

Article

## Study of Topological Behavior of Some Computer Related Graphs

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**Abstract:** Network theory is the study of graphs such as representing equilibrium relationships or unequal relationships between different objects. A network can be defined as a graph where nodes and / or margins have attributes (e.g. words). Topological index of a graph is a number that helps to understand its topology and a topological index is known as irregularity index if it is greater than zero and topological index of graph is equal to zero if and only if graph is regular. The irregularity indices are used for computational analysis of nonregular graph topological composition. In this paper, we aim to compute topological invariants of some computer related graph networks. We computed various irregularities indices for the graphs of OTIS swapped network  $OP_a$  and Biswapped Networks  $Bsw(Pa)$ .

**Keywords:** network, graph, topological index.

**Mathematics Subject Classification:** 05C15, 05C05.

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### 1. Introduction

A topology index is a number associated with the graph network and helps to understand the topology of concerned network [1,2]. The topological indices remain same upto graph isomorphism. The theory of topological indices begun with Wiener, when he introduced the very first topological index, named as Wiener index [3]. After Wiener, Milan Randic in the year 1975, introduced Randic index [4]. Due to huge applications and various applications of Randic index many topological indices are introduced and studied in recent years, for example, Zagreb indices [5,6], augmented Zagreb index [7], multiplicative versions of first Zagreb index [8], hyper Zagreb index [9], harmonic index [10] and so on [11–13].

The mainly studied classes of topological indices are distance based [14, 15] and degree based [16, 17]. Like distance based and degree based topological indices, irregularities indices are also very important and well studied class of indices [18]. A topological index is called as irregularity index, if its value is either greater or equal to 0 and a topological index of graph is 0 iff it is a regular graph. Irregularity indices are typically used for quantitative characterization of the topological structures of non-regular graphs. In varied issues and applications, particularly within the fields of chemistry and material engineering, it's helpful to bear in mind of the irregularity of a molecular structure.

Various irregularity indices for dendrimer structures were established in [18] and for Probabilistic Neural Networks are established in [19]. Abdo et al. [20] presented the graphs with maximal

irregularity and the graphs with equal irregularity indices are presented by Dimitrov and Réti [21]. Moreover, Gutman presented irregularity for the various molecular graphs in [22].

In this paper, we consider all graphs are simple, finite and undirected. Let us consider  $G$  be such a graph,  $V$  be the set of vertices of  $G$  and  $E$  be the set of edges of  $G$ . For any vertex  $v$ , its degree is the number of vertices that are at distance one from  $v$  and is denoted by  $d_v$ . For the other notions that are used in this paper but not defined, we refer to the readers [23, 24]. The irregularities are presented in Table 1 [25, 26] below;

**Table 1.** Irregularity indices [25, 26]

$VAR(G) = \frac{M_1(G)}{n} - \left(\frac{2m}{n}\right)^2$	$IRC(G) = \frac{RR(G)}{m} - \frac{2m}{n}$
$AL(G) = \sum_{uv \in E(G)}  d_u - d_v $	$IRDIF(G) = \sum_{uv \in E(G)} \left  \frac{d_u}{d_v} - \frac{d_v}{d_u} \right $
$IR1(G) = F(G) - \frac{2m}{n} M_1(G)$	$IRL(G) = \sum_{uv \in E(G)}  \ln(d_u) - \ln(d_v) $
$IR2(G) = \sqrt{\frac{M_2(G)}{m}} - \frac{2m}{n}$	$IRLU(G) = \sum_{uv \in E(G)} \frac{ (d_u)-(d_v) }{\min(d_u, d_v)}$
$IRF(G) = F(G) - 2M_2(G)$	$IRLF(G) = \sum_{uv \in E(G)} \frac{ d_u-d_v }{\sqrt{(d_u \times d_v)}}$
$IRFW(G) = \frac{IRF(G)}{M_2(G)}$	$IRLA(G) = \sum_{uv \in E(G)} 2 \frac{ d_u-d_v }{d_u+d_v}$
$IRA(G) = \sum_{uv \in E(G)} \left( d_u^{-\frac{1}{2}} - d_v^{-\frac{1}{2}} \right)^2$	$IRD1(G) = \sum_{uv \in E(G)} \ln[1 +  d_u - d_v ]$
$IRB(G) = \sum_{uv \in E(G)} \left( d_u^{\frac{1}{2}} - d_v^{\frac{1}{2}} \right)^2$	$IRGA(G) = \sum_{uv \in E(G)} \ln \left( \frac{d_u+d_v}{2 \sqrt{(d_u \times d_v)}} \right)$

It can be observed from this Table 1 that all the presented irregularities are related with degree-based topological indices.

The main applications of OTIS networks (optical transpose interconnection system) is in the new structure design of computers and in electronic gadget [27]. In these networks, the processors are taken as clusters and the similar clusters are connected through electronic links, whereas optical links are utilized for intercluster communication. The algorithms are devised for routing, selection/sorting [28, 29], bound numerical-computations [30], Fourier-remodel [31], matrix-operation [32], and image-process [33].

The structure of associate interconnection network are often mathematically shaped as graph. In this graph, processors are taken as vertices and the links between these processors are taken as edges. By studying the topological index we determine the topology of these networks which inform us about the arrangement of vertices(processors) and edges(links between processors). Furthermore, the properties of these networks can be determined with the topological indices and one can get the desired properties by rearranging the processors and links between the processors. The diameter of the graph tells us the largest distance between any two nodes and the degree of any vertex is the total links connected with a single node. If the number of links of each node is same than the network is called as regular network(graph).

The aim of present note is to compute 16 kind of irregularities for the OTIS swapped network  $OP_a$  and biswapped networks  $Bsw(Pa)$ .

## 2. Irregularities of OTIS swapped network $OP_a$

The OTIS swapped networks are extensively studied due to its huge applications [34,35]. Different kinds of OTIS swapped networks are recently introduced for betterment of mankind [36,37]. Below is the definition of OTIS swapped network;

**Definition 1** (OTIS swapped network [38]). *The OTIS swapped network denoted by  $O_G$  (can be*

deduced from any graph  $G$ ) is a graph with the vertex set  $V(O_G) = \{< u, v > | u, v \in V(G)\}$  and the edge set  $E(O_G) = \{(u, v_1 >, < u, v_2 >) | g \in V(G), (v_1, v_2) \in E(G)\} \cup \{(u, v >, < v, u >) | u, v \in V(G) \text{ and } u \neq v\}$ .

Here, we studied the OTIS swapped network which is generated by a path graph  $P_a$ , where the natural number  $a$  denotes the total number of vertices in  $P_a$ . The graph for  $O_{P_a}$  for  $a + 6$  is given in Figure 1. It is easy to observe that the vertex set of  $O_{P_a}$  contain  $a^2$  elements and the edge set of it contains  $\frac{3a(a-1)}{2}$  elements.

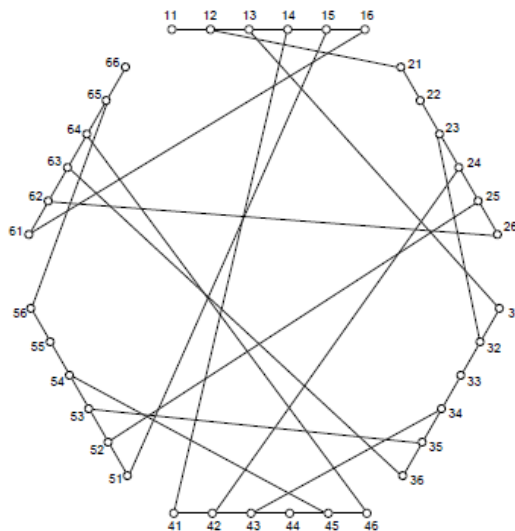


Figure 1. Graph of  $O_{P_a}$  for  $a = 6$ .

**Theorem 1.** For the graph of  $O_{P_a}$ , we have following irregularities;

1.  $VAR(O_{P_a}) = \frac{3a-5}{a^2}$ .
2.  $AL(O_{P_a}) = 6a - 10$ .
3.  $IR1(O_{P_a}) = \frac{3(5a^2-11a+4)}{a}$ .
4.  $IR2(O_{P_a}) = \sqrt{\frac{2\left(\frac{27a^2}{2} - \frac{63a}{2} + 15\right)}{3a(a-1)}} - \frac{3(a-1)}{a}$ .
5.  $IRF(O_{P_a}) = 6a - 6$ .
6.  $IRFW(O_{P_a}) = \frac{4a-4}{9a^2-21a+10}$ .
7.  $IRA(O_{P_a}) = \frac{3639647609281071a}{36028797018963968} + \frac{4379338332339151}{36028797018963968}$ .
8.  $IRB(O_{P_a}) = \frac{2729735706960801a}{4503599627370496} - \frac{192804972936719}{562949953421312}$ .
9.  $IRC(O_{P_a}) = \frac{12\sqrt{6a-27a+4}\sqrt{3-28}\sqrt{6+57}}{3a(a-1)}$ .
10.  $IRDIF(O_{P_a}) = 5a - \frac{19}{3}$ .
11.  $IRL(O_{P_a}) = \frac{10956315058725999a}{4503599627370496} - \frac{15669315349286639}{4503599627370496}$ .
12.  $IRLU(O_{P_a}) = 3a - 3$ .
13.  $IRLF(O_{P_a}) = \frac{4\sqrt{3}}{3} + \frac{\sqrt{6}(6a-14)}{6}$ .
14.  $IRLA(O_{P_a}) = \frac{12a}{5} - \frac{18}{5}$ .
15.  $IRD1(O_{P_a}) = \frac{18729944304496077a}{4503599627370496} - \frac{33807783589416821}{4503599627370496}$ .
16.  $IRGA(O_{P_a}) = \frac{8824604126877213a}{72057594037927936} + \frac{138935026006723}{72057594037927936}$ .

*Proof.* The vertices of  $O_{P_a}$  consists of following three classes w.r.t the degrees of vertices;

$$\begin{aligned} V_1(O_{P_a}) &= \{v \in V(O_{P_a}) : d_u = 1\}, \\ V_2(O_{P_a}) &= \{v \in V(O_{P_a}) : d_u = 2\}, \\ V_3(O_{P_a}) &= \{v \in V(O_{P_a}) : d_u = 3\}. \end{aligned}$$

The edges of  $O_{P_a}$  consists of following classes w.r.t the degrees of end vertices;

$$E_1(O_{P_a}) = \{uv = e \in E(O_{P_a}) : d_u = 3, \text{ and } d_v = 3\},$$

$$\begin{aligned} E_2(O_{P_a}) &= \{uv = e \in E(O_{P_a}) : d_u = 2, \text{ and } d_v = 2\}, \\ E_3(O_{P_a}) &= \{uv = e \in E(O_{P_a}) : d_u = 1, \text{ and } d_v = 3\}, \\ E_4(O_{P_a}) &= \{uv = e \in E(O_{P_a}) : d_u = 2, \text{ and } d_v = 3\}. \end{aligned}$$

The cordialities of above edge classes are;

$$\begin{aligned} |E_1(O_{P_a})| &= \frac{(3a - 6)(a - 3)}{2}, \\ |E_2(O_{P_a})| &= 3, \\ |E_3(O_{P_a})| &= 2, \\ |E_4(O_{P_a})| &= 6a - 14. \end{aligned}$$

Firstly, we compute the indices that are useful in computing the irregularities for  $O_{P_a}$ .

$$\begin{aligned} M_1(O_{P_a}) &= \sum_{uv \in E(O_{P_a})} (d_u + d_v) = 9a^2 - 15a + 4. \\ M_2(O_{P_a}) &= \sum_{uv \in E(O_{P_a})} (d_u \times d_v) = \frac{27a^2}{2} - \frac{63a}{2} + 15. \\ F(O_{P_a}) &= \sum_{uv \in E(O_{P_a})} ((d_u)^2 + (d_v)^2) = 27a^2 - 57a + 24. \\ RR(O_{P_a}) &= \sum_{uv \in E(O_{P_a})} \sqrt{d_u d_v} = \frac{3(3a - 6)(a - 3)}{2} + 2\sqrt{3} + \sqrt{6}(6a - 14) + 6. \end{aligned}$$

Now,

$$\begin{aligned} VAR(O_{P_a}) &= \frac{M_1(O_{P_a})}{n} - \left(\frac{2m}{n}\right)^2 \\ &= \frac{9a^2 - 15a + 4}{a^2} - \left(\frac{2\left(\frac{3a(a-1)}{2}\right)}{a^2}\right)^2 \\ &= \frac{3a - 5}{a^2}. \\ AL(O_{P_a}) &= \sum_{uv \in E(O_{P_a})} |d_u - d_v| \\ &= \sum_{uv \in E_1(O_{P_a})} (0) + \sum_{uv \in E_2(O_{P_a})} (0) + \sum_{uv \in E_3(O_{P_a})} |(1) - (3)| + \sum_{uv \in E_4(O_{P_a})} |(2) - (3)| \\ &= 6a - 10. \\ IR1(O_{P_a}) &= F(O_{P_a}) - \frac{2m}{n} M_1(O_{P_a}) \\ &= 27a^2 - 57a + 24 - \frac{2\left(\frac{3a(a-1)}{2}\right)}{a^2} (9a^2 - 15a + 4) \\ &= \frac{3(5a^2 - 11a + 4)}{a}. \\ IR2(O_{P_a}) &= \sqrt{\frac{M_2(O_{P_a})}{m} - \frac{2m}{n}} \\ &= \sqrt{\frac{\frac{27a^2}{2} - \frac{63a}{2} + 15}{\frac{3a(a-1)}{2}} - \frac{2\left(\frac{3a(a-1)}{2}\right)}{a^2}} \end{aligned}$$

$$= \sqrt{\frac{2 \left( \frac{27a^2}{2} - \frac{63a}{2} + 15 \right)}{3a(a-1)}} - \frac{3(a-1)}{a}.$$

$$IRF(O_{P_a}) = F(O_{P_a}) - 2M_2(O_{P_a})$$

$$= 27a^2 - 57a + 24 - 2 \left( \frac{27a^2}{2} - \frac{63a}{2} + 15 \right)$$

$$= 6a - 6.$$

$$IRFW(O_{P_a}) = \frac{IRF(O_{P_a})}{M_2(O_{P_a})}$$

$$= \frac{6a - 6}{\frac{27a^2}{2} - \frac{63a}{2} + 15}$$

$$= 6a - 6.$$

$$IRA(O_{P_a}) = \sum_{uv \in E(O_{P_a})} \left( d_u^{-\frac{1}{2}} - d_v^{-\frac{1}{2}} \right)^2$$

$$= \sum_{uv \in E_1(O_{P_a})} (0) + \sum_{uv \in E_2(O_{P_a})} (0) + \sum_{uv \in E_3(O_{P_a})} \left( (1)^{-\frac{1}{2}} - (3)^{-\frac{1}{2}} \right)^2 + \sum_{uv \in E_4(O_{P_a})} \left( (2)^{-\frac{1}{2}} - (3)^{-\frac{1}{2}} \right)^2$$

$$= \frac{3639647609281071a}{36028797018963968} + \frac{4379338332339151}{36028797018963968}.$$

$$IRB(O_{P_a}) = \sum_{uv \in E(O_{P_a})} \left( d_u^{\frac{1}{2}} - d_v^{\frac{1}{2}} \right)^2$$

$$= \sum_{uv \in E_1(O_{P_a})} (0) + \sum_{uv \in E_2(O_{P_a})} (0) + \sum_{uv \in E_3(O_{P_a})} \left( (1)^{\frac{1}{2}} - (3)^{\frac{1}{2}} \right)^2 + \sum_{uv \in E_4(O_{P_a})} \left( (2)^{\frac{1}{2}} - (3)^{\frac{1}{2}} \right)^2$$

$$= \frac{2729735706960801a}{4503599627370496} - \frac{192804972936719}{562949953421312}.$$

$$IRC(O_{P_a}) = \frac{RR(O_{P_a})}{m} - \frac{2m}{n}$$

$$= \frac{\frac{3(3a-6)(a-3)}{2} + 2\sqrt{3} + \sqrt{6}(6a-14) + 6}{\frac{3a(a-1)}{2}} - \frac{2 \left( \frac{3a(a-1)}{2} \right)}{a^2}$$

$$= \frac{12\sqrt{6}a - 27a + 4\sqrt{3} - 28\sqrt{6} + 57}{3a(a-1)}.$$

$$IRDIF(O_{P_a}) = \sum_{uv \in E(O_{P_a})} \left| \frac{d_u}{d_v} - \frac{d_v}{d_u} \right|$$

$$= \sum_{uv \in E_1(O_{P_a})} (0) + \sum_{uv \in E_2(O_{P_a})} (0) + \sum_{uv \in E_3(O_{P_a})} \left| \frac{1}{3} - \frac{3}{1} \right| + \sum_{uv \in E_4(O_{P_a})} \left| \frac{2}{3} - \frac{3}{2} \right|$$

$$= 5a - \frac{19}{3}.$$

$$IRL(O_{P_a}) = \sum_{uv \in E(O_{P_a})} |\ln(d_u) - \ln(d_v)|$$

$$= \sum_{uv \in E_1(O_{P_a})} (0) + \sum_{uv \in E_2(O_{P_a})} (0) + \sum_{uv \in E_3(O_{P_a})} |\ln(1) - \ln(3)| + \sum_{uv \in E_4(O_{P_a})} |\ln(2) - \ln(3)|$$

$$= \frac{10956315058725999a}{4503599627370496} - \frac{15669315349286639}{4503599627370496}.$$

$$IRLU(O_{P_a}) = \sum_{uv \in E(O_{P_a})} \frac{|(d_u) - (d_v)|}{\min(d_u, d_v)}$$

$$\begin{aligned}
 &= \sum_{uv \in E_1(O_{P_a})} (0) + \sum_{uv \in E_2(O_{P_a})} (0) + \sum_{uv \in E_3(O_{P_a})} \frac{|(1) - (3)|}{\min(1, 3)} + \sum_{uv \in E_4(O_{P_a})} \frac{|(2) - (3)|}{\min(2, 3)} \\
 &= 3a - 3. \\
 IRLF(O_{P_a}) &= \sum_{uv \in E(O_{P_a})} \frac{|d_u - d_v|}{\sqrt{(d_u \times d_v)}} \\
 &= \sum_{uv \in E_1(O_{P_a})} (0) + \sum_{uv \in E_2(O_{P_a})} (0) + \sum_{uv \in E_3(O_{P_a})} \frac{|1 - 3|}{\sqrt{(1 \times 3)}} + \sum_{uv \in E_4(O_{P_a})} \frac{|2 - 3|}{\sqrt{(2 \times 3)}} \\
 &= \frac{4\sqrt{3}}{3} + \frac{\sqrt{6}(6a - 14)}{6}. \\
 IRLA(O_{P_a}) &= \sum_{uv \in E(O_{P_a})} 2 \frac{|d_u - d_v|}{d_u + d_v} \\
 &= \sum_{uv \in E_1(O_{P_a})} (0) + \sum_{uv \in E_2(O_{P_a})} (0) + \sum_{uv \in E_3(O_{P_a})} 2 \frac{|1 - 3|}{1 + 3} + \sum_{uv \in E_4(O_{P_a})} 2 \frac{|2 - 3|}{2 + 3} \\
 &= \frac{12a}{5} - \frac{18}{5}. \\
 IRD1(O_{P_a}) &= \sum_{uv \in E(O_{P_a})} \ln[1 + |d_u - d_v|] \\
 &= \sum_{uv \in E_1(O_{P_a})} (0) + \sum_{uv \in E_2(O_{P_a})} (0) + \sum_{uv \in E_3(O_{P_a})} \ln[1 + |(1) - (3)|] + \sum_{uv \in E_4(O_{P_a})} \ln[1 + |(2) - (3)|] \\
 &= \frac{18729944304496077a}{4503599627370496} - \frac{33807783589416821}{4503599627370496}. \\
 IRGA(O_{P_a}) &= \sum_{uv \in E(O_{P_a})} \ln\left(\frac{d_u + d_v}{2\sqrt{(d_u \times d_v)}}\right) \\
 &= \sum_{uv \in E_1(O_{P_a})} (0) + \sum_{uv \in E_2(O_{P_a})} (0) + \sum_{uv \in E_3(O_{P_a})} \ln\left(\frac{1 + 3}{2\sqrt{(1 \times 3)}}\right) + \sum_{uv \in E_4(O_{P_a})} \ln\left(\frac{2 + 3}{2\sqrt{(2 \times 3)}}\right) \\
 &= \frac{8824604126877213a}{72057594037927936} + \frac{138935026006723}{72057594037927936}.
 \end{aligned}$$

□

### 3. Irregularities for Biswapped Networks $B_{sw}(P_a)$

**Definition 2** (Biswapped Networks [38]). *The biswapped network is denoted by  $B_{sw}(G)$  (can be obtained from any graph  $G$ ), is a graph with the vertex set;*

$$V(B_{sw}(G)) = \{ \langle 0, u, v \rangle, \langle 1, u, v \rangle \mid u, v \in V(G) \},$$

and the edge set;

$$\begin{aligned}
 E(B_{sw}(G)) &= \{ (\langle 0, u, v_1 \rangle, \langle 0, u, v_2 \rangle), (\langle 1, u, v_1 \rangle, \langle 1, u, v_2 \rangle) \mid (v_1, v_2) \in E(G), u \in V(G) \} \\
 &\cup \{ (\langle 0, u, v \rangle, \langle 1, v, u \rangle) \mid u, v \in V(G) \}.
 \end{aligned}$$

The biswapped networks  $B_{sw}(P_a)$  studied in this paper are obtained from the path  $P_a$  and the plot of  $B_{sw}(P_a)$  for  $a=5$  is given in Figure 2. It can be observed from Figure 2 that  $B_{sw}(P_a)$  has  $2a^2$  vertices and  $3a^2 - 2a$  edges [39].

**Theorem 2.** *For the Biswapped network  $B_{sw}(P_a)$ , we have the following irregularities;*

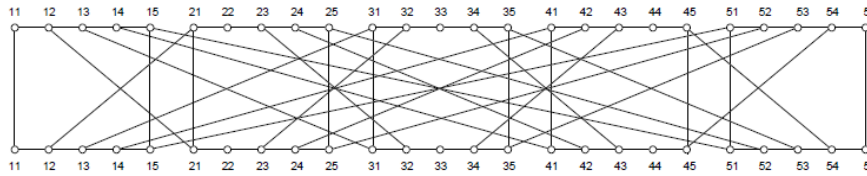


Figure 2.  $Bsw(P_n)$ .

1.  $VAR(BSN - 1) = \frac{2(a-2)}{a^2}$ .
2.  $AL(Bsw(P_n)) = 8a - 8$ .
3.  $IR1(Bsw(P_n)) = 20a - 40$ .
4.  $IR2(Bsw(P_n)) = \frac{4a-6a^2}{2a^2} + \sqrt{\frac{-27a^2-42a+4}{2a-3a^2}}$ .
5.  $IRF(Bsw(P_n)) = 8a - 8$ .
6.  $IRFW(Bsw(P_n)) = \frac{8a-8}{27a^2-42a+4}$ .
7.  $IRA(Bsw(P_n)) = \frac{1213215869760357a}{9007199254740992} - \frac{1213215869760357}{9007199254740992}$ .
8.  $IRB(Bsw(P_n)) = \frac{909911902320267a}{1125899906842624} - \frac{909911902320267}{1125899906842624}$ .
9.  $IRC(Bsw(P_n)) = -\frac{18a-8\sqrt{6a+8}\sqrt{6-16}}{a(3a-2)}$ .
10.  $IRDIF(Bsw(P_n)) = \frac{20a}{3} - \frac{20}{3}$ .
11.  $IRL(Bsw(P_n)) = \frac{3652105019575333a}{1125899906842624} - \frac{3652105019575333}{1125899906842624}$ .
12.  $IRLU(Bsw(P_n)) = 4a - 4$ .
13.  $IRLF(Bsw(P_n)) = \frac{\sqrt{6}(8a-8)}{6}$ .
14.  $IRLA(Bsw(P_n)) = \frac{16a}{5} - \frac{16}{5}$ .
15.  $IRD1(Bsw(P_n)) = \frac{6243314768165359a}{1125899906842624} - \frac{6243314768165359}{1125899906842624}$ .
16.  $IRGA(Bsw(P_n)) = \frac{2941534708959071a}{18014398509481984} - \frac{2941534708959071}{18014398509481984}$ .

Proof. The vertices of  $Bsw(P_a)$  has following two classes w.r.t the degrees of vertices;

$$V1(Bsw(P_a)) = \{v \in V(Bsw(P_a)) : d_u = 2\},$$

$$V2(Bsw(P_a)) = \{v \in V(Bsw(P_a)) : d_u = 3\}.$$

The edges of  $Bsw(P_a)$  consists of following three classes;

$$E_1(Bsw(P_a)) = \{uv \in E(Bsw(P_a)) : d_u = d_v = 3\},$$

$$E_2(Bsw(P_a)) = \{uv \in E(Bsw(P_a)) : d_u = d_v = 2\},$$

$$E_3(Bsw(P_a)) = \{uv \in E(Bsw(P_a)) : d_u = 2, d_v = 3\}.$$

The cardinalities of above mentioned classes of edges are;

$$|E_1(Bsw(P_a))| = 3a^2 - 10a + 4,$$

$$|E_2(Bsw(P_a))| = 4,$$

$$|E_3(Bsw(P_a))| = 8a - 8.$$

The indices of  $Bsw(P_a)$  related to our irregularities are;

$$M_1(Bsw(P_a)) = \sum_{uv \in E(Bsw(P_a))} (d_u + d_v) = 2a(9a - 10).$$

$$M_2(Bsw(P_a)) = \sum_{uv \in E(Bsw(P_a))} (d_u \times d_v) = 27a^2 - 42a + 4.$$

$$F(Bsw(P_a)) = \sum_{uv \in E(Bsw(P_a))} ((d_u)^2 + (d_v)^2) = 2a(27a - 38).$$

$$RR(Bsw(P_a)) = \sum_{uv \in E(Bsw(P_a))} \sqrt{d_u d_v} = 9a^2 - 30a + \sqrt{6}(8a - 8) + 20.$$

Now,

$$\begin{aligned} VAR(Bsw(P_a)) &= \frac{M_1(Bsw(P_a))}{n} - \left(\frac{2m}{n}\right)^2 \\ &= \frac{2a(9a - 10)}{2a^2} - \left(\frac{23a^2 - 2a}{2a^2}\right)^2 \\ &= \frac{2(a - 2)}{a^2}. \end{aligned}$$

$$\begin{aligned} AL(Bsw(P_a)) &= \sum_{uv \in E(Bsw(P_a))} |d_u - d_v| \\ &= \sum_{uv \in E_1(Bsw(P_a))} (0) + \sum_{uv \in E_2(Bsw(P_a))} (0) + \sum_{uv \in E_3(Bsw(P_a))} |(2) - (3)| \\ &= 8a - 8. \end{aligned}$$

$$\begin{aligned} IR1(Bsw(P_a)) &= F(Bsw(P_a)) - \frac{2m}{n}M_1(Bsw(P_a)) \\ &= 2a(27a - 38) - \frac{2(3a^2 - 2a)}{2a^2}2a(9a - 10) \\ &= 20a - 40. \end{aligned}$$

$$\begin{aligned} IR2(Bsw(P_a)) &= \sqrt{\frac{M_2(Bsw(P_a))}{m} - \frac{2m}{n}} \\ &= \sqrt{\frac{27a^2 - 42a + 4}{3a^2 - 2a} - \frac{2(3a^2 - 2a)}{2a^2}} \\ &= \frac{4a - 6a^2}{2a^2} + \sqrt{-\frac{27a^2 - 42a + 4}{2a - 3a^2}}. \end{aligned}$$

$$\begin{aligned} IRF(Bsw(P_a)) &= F(Bsw(P_a)) - 2M_2(Bsw(P_a)) \\ &= 2a(27a - 38) - 2(27a^2 - 42a + 4) \\ &= 8a - 8. \end{aligned}$$

$$\begin{aligned} IRFW(Bsw(P_a)) &= \frac{IRF(Bsw(P_a))}{M_2(Bsw(P_a))} \\ &= \frac{8a - 8}{27a^2 - 42a + 4} \\ &= 8a - 8. \end{aligned}$$

$$\begin{aligned} IRA(Bsw(P_a)) &= \sum_{uv \in E(Bsw(P_a))} \left(d_u^{-\frac{1}{2}} - d_v^{-\frac{1}{2}}\right)^2 \\ &= \sum_{uv \in E_1(Bsw(P_a))} (0) + \sum_{uv \in E_2(Bsw(P_a))} (0) + \sum_{uv \in E_3(Bsw(P_a))} \left((2)^{-\frac{1}{2}} - (3)^{-\frac{1}{2}}\right)^2 \\ &= \frac{1213215869760357a}{9007199254740992} - \frac{1213215869760357}{9007199254740992}. \end{aligned}$$

$$\begin{aligned} IRB(Bsw(P_a)) &= \sum_{uv \in E(Bsw(P_a))} \left(d_u^{\frac{1}{2}} - d_v^{\frac{1}{2}}\right)^2 \\ &= \sum_{uv \in E_1(Bsw(P_a))} (0) + \sum_{uv \in E_2(Bsw(P_a))} (0) + \sum_{uv \in E_3(Bsw(P_a))} \left((2)^{\frac{1}{2}} - (3)^{\frac{1}{2}}\right)^2 \end{aligned}$$



$$\begin{aligned}
 &= \frac{909911902320267 a}{1125899906842624} - \frac{909911902320267}{1125899906842624} \\
 IRC(Bsw(P_a)) &= \frac{RR(Bsw(P_a))}{m} - \frac{2m}{n} \\
 &= \frac{9 a^2 - 30 a + \sqrt{6} (8 a - 8) + 20}{3 a^2 - 2 a} - \frac{2 (3 a^2 - 2 a)}{2 a^2} \\
 &= -\frac{18 a - 8 \sqrt{6} a + 8 \sqrt{6} - 16}{a (3 a - 2)} \\
 IRDIF(Bsw(P_a)) &= \sum_{uv \in E(Bsw(P_a))} \left| \frac{d_u}{d_v} - \frac{d_v}{d_u} \right| \\
 &= \sum_{uv \in E_1(Bsw(P_a))} (0) + \sum_{uv \in E_2(Bsw(P_a))} (0) + \sum_{uv \in E_3(Bsw(P_a))} \left| \frac{2}{3} - \frac{3}{2} \right| \\
 &= \frac{20 a}{3} - \frac{20}{3} \\
 IRL(Bsw(P_a)) &= \sum_{uv \in E(Bsw(P_a))} |\ln(d_u) - \ln(d_v)| \\
 &= \sum_{uv \in E_1(Bsw(P_a))} (0) + \sum_{uv \in E_2(Bsw(P_a))} (0) + \sum_{uv \in E_3(Bsw(P_a))} |\ln(2) - \ln(3)| \\
 &= \frac{3652105019575333 a}{1125899906842624} - \frac{3652105019575333}{1125899906842624} \\
 IRLU(Bsw(P_a)) &= \sum_{uv \in E(Bsw(P_a))} \frac{|(d_u) - (d_v)|}{\min(d_u, d_v)} \\
 &= \sum_{uv \in E_1(Bsw(P_a))} (0) + \sum_{uv \in E_2(Bsw(P_a))} (0) + \sum_{uv \in E_3(Bsw(P_a))} \frac{|(2) - (3)|}{\min(2, 3)} \\
 &= 4 a - 4 \\
 IRLF(Bsw(P_a)) &= \sum_{uv \in E(Bsw(P_a))} \frac{|d_u - d_v|}{\sqrt{(d_u \times d_v)}} \\
 &= \sum_{uv \in E_1(Bsw(P_a))} (0) + \sum_{uv \in E_2(Bsw(P_a))} (0) + \sum_{uv \in E_3(Bsw(P_a))} \frac{|2 - 3|}{\sqrt{(2 \times 3)}} \\
 &= \frac{\sqrt{6} (8 a - 8)}{6} \\
 IRLA(Bsw(P_a)) &= \sum_{uv \in E(Bsw(P_a))} 2 \frac{|d_u - d_v|}{d_u + d_v} \\
 &= \sum_{uv \in E_1(Bsw(P_a))} (0) + \sum_{uv \in E_2(Bsw(P_a))} (0) + \sum_{uv \in E_3(Bsw(P_a))} 2 \frac{|2 - 3|}{2 + 3} \\
 &= \frac{16 a}{5} - \frac{16}{5} \\
 IRD1(Bsw(P_a)) &= \sum_{uv \in E(Bsw(P_a))} \ln[1 + |d_u - d_v|] \\
 &= \sum_{uv \in E_1(Bsw(P_a))} (0) + \sum_{uv \in E_2(Bsw(P_a))} (0) + \sum_{uv \in E_3(Bsw(P_a))} \ln[1 + |(2) - (3)|] \\
 &= \frac{6243314768165359 a}{1125899906842624} - \frac{6243314768165359}{1125899906842624}
 \end{aligned}$$

$$\begin{aligned}
IRGA(B_{sw}(P_a)) &= \sum_{uv \in E(B_{sw}(P_a))} \ln \left( \frac{d_u + d_v}{2\sqrt{(d_u \times d_v)}} \right) \\
&= \sum_{uv \in E_1(B_{sw}(P_a))} (0) + \sum_{uv \in E_2(B_{sw}(P_a))} (0) + \sum_{uv \in E_3(B_{sw}(P_a))} \ln \left( \frac{2+3}{2\sqrt{(2 \times 3)}} \right) \\
&= \frac{2941534708959071}{18014398509481984} a - \frac{2941534708959071}{18014398509481984}.
\end{aligned}$$

□

#### 4. Concluding Remarks

Topological index, "the mathematical formula that can be calculated for any simple connected graph network" is helpful to analyze the concerned network. With the help of these indices, we can get properties of OTIS networks and processors can be increased or decreased to get our required properties. Furthermore we can also rearrange the links between processors to enhance the ability of OTIS networks which help us to better design the computers and electronic tools. We can say that with the help of topological indices, we can efficiently get our desired network with any lose of time and expensive experiments. Topological indices also have an important role in chemistry and physics. These indices with quantitative structure-property relationship (QSPR) and quantitative structure-activity relationship (QSAR) give us boiling points any other properties. Irregularities are also important to understand the topology of OTIS swapped and OTIS biswapped networks. In the present research, we presented sixteen irregularities for these networks. Our results are very helpful in determining the properties of concerned networks and also can be used to enhance their ability.

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#### Authors Contribution

X. Ren proved the main results, I. Ahmed wrote the first version of the paper and analyzed the results and R. Liu proposed the problem, supervised this work and arranged the funding for this paper.

#### Conflict of interest

The authors declare no conflict of interests.

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