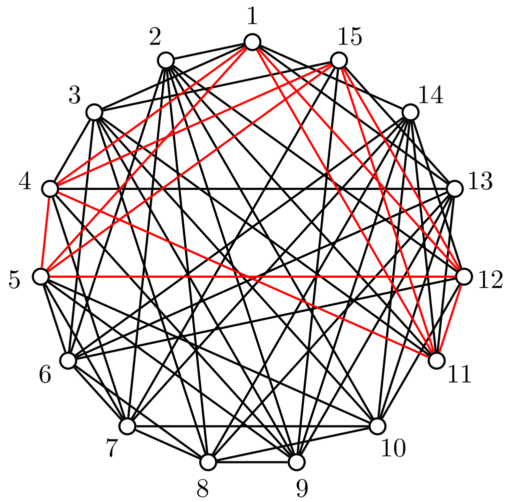

Figure 5. An 8-regular distance magic graph with 15 vertices and $k = 64$.

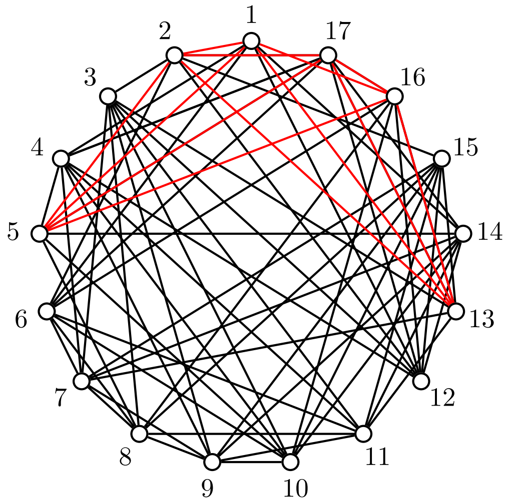

Figure 6. An 8-regular distance magic graph with 17 vertices and $k = 72$.

Proof. Suppose G is a 10-regular distance magic graph of order n with a distance magic labeling f and the specified subgraph $C_4[\overline{K_2}]$ ($C_4[\overline{K_2}]$ is isomorphic to $K_{4,4}$). We denote the vertices in each partite set of the subgraph $C_4[\overline{K_2}]$ by u_i, v_i, w_i, and z_i, respectively, where $i = 1, 2$ and the sums of labels $f(u_1) + f(u_2) = f(v_1) + f(v_2) = f(w_1) + f(w_2) = f(z_1) + f(z_2) = n + 1$. In Figure 7, the edges of the subgraph $C_4[\overline{K_2}]$ are highlighted in red.

Now, we show that there exists a distance magic labeling of G' with $n+4$ vertices, where G' arises from G by removing all 16 edges of the $C_4[\overline{K_2}]$ subgraph, by adding 4 new vertices x_1, x_2, y_1, and y_2, by adding 16 new edges of $K_{4,4}$ with partite sets $\{u_1, u_2, v_1, v_2\}$ and $\{x_1, x_2, y_1, y_2\}$ (drawn in green), by adding 4 edges of a $K_{2,2}$ with partite sets $\{x_1, x_2\}$ and $\{y_1, y_2\}$ (drawn in black), and by adding 16 edges of another $K_{4,4}$ with partite sets $\{w_1, w_2, z_1, z_2\}$ and $\{x_1, x_2, y_1, y_2\}$ (drawn in blue). We observe that G' is a connected 10-regular graph with $n + 4$ vertices. Consider the following labeling f' of G'.

$$f'(v) = \begin{cases} f(v) + 2 & \text{for } v \in V(G), \\ 1 & \text{for } v = x_1, \\ n + 4 & \text{for } v = x_2, \\ 2 & \text{for } v = y_1, \\ n + 3 & \text{for } v = y_2. \end{cases}$$

Again, one can check that f' is a distance magic labeling of G'. Moreover, all the green edges (as

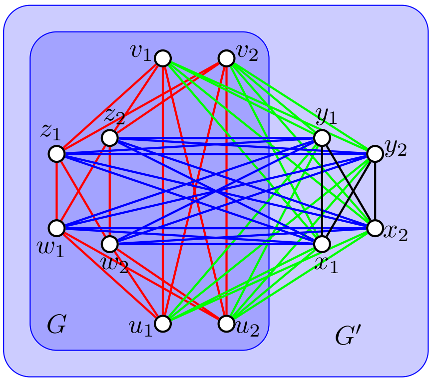

Figure 7. Adding four vertices to a 10-regular distance magic graph.

well as the blue edges) form a subgraph $K_{4,4}$ of G', which satisfies the properties required in Lemma 3, and the claim follows by induction.

□

Theorem 3. *There exists a 10-regular distance magic graph of odd order n if and only if* $n \geq 15$.

Proof. For odd $n < 13$ clearly no such graph exists. For $n = 13$ it does not exist according to Proposition 2. Regular distance magic graphs for $n = 15$ and $n = 17$ are shown in Figures 8 and 9. The subgraphs required in Lemma 3 are highlighted in red. The required sums are $f(u_1) + f(u_2) = f(v_1)+f(v_2) = f(w_1)+f(w_2) = f(z_1)+f(z_2) = 3+13 = 4+12 = 5+11 = 6+10 = 16$ in Figure 8 and $f(u_1)+f(u_2) = f(v_1)+f(v_2) = f(w_1)+f(w_2) = f(z_1)+f(z_2) = 2+16 = 4+14 = 5+13 = 8+11 = 18$ in Figure 9. For all remaining odd n the result follows by Lemma 3.

□

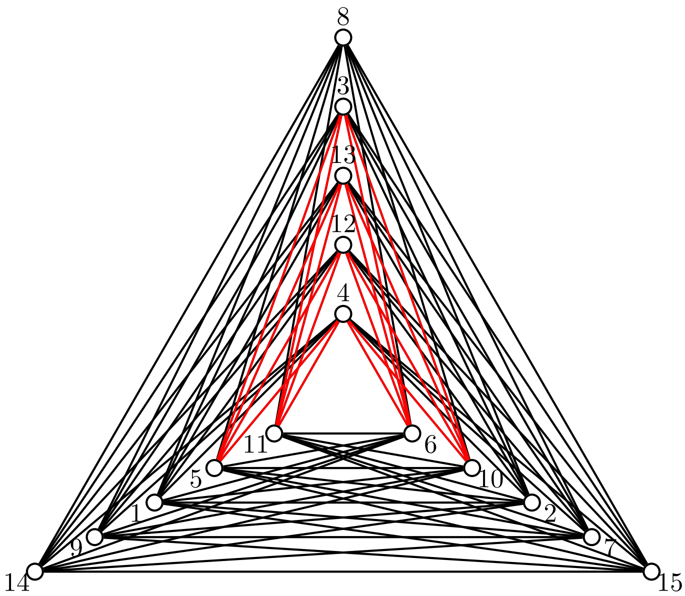

Figure 8. A 10-regular distance magic graph $C_3[\overline{K}_5]$ with 15 vertices and $k = 80$.

5. On 12-regular graphs

Finally, we characterize all orders of 12-regular distance magic graphs. By H we denote the graph which arises by taking the five cycle u, v, p, w, q with one additional edge pq (one chordal) and composing it with $\overline{K}_2$. We denote the vertices of H by u_i, v_i, p_i, w_i, q_i for $i = 1, 2$, respectively.

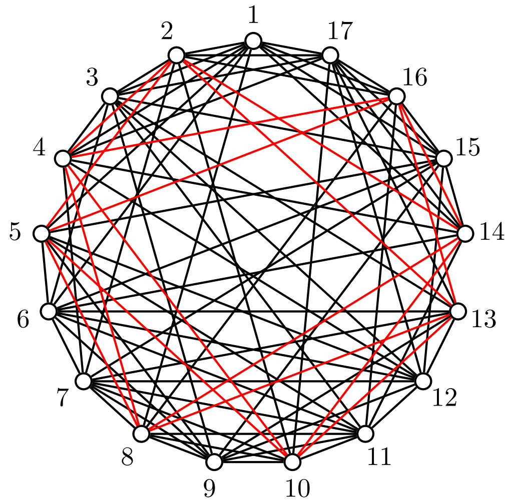

Figure 9. A 10-regular distance magic graph with 17 vertices and $k = 90$.

Lemma 4. *Let G be a 12-regular graph of order n with a subgraph H. If G allows a distance magic labeling f such that the sum of vertex labels in each pair corresponding to the $\overline{K}_2$ in H is $n + 1$, then there exists a 12-regular distance magic graph with $n + 4t$ vertices for all non-negative integers t.*

Proof. Let G be a 12-regular distance magic graph of order n with a distance magic labeling f and the specified subgraph H. Suppose $f(u_1) + f(u_2) = f(v_1) + f(v_2) = f(p_1) + f(p_2) = f(w_1) + f(w_2) = f(q_1) + f(q_2) = n + 1$. In Figure 10, the edges of the subgraph H are highlighted in red (the cycle $C_5[\overline{K_2}]$) and yellow (the blown up chordal edge pq), respectively.

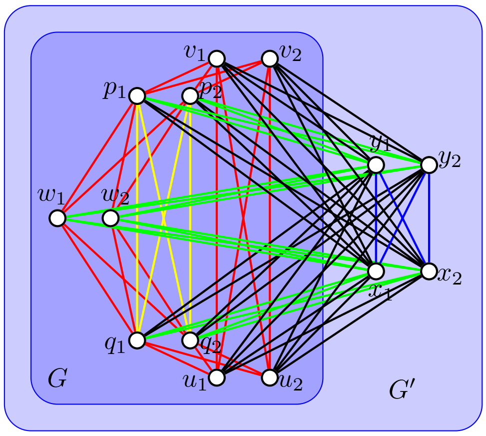

Figure 10. Adding four vertices to a 12-regular distance magic graph.

There exists a distance magic labeling of G', where G' arises from G by removing all 20 edges of the $C_5[\overline{K_2}]$ subgraph (drawn in red in Figure 10), by adding 4 new vertices x_1, x_2, y_1, and y_2, by adding 16 new edges of $P_5[\overline{K_2}]$ with pairs $\{q_1, q_2\}$, $\{x_1, x_2\}$, $\{w_1, w_2\}$, $\{y_1, y_2\}$ and $\{p_1, p_2\}$ (drawn in green), by adding 4 edges of a $K_{2,2}$ with partite sets $\{x_1, x_2\}$ and $\{y_1, y_2\}$ (blue), and by adding all remaining 24 edges that are missing in a $K_{8,4}$ with partite sets $\{u_1, u_2, q_1, q_2, p_1, p_2, v_1, v_2\}$ and $\{x_1, x_2, y_1, y_2\}$ (drawn in black).

Observe that G' is a connected 12-regular graph with $n+4$ vertices. Moreover, if the subgraph $K_{2,2}$ with partite sets $\{p_1, p_2\}$ and $\{q_1, q_2\}$ was in G (i.e. the blown up chordal edge pq), then G' contains another subgraph H' with vertices q_1, q_2, x_1, x_2, w_1, w_2, y_1, y_2, p_1, and p_2, where the subgraph $K_{2,2}$ with partite sets $\{x_1, x_2\}$ and $\{y_1, y_2\}$ corresponds to a blown up chordal edge xy (drawn in blue in

Figure 10). Consider the following labeling f' of G'.

$$f'(v) = \begin{cases} f(v) + 2 & \text{for } v \in V(G), \\ 1 & \text{for } v = x_1, \\ n + 4 & \text{for } v = x_2, \\ 2 & \text{for } v = y_1, \\ n + 3 & \text{for } v = y_2. \end{cases}$$

It is easy to check that f' is a distance magic labeling of G'. Also all the green edges along with the yellow and blue edges form a subgraph H', which satisfies the properties required in Lemma 4, and the claim follows by induction.

□

Theorem 4. *There exists a 12-regular distance magic graph of odd order n if and only if $n \geq 15$.*

Proof. For odd $n \leq 13$ clearly no such graph exists. Regular distance magic graphs for $n = 15$ and $n = 17$ with highlighted subgraphs H are shown in Figures 11 and 12. The required sums are $1+15 = 2+14 = 3+13 = 4+12 = 5+11 = 16$ in Figure 11 and $2+16 = 3+15 = 4+14 = 5+13 = 6+12 = 18$ in Figure 9. For all remaining odd $n \geq 19$ the result follows by Lemma 4.

□

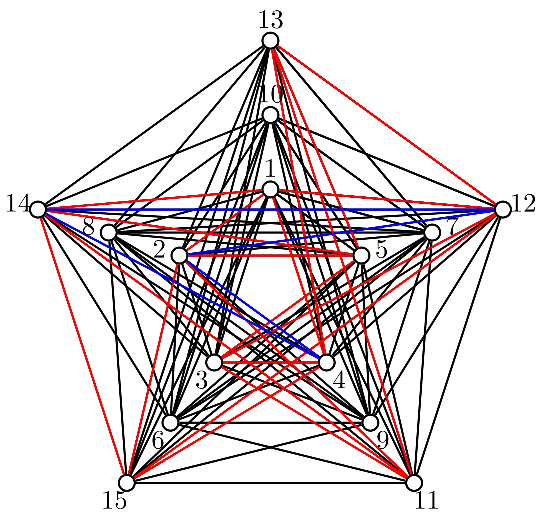

Figure 11. A 12-regular distance magic graph $C_5[\overline{K}_3]$ with 15 vertices and $k = 96$.

6. Conclusion

Notice that Theorem 1 (and similarly Theorems 2, 3, and 4) do not provide a full list of all r-regular distance magic graphs for each r but together with Proposition 1 they fully characterize all orders for which such graphs exist. In fact, a brute force search gave two differently labeled 6-regular distance magic graphs with 9 vertices, 36 differently labeled 6-regular distance magic graphs with 13 vertices and 2078 differently labeled 6-regular distance magic graphs with 15 vertices.

As in the previous sections, one can try to construct other even-regular graphs, however the constructions become more technical. Potentially more than four additional vertices would be required in the inductive step. This would at the same time require the finding of additional regular distance magic graphs with a special subgraph in the basis of the inductive construction. In some sense, this disqualifies such an approach for a complete characterization of all odd orders and (even) regularities

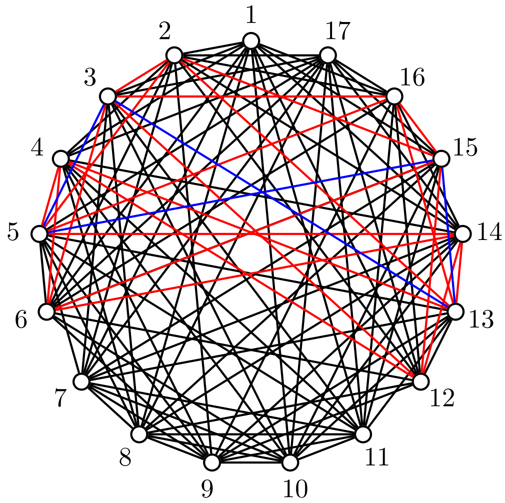

Figure 12. A 12-regular distance magic graph with 17 vertices and $k = 108$.

$r \geq 14$. Considering the practical motivation of fair incomplete tournaments, the existence is now settled completely when n is even or when n is odd and $2 \leq r \leq 12$.

Recall that for even values of n the existence of regular distance magic graphs was completely settled by Proposition 1. Notice that for n odd there are some exceptional values of n and r for which no regular distance magic graph exists, namely for $r = 4$ when $n \leq 15$ is odd and for $r = 6$ when $n = 11$. Apparently the graphs are too small.

For all higher orders we expect the graphs to exist, and we post it as a possible research problem:

Conjecture 1. *For all odd n, $n \geq 14$ an r-regular distance magic graph of order n exists*

- *for all even r, $14 \leq r \leq n-3$ if $n \equiv 0 \pmod 3$,*
- *for all even r, $14 \leq r \leq n-5$ otherwise.*

References

1. Gallian, J.A., 2018. A dynamic survey of graph labeling. *Electronic Journal of combinatorics, 1*(DynamicSurveys), p.DS6.
2. Fronček, D., Kovář, P., and Kovářová, T., 2006. Fair incomplete tournaments. *Bulletin of the ICA, 48*(1), pp.31–33.
3. Arumugam, S., Fronček, D., and Kamatchi, N., 2011. Distance Magic Graphs – A Survey. *Journal of the Indonesian Mathematical Society, Special Edition*, pp.11–26.
4. Fronček, D., 2007. Fair incomplete tournaments with odd number of teams and large number of games. *Congressus Numerantium, 187*, pp.83–89.
5. Fronček, D., Kovář, P., and Kovářová, T., 2011. Constructing distance magic graphs from regular graphs. *JCMCC, 78*(4), pp.349–354.
6. Kovář, P., and Silber, A., 2012. Distance magic graphs of high regularity. In *AKCE International Journal of Graphs and Combinatorics, 9*, pp.213–219.
7. Kovář, P., Fronček, D., and Kovářová, T., 2012. A note on 4-regular distance magic graphs. *Australasian Journal of Combinatorics, 54*, pp.127–132.
8. Arumugam, S., Kovář, P., and Kamatchi, N., 2016. Distance magic graphs. *Utilitas Mathematica, 99*, pp.131–142.