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Article

The Appropriate Scale of Competition Between Online Taxis and Taxis Based on the Lotka-Volterra Evolutionary Model

Yi Zeng[1] **and Beijun Chu**[1,*]

[1] School of Transportation Engineering, East China Jiaotong University, 330000, Jiangxi Province, China

* **Correspondence:** cbj19990929@163.com.

Abstract: In order to determine the optimal scale for urban ride-hailing services and taxis while promoting their sustainable growth, we have developed a Lotka-Volterra evolutionary model that accounts for the competitive, cooperative, and mixed dynamics between these two entities. This model is rooted in the theory of synergistic evolution and is supported by data simulation and analysis. By employing this model, we can identify the appropriate size for urban ride-hailing services and taxis when they reach equilibrium under different environmental conditions. The study's findings reveal that the evolutionary outcomes of online ride-hailing services and traditional taxis are closely linked to the competitive impact coefficient and the cooperative effect coefficient. In highly competitive environments, intense rivalry can lead to the elimination of the less competitive party, while the dominant player ultimately attains a specific size threshold. As competition moderates, both entities can achieve a balanced and stable coexistence in the market. In cooperative environments, both online ride-hailing services and traditional taxis have more room for development, which facilitates the integration of existing and innovative business models. In environments marked by competition, the development trends of both entities mirror those in competitive settings, but cooperation can slow down the decline of the less competitive party. In conclusion, we propose strategies to foster fair competition between online ride-hailing services and traditional taxis, consider the coexistence of old and new business models, and promote their integrated development.

Keywords: urban transport; right-sizing; Lotka-Volterra model; net cars; taxis

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1. Introduction

As integral components of urban public transportation, taxis play a pivotal role in fulfilling the commuting needs of the populace. They offer passengers a convenient, comfortable, and flexible means of personalized transportation, effectively complementing urban public transit systems [1]. The evolution of the taxi industry has, however, given rise to monopolistic tendencies, resulting in issues such as subpar service quality, refusals of service, circuitous routes, and arbitrary fare quotations by select taxi drivers [2].

Simultaneously, the rapid emergence of the net-contracted vehicle sector, driven by developments in mobile internet, big data, and the sharing economy, has significantly transformed the urban transportation landscape. Despite public concerns regarding passenger safety, the legal framework governing online taxis has gradually solidified through the introduction of pertinent regulations by the Ministry of Transport. Leading platforms such as Drip, Shenzhou, and Cao Travel have rapidly gained prominence in this domain [3]. The ascendancy of online taxis has caused a substantial decline in traditional taxi ridership, thereby adversely impacting the conventional taxi industry and inciting a series of driver strikes.

Moreover, recent concerns include the substantial commissions levied by online taxi platforms, the non-transparent allocation mechanism, and arbitrary fare rule adjustments, which have prompted community apprehensions [4]. To forestall long-term platform dominance and the infringement of driver and passenger interests, and to stimulate the healthy co-development of online taxis and traditional taxis, it is imperative to investigate their scale of development within an environment marked by competition, cooperation, and rivalry.

Numerous scholars have conducted extensive research on the dynamics of online taxis and traditional taxis. Some have devised game models to analyze the market dynamics and equilibrium conditions, subsequently estimating the appropriate scale for taxi deployment [5]. Others have proposed government regulatory measures to oversee online taxi platforms, framed within a two-dimensional game model involving the government and online taxi providers [6]. Additional studies have investigated the coexistence of online taxis and traditional taxis within competitive markets, probing their future trajectories and service adaptations [7]. Furthermore, research models have explored the game between online taxis and traditional taxis, considering platform subsidies, with results suggesting that traditional taxis are unlikely to exit the market due to rapid internet development and substantial subsidies from online taxi platforms [8]. Studies on the impact of online taxis on traditional taxis also argue that traditional taxis will continue to persist in the market [9]. Some scholars advocate a synergistic developmental approach for online taxis and traditional taxis within a competitive framework, emphasizing passenger travel perspectives and suggesting the implementation of a dual-track system to foster industry reform, upgrade, and long-term stability [10].

Competitive evolutionary models have widespread applications in fields like ecology and business competition. The Lotka-Volterra model, typically utilized in these contexts, has found relevance in transportation studies. For instance, it was employed from a biomechanical viewpoint to evaluate the stability and evolutionary trends of road and rail transport [11]. Other scholars have considered various urban residential travel cohorts and built competition models between public and private transportation systems grounded in the Lotka-Volterra framework, optimizing transport structures through competition intensity adjustments [12].

Existing studies predominantly revolve around revenue competition between online taxis and traditional taxis, with relatively less exploration of evolutionary laws and optimal scales for their coexistence [13]. Therefore, this paper embraces the theory of synergistic evolution, introduces the cooperative effect coefficient, and establishes a Lotka-Volterra model for online taxis and traditional taxis. Through data simulation and analysis, the study determines the suitable scale for their coexistence, thereby fostering the stable and healthy development of the industry. The paper also presents sensible recommendations for integrating old and new business models in the context of online taxis and traditional taxis.

2. Analysis of the Evolution of Competitive Synergies

2.1. *Modeling the Evolution of Competitive Synergy*

As an important supplement to urban public transport, the development of online taxis and taxis has a strong interaction with each other and the development of the social economy. They both rely on a certain volume of passenger traffic as the basis for their growth. Therefore, the scale of development for both types of transportation should be limited within the limits of the traffic environment and resource capacity. The growth of their vehicle ownership is characterized by a logistic curve [14]. Since the logistic model can better reveal the development trend of urban transportation, this paper assumes that the development of online taxis and taxis conforms to the logistic model and expresses their development scale in terms of vehicle ownership [15].

The evolution of competition between net cars and taxis can be analyzed using an improved logistic model, i.e. the Lotka-Volterra model satisfying multiple systems, with a set of coevolution equations as:

$$\begin{cases} \frac{\mathrm{d}X_1(t)}{\mathrm{d}t} = f(X_1, X_2) = \gamma_1 X_1(t)\left[1 - \frac{X_1(t)}{N_1} - \alpha_{12}\frac{X_2(t)}{N_2}\right] \\ \frac{\mathrm{d}X_2(t)}{\mathrm{d}t} = g(X_1, X_2) = \gamma_2 X_2(t)\left[1 - \frac{X_2(t)}{N_2} - \alpha_{21}\frac{X_1(t)}{N_1}\right] \end{cases}, \quad (1)$$

where $\frac{\mathrm{d}X_1(t)}{\mathrm{d}t}, \frac{\mathrm{d}X_2(t)}{\mathrm{d}t}$ is the growth rate of net cars and taxis at time t respectively; $X_1(t), X_2(t)$ is the number of net cars and taxis respectively, which varies with time t and is a function of time t; $f(X_1, X_2)$ is the growth rate of net cars at time t when the number of taxis is X_2; $g(X_1, X_2)$ is the growth rate of taxis at time t when the number of net cars is X_1; γ_1, γ_2 is the fixed growth rate of net cars and taxis respectively, which is assumed to be a constant greater than 0; N_1, N_2 is the size threshold of net cars and taxis respectively under certain socio-economic conditions, which is assumed to be constant; α_{12} is the effect of competition of taxis on net cars, α_{21} is the effect of competition of net cars on taxis, i.e. the degree of competition, $\alpha_{12}, \alpha_{21} \geq 0$.

Net cars and taxis compete with each other in a competitive environment and reach equilibrium with each other when time t $\rightarrow \infty$ satisfies $\frac{\mathrm{d}X_1(t)}{\mathrm{d}t} = \frac{\mathrm{d}X_2(t)}{\mathrm{d}t} = 0$, when the number of net cars and taxis is X_1^*, X_2^*, respectively, and four local equilibrium points are obtained: $M_0(0,0), M_1(0,N_2), M_2(N_1,0), M_3\left(\frac{N_1(1-\alpha_{12})}{1-\alpha_{12}\alpha_{21}}, \frac{N_2(1-\alpha_{21})}{1-\alpha_{12}\alpha_{21}}\right)$.

2.2. *Competitive Coevolutionary Stability Analysis*

The stability of the 4 equilibria can be judged by the Jacobian matrix if $\det(J) \neq 0$, the balance points to meet $\det(J) > 0$ and also $-tr(J) = \left(\frac{\partial f}{\partial X_1} + \frac{\partial f}{\partial X_2}\right) > 0$, which is the stability point. The Jacobian matrix is:

$$\mathbf{J} = \begin{bmatrix} f_{X_1} & f_{X_2} \\ g_{X_1} & g_{X_2} \end{bmatrix}_{\mu_1(X_1', X_2')} = \begin{bmatrix} \gamma_1\left(1 - \frac{2X_1}{N_1} - \alpha_{12}\frac{X_2}{N_2}\right) & -\gamma_1\alpha_{12}\frac{X_1}{N_2} \\ -\gamma_2\alpha_{21}\frac{X_2}{N_1} & \gamma_2\left(1 - \frac{2X_2}{N_2} - \alpha_{21}\frac{X_1}{N_1}\right) \end{bmatrix} \quad (2)$$

where $f_{X_1} = \frac{\partial f}{\partial X_1}, f_{X_2} = \frac{\partial f}{\partial X_2}, g_{X_1} = \frac{\partial f}{\partial X_1}, g_{X_2} = \frac{\partial f}{\partial X_2}$.

Substituting the four equilibrium points M_0, M_1, M_2, M_3 into Eq. (2) and calculating $\det(J)$ and $-tr(J)$, the stability or stability conditions can be found, as shown in Table 1.

M_i	$\det(J)$	$-tr(J)$	Stable condition
M_0	$\gamma_1\gamma_2$	$-(\gamma_1 + \gamma_2)$	Instability
M_1	$-\gamma_1\gamma_2(1 - a_{12})$	$\gamma_2 - \gamma_1(1 - a_{12})$	$a_{12} > 1, a_{21} < 1$
M_2	$-\gamma_1\gamma_2(1 - a_{21})$	$\gamma_1 - \gamma_2(1 - a_{21})$	$a_{21} > 1, a_{12} < 1$
M_3	$\frac{\gamma_1\gamma_2(1-a_{12})(1-a_{21})}{1-a_{12}a_{21}}$	$\frac{\gamma_1(1-a_{12})+\gamma_2(1-a_{21})}{1-a_{12}a_{21}}$	$a_{12} < 1, a_{21} < 1$

Table 1. Analysis of the stability of the evolution of competitive synergies

According to Table 1, $M_0(0,0)$ is the unstable point; $\det(J) > 0, -tr(J) > 0, M_1(0,N_2)$ is the stable point when $a_{12} > 1$; When $a_{21} > 1$, $\det(J) > 0, -tr(J) > 0, M_2(N_1,0)$ is the stability point; When $a_{12} < 1, a_{21} < 1$, $\det(J) > 0, -tr(J) > 0, M_3\left(\frac{N_1(1-a_{12})}{1-a_{12}a_{21}}, \frac{N_2(1-a_{21})}{1-a_{12}a_{21}}\right)$ is the stability point.

As shown in Figure 1, the evolution of the competitive equilibrium between online taxis and taxis is generated with $X_1(t)$ as the horizontal axis and $X_2(t)$ as the vertical axis. As shown in Figure 1(a), when $\alpha_{12} > 1$ and $\alpha_{21} < 1$, the plane is divided into 3 regions: I for $\frac{dX_1(t)}{dt} > 0, \frac{dX_2(t)}{dt} > 0$; II for $\frac{dX_1(t)}{dt} < 0, \frac{dX_2(t)}{dt} > 0$; and III for $\frac{dX_1(t)}{dt} < 0, \frac{dX_2(t)}{dt} < 0$.

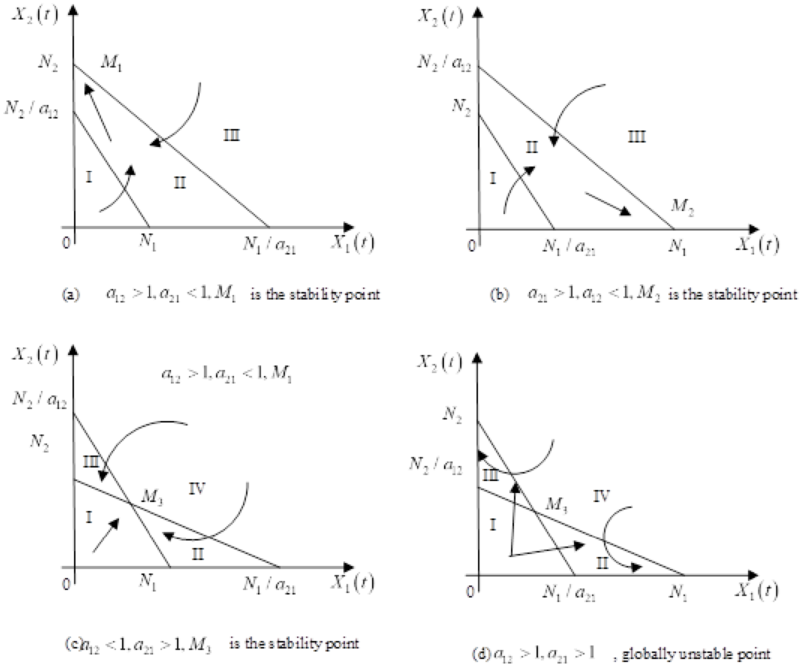


Figure 1. Evolution of the competitive equilibrium

According to the above regional characteristics, the evolutionary trajectory line converges to the equilibrium point M_1 when $t \to \infty$, regardless of the region from which it starts, and the net car will be eliminated [16]. When starting from I, the growth rates γ_1, γ_2 of both net cars and taxis are positive, their size will expand over time and the trajectory line moves towards II; When starting from II, the growth rate of net cars is negative and the growth rate of taxis is positive. Over time, the size of net cars gradually decreases and the size of taxis gradually expands, eventually moving towards M_1; when starting from III, the growth rates of both net cars and taxis are negative, which in turn move towards II and eventually also tend to M_1.

As shown in Figure 1(b), the analysis of the competition evolution trend is similar to Figure 1(a) when $a_{21} > 1, a_{12} < 1$, which eventually tends to the equilibrium point M_2. At this time, the competition impact brought by the online taxi to the taxi is much larger than the competition within the taxi itself, and the taxi will be eliminated in the market competition [17]. As shown in Figure 1(c), when $a_{12} < 1, a_{21} < 1$, the final competitive equilibrium point tends to be M_3 according to the analysis in Figure 1(a). At this point, the competition between online taxis and taxis is more moderate, and they can co-exist with each other in the market, and eventually reach an equilibrium state.

As shown in Figure 1(d), when $a_{12} > 1, a_{21} > 1$, M_3 is the saddle point and there are no stable points of competition between each other.

3. Analysis of Cooperative Co-evolution

3.1. Cooperative Evolutionary Model Construction

If the net car and taxis seek cooperation to achieve the high-end travel of net cars and taxi service standards, price unification, and other complementary advantages, mutual benefit and symbiosis, the domestic has been constantly exploring the cooperation between net cars and taxis, such as taxis joining the net car platform, the taxi vehicle net upgrading to achieve service standardization, and taxis can also join the net car platform to accept the platform's allocation of orders, the two sides jointly develop market plans of the cooperation model [18]. Therefore, the introduction of the influence effect under the cooperation condition β_{ij}, and its synergistic evolution equation system is:

$$\begin{cases} \frac{dX_1(t)}{dt} = \gamma_1 X_1(t)\left[1 - \frac{X_1(t)}{N_1} + \beta_{12}\frac{X_2(t)}{N_2}\right] \\ \frac{dX_2(t)}{dt} = \gamma_2 X_2(t)\left[1 - \frac{X_2(t)}{N_2} + \beta_{21}\frac{X_1(t)}{N_1}\right] \end{cases}, \tag{3}$$

where β_{12},β_{21} is the cooperative influence effect of taxis on net cars, net cars on taxis, and $\beta_{ij} \geq 0$ respectively.

Similarly, 4 local equilibrium points can be obtained: $M_0(0,0), M_1(0,N_2), M_2(N_1,0)$, $M_3\left(\frac{N_1(1+\beta_{12})}{1-\beta_{12}\beta_{21}}, \frac{N_2(1+\beta_{21})}{1-\beta_{12}\beta_{21}}\right)$.

3.2. Analysis of Cooperative Synergistic Evolutionary Stability

According to the analysis idea of the evolutionary stability of competitive synergy, the stability or stability conditions of four local equilibrium points in the cooperative environment between online taxis and taxis can be obtained, as shown in Table 2.

M_i	$\det(J)$	$-tr(J)$	Stable condition
M_0	$\gamma_1\gamma_2$	$-(\gamma_1+\gamma_2)$	Instability
M_1	$-\gamma_1\gamma_2(1+\beta_{12})$	$\gamma_2-\gamma_1(1+a_{21})$	Instability
M_2	$-\gamma_1\gamma_2(1+\beta_{21})$	$\gamma_1-\gamma_2(1+\beta_{21})$	Instability
M_3	$\frac{\gamma_1\gamma_2(1+\beta_{12})(1+\beta_{21})}{1-\beta_{12}\beta_{21}}$	$\frac{\gamma_1(1+\beta_{12})+\gamma_2(1+\beta_{21})}{1-\beta_{12}\beta_{21}}$	$\beta_{12}\beta_{21}<1$

Table 2. Analysis of the stability of cooperative synergistic evolution

According to Table 2, the equilibrium point $M_3\left(\frac{N_1(1+\beta_{12})}{1-\beta_{12}\beta_{21}}, \frac{N_2(1+\beta_{21})}{1-\beta_{12}\beta_{21}}\right)$ is the stable point when $\beta_{12},\beta_{21} < 1$ is satisfied. Combining with the characteristics of the region depicted in Figure 1(a), the equilibrium evolution diagram of cooperation between online taxis and taxis can be obtained, as shown in Figure 2.

When $\beta_{12},\beta_{21} < 1$, regardless of the initial state, the trajectory line of cooperative evolution converges to the equilibrium point M_3, M_3 over time as a stable solution for the symbiotic evolution of net cars and taxis. Under such conditions, they complement each other's strengths through cooperation and expand their own development scale.

4. Analysis of the Evolution of Competitive Synergy

4.1. Competitive Synergy Evolutionary Model Construction

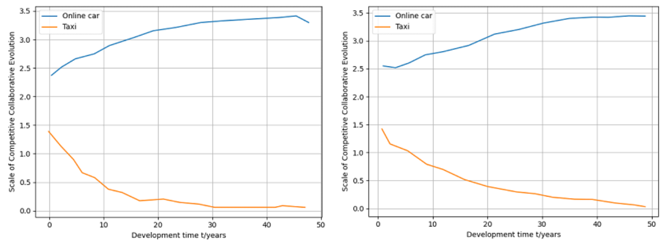
The essence of the competition between online cars and taxis is the competition and occupation of passenger flow. The party that cannot adapt to the fierce competition in the market will be eliminated or reform and upgrade itself to improve its competitiveness and achieve the separation of market positioning, and finally achieve the state of mutual competition and symbiosis [19]. Therefore, introducing both the competitive influence effect coefficient a_{ij} and the cooperative influence effect coefficient β_{ij} , the set of synergistic evolution equations is

$$\begin{cases} \frac{dX_1(t)}{dt} = \gamma_1 X_1(t)\left[1 - \frac{X_1(t)}{N_1} - \alpha_{12}\frac{X_2(t)}{N_2} + \beta_{12}\frac{X_2(t)}{N_2}\right] \\ \frac{dX_2(t)}{dt} = \gamma_2 X_2(t)\left[1 - \frac{X_2(t)}{N_2} - \alpha_{21}\frac{X_1(t)}{N_1} + \beta_{21}\frac{X_1(t)}{N_1}\right] \end{cases} \tag{4}$$

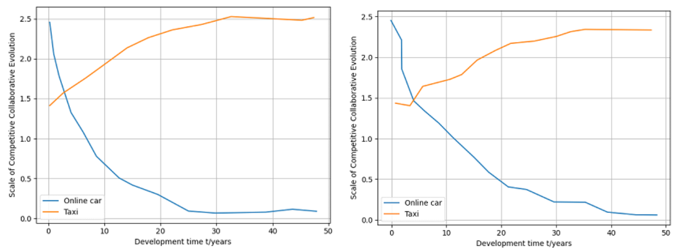
The four equilibrium points in Eq. (4) are: $M_0(0,0), M_1(0,N_2), M_2(N_1,0)$, $M_3\left(\frac{N_1(1+\beta_{12}-\alpha_{12})}{1-(\beta_{12}-\alpha_{12})(\beta_{21}-\alpha_{21})}, \frac{N_2(1+\beta_{21}-\alpha_{21})}{1-(\beta_{12}-\alpha_{12})(\beta_{21}-\alpha_{21})}\right)$, the evolution of which depends on the interrelationship of $\alpha_{12},\alpha_{21},\beta_{12},\beta_{21}$.

4.2. Stability Analysis of Competing Synergistic Evolution

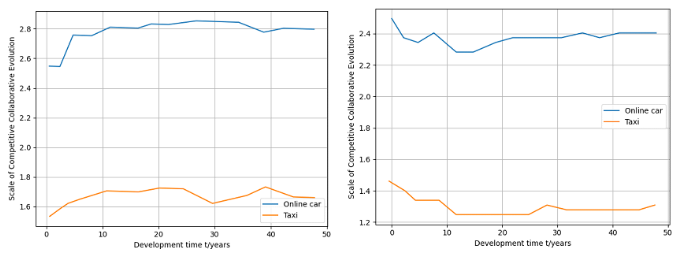
As before, the stability and stability conditions of the four local equilibrium points under the competing relationship between online taxis and taxis can be obtained according to the criteria of the



a $a_{12} = 0.5, a_{21} = 1.5$ b $a_{12} = 0.6, a_{21} = 1.3$



c $a_{12} = 1.5, a_{21} = 0.5$ d $a_{12} = 1.3, a_{21} = 0.6$



e $a_{12} = 0.3, a_{21} = 0.4$ f $a_{12} = 0.6, a_{21} = 0.7$

Figure 2. Evolution of cooperative equilibrium

Jacobi matrix for stability judgment, as shown in Table 3.

According to Table 3, $M_0(0,0)$ is not a stable point; when $\alpha_{12}-\beta_{12}>1$ and $\alpha_{21}-\beta_{21}<1$, the $\det(J)>0, -tr(J)>0, M_1(0,N_2)$ is the stability point; when $\alpha_{21}-\beta_{21}>1$ and $\alpha_{12}-\beta_{12}<1$, $\det(J)>0, -tr(J)>0, M_2(N_1,0)$ is the stability point; when $\alpha_{12}-\beta_{12}<1$ and $\alpha_{21}-\beta_{21}<1$, $\det(J)>0, -tr(J)>0, M_3\left(\frac{N_1(1+\beta_{12}-\alpha_{12})}{1-(\beta_{12}-\alpha_{12})(\beta_{21}-\alpha_{21})}, \frac{N_2(1+\beta_{21}-\alpha_{21})}{1-(\beta_{12}-\alpha_{12})(\beta_{21}-\alpha_{21})}\right)$ is the stability point.

M_i	$\det(J)$	$-tr(J)$	Stable condition
M_0	$\gamma_1\gamma_2$	$-(\gamma_1+\gamma_2)$	Instability
M_1	$-\gamma_1\gamma_2(1+\beta_{12}-a_{12})$	$\gamma_2-\gamma_1(1+\beta_{12}-a_{12})$	$a_{12}-\beta_{12}>1$, $a_{21}-\beta_{21}<1$
M_2	$-\gamma_1\gamma_2(1+\beta_{21}-a_{21})$	$\gamma_1-\gamma_2(1+\beta_{21}-a_{21})$	$a_{21}-\beta_{21}>1$, $a_{12}-\beta_{12}<1$
M_3	$\frac{\gamma_1\gamma_2(1+\beta_{12}-a_{12})(1+\beta_{21}-a_{21})}{1-(\beta_{12}-a_{12})(\beta_{21}-a_{21})}$	$\frac{\gamma_1(1+\beta_{12}-a_{12})\gamma_2(1+\beta_{21}-a_{21})}{1-(\beta_{12}-a_{12})(\beta_{21}-a_{21})}$	$a_{12}-\beta_{12}<1$, $a_{21}-\beta_{21}>1$

Table 3. Analysis of the stability of the evolution of competing synergies

The evolutionary trend of competition and cooperation between net cars and taxis is shown in Figure 3, with different competition and cooperation impact effect coefficients, and the evolutionary dynamics of competition and cooperation between net cars and taxis tending to different equilibrium points [20]. As shown in Figure 3(a), when $\alpha_{12}-\beta_{12}>1$ and $\alpha_{21}-\beta_{21}<1$, Region I is $\frac{dX_1(t)}{dt}>0, \frac{dX_2(t)}{dt}>0$; Region II is $\frac{dX_1(t)}{dt}<0, \frac{dX_2(t)}{dt}>0$; and Region III is $\frac{dX_1(t)}{dt}<0, \frac{dX_2(t)}{dt}<0$. Over time, the evolutionary trajectory lines eventually converge to the equilibrium point M_1. If the initial state is in I, both net cars and taxis have a growth rate greater than zero, the evolutionary trajectory will move towards II; if the initial state is in II, the growth rate of taxis is positive, the growth rate of net cars is negative, the scale of net cars gradually shrinks and the scale of taxis gradually expands, eventually moving towards M_1; if the initial state is in III, the growth rate of both net cars and taxis is negative, first moving towards II and eventually reaching the equilibrium point M_1 .

As shown in Figure 3(b), when $\alpha_{21}-\beta_{21}>1$ and $\alpha_{12}-\beta_{12}<1$, the regional characteristics are similar to Figure 3(a), and the competing evolutionary trend eventually tends to the equilibrium point M_2 . At this point the competitive impact of mutual cooperation and taxis on the market is less than the competitive impact of net taxis on the market, and net taxis will gradually dominate the whole market.

As shown in Figure 3(c), when$\alpha_{12}-\beta_{12}<1$ and $\alpha_{21}-\beta_{21}<1$, similarly, the competing evolutionary trends eventually converge towards the equilibrium point M_3 . At this point, the competition between online taxis and taxis is more moderate, which, together with the cooperative relationship between them, can lead to an equilibrium state in the market.

As shown in Figure 3(d), when $\alpha_{12}-\beta_{12}>1$ and $\alpha_{21}-\beta_{21}>1$, M_3 is the saddle point and there is no stable point of competition between them [21–23].

5. Co-evolutionary simulation analysis

5.1. Simulation analysis of competitive co-evolution

In order to visualize the development trend of urban online taxis and taxis under different relationships, data simulation analysis was carried out using Matlab simulation software, and the model parameters were used to determine the initial scale with the number of online taxis and taxis in Xi'an in 2020. respectively $X_{10}=2.5$, $X_{20}=1.5$ Other parameters are assumed to be $\gamma_1=0.3, \gamma_2=0.2, N_1=3.5, N_2=2.5$ [21]. With these parameters set constant, the evolution of the competitive synergy between net cars and taxis varies with the change of the competitive influence coefficient. Assuming that the two sets of competitive influence coefficients are $a_{12}=0.5$, $a_{21}=1.5$,

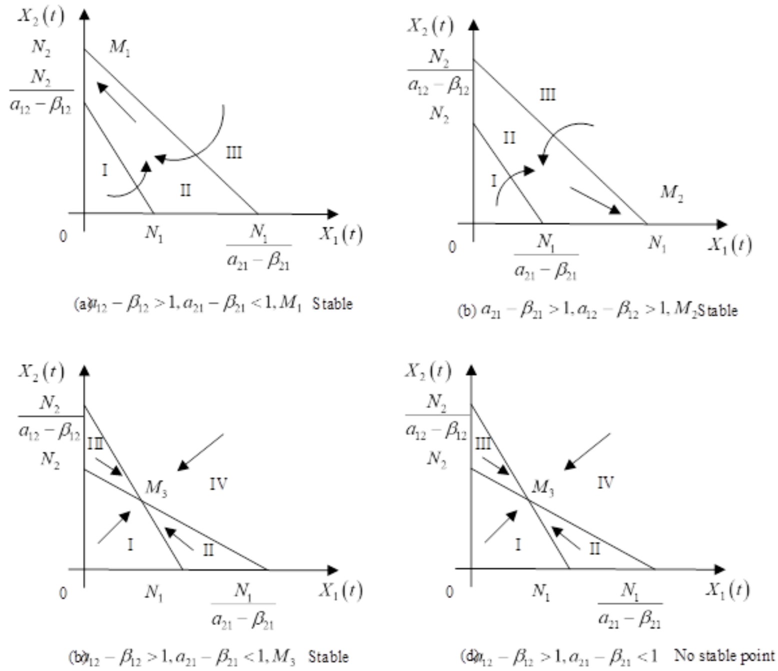


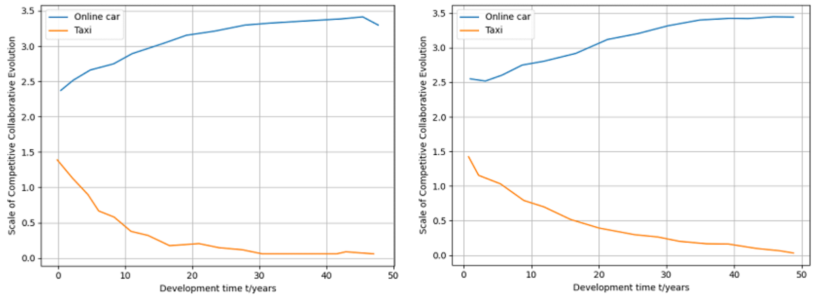
Figure 3. Diagram of the evolution of competing equilibria

$a_{12} = 1.5$, $a_{21} = 0.5$, $a_{12} = 0.3$, $a_{21} = 0.4$, $a_{12} = 0.6$, $a_{21} = 1.3$, $a_{12} = 1.3$, $a_{21} = 0.6$, $a_{12} = 0.6$, $a_{21} = 0.7$ respectively, the competitive synergy evolution curve obtained using Matlab simulation software is shown in Figure 4.

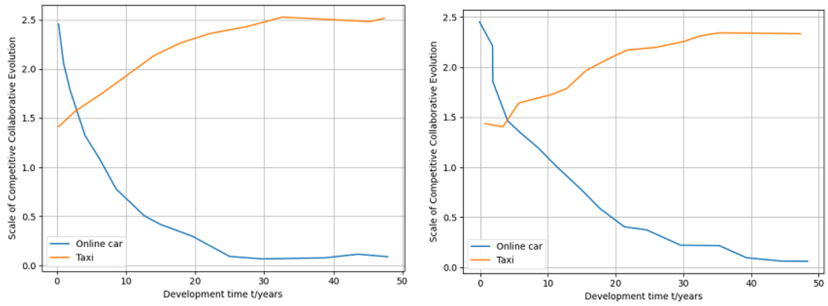
It can be seen from the simulation curves: in Figure 4(a) and (b), $a_{21} > 1$, indicating that the net car has a stronger ability to compete for and occupy the market, which inevitably causes taxis at a disadvantage to gradually lose space for development, leading to the gradual elimination of taxis in the process of competitive co-evolution; On the contrary in Figure 4(c) and (d), $a_{12} > 1$, net taxis will be eliminated in the market competition [24]. Online taxis and taxis have their own advantages, and both maintain a reasonable number structure and development scale can better meet the travel needs of urban residents [25]. If the intensity of competition between the two in the market is mitigated, as in Figure 4(e) and (f), the competition impact coefficients a_{12} and a_{21} are both less than 1. The competition between the two is more moderate and reaches a symbiotic state after some development in the market.

5.2. Simulation analysis of cooperative co-evolution

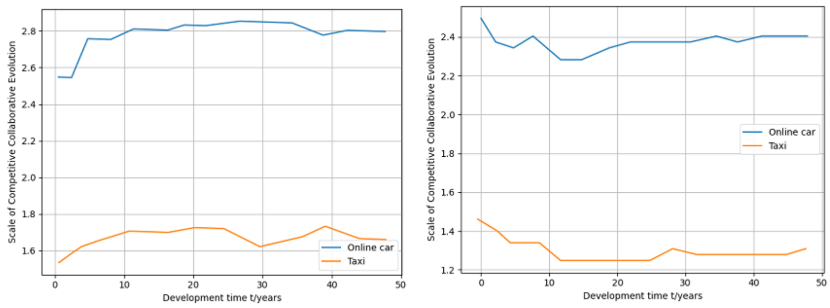
In the cooperative environment, the other parameters of the model remain unchanged, and it is assumed that the coefficients of the cooperative effect of the two groups of net cars and taxis are$\beta_{12} = 0.3$,$\beta_{21} = 0.3$,$\beta_{12} = 0.1$,$\beta_{21} = 0.1$ respectively, i.e. the stability condition of $\beta_{12}\beta_{21} < 1$ is satisfied. The data simulation using Matlab software is used to obtain the cooperative evolution curve under the cooperative environment, as shown in Figure 5. From the simulation results, it can be seen that both online taxis and taxis are able to get a larger development space beyond their scale thresholds, but under the cooperation model of a large number of taxis joining the online platform, the part of taxis undergoing online upgrading is included in the development scale of both online taxis and taxis, so their actual development scale is smaller than the development scale of the simulation results, however both still have a larger development space in general [26]. In a collaborative environment, it is possible to reduce losses due to excessive competition, while at the same time making full use of each other's strengths to serve different travel groups and improve the travel experience of city dwellers.



a $a_{12} = 0.5, a_{21} = 1.5$ b $a_{12} = 0.6, a_{21} = 1.3$

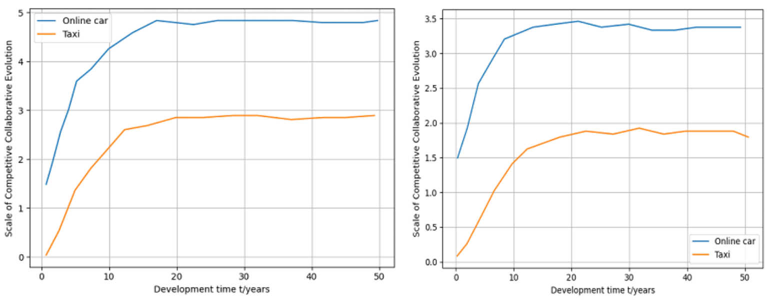


c $a_{12} = 1.5, a_{21} = 0.5$ d $a_{12} = 1.3, a_{21} = 0.6$



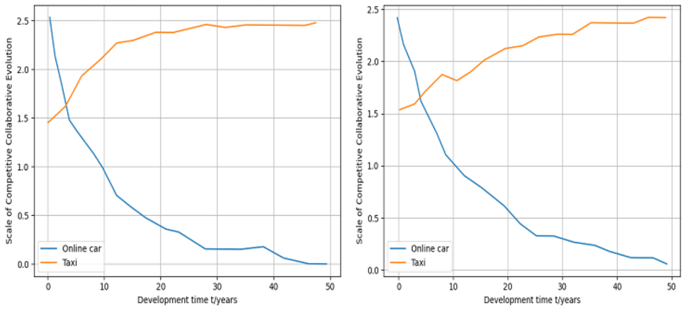
e $a_{12} = 0.3, a_{21} = 0.4$ f $a_{12} = 0.6, a_{21} = 0.7$

Figure 4. Evolutionary curve of competitive synergy

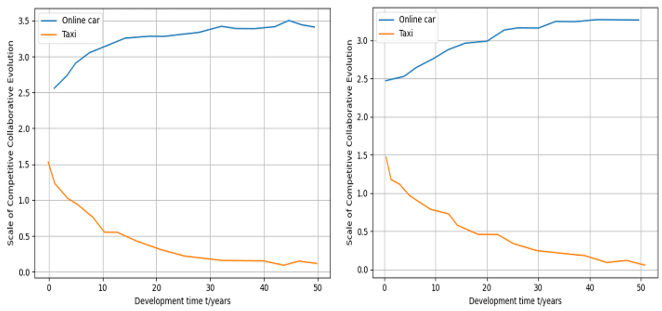


e $\beta_{12} = 0.3, \beta_{21} = 0.3$ b $\beta_{12} = 0.1, \beta_{21} = 0.1$

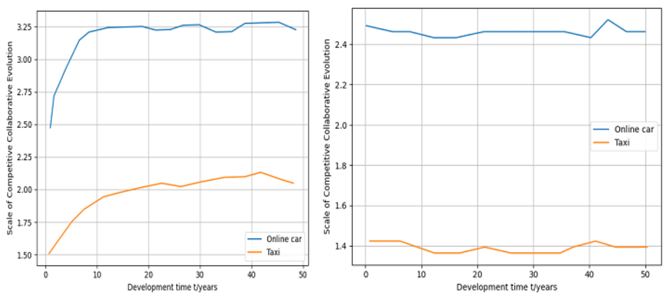
Figure 5. Collaborative synergy evolutionary curve



a $\alpha_{12} = 0.5, \alpha_{21} = 1.5, \beta_{12} = 0.3, \beta_{21} = 0.3$ b $\alpha_{12} = 1.3, \alpha_{21} = 0.6, \beta_{12} = 0.1, \beta_{21} = 0.1$



c $\alpha_{12} = 1.5, \alpha_{21} = 0.5, \beta_{12} = 0.3, \beta_{21} = 0.3$ d $\alpha_{12} = 0.6, \alpha_{21} = 1.3, \beta_{12} = 0.1, \beta_{21} = 0.1$



e $\alpha_{12} = 0.4, \alpha_{21} = 0.5, \beta_{12} = 0.3, \beta_{21} = 0.3$ f $\alpha_{12} = 0.6, \alpha_{21} = 0.7, \beta_{12} = 0.1, \beta_{21} = 0.1$

Figure 6. Competitive synergy evolutionary curve

5.3. *Simulation analysis of the evolution of competition and cooperation synergy*

In the competing environment, the coefficients of the competitive and cooperative effects are the same as before, and the Matlab software is used to simulate and analyze the evolution law under different model parameters, as shown in Figure 6.

In Figure 6(a) to Figure 6(d), when one party is more competitive, it is sufficient to dominate the whole market and eliminate the other party. Although this law of evolution is also satisfied in a competitive environment, the demise of the inferior party is greatly delayed because there is some cooperation between them [27]. In Figure 6(e) and (f), compared with the competition simulation results, when the competition between online taxis and taxis is more moderate, together with their cooperative relationship, there is more room for both to grow, but not beyond the size threshold of urban online taxis and taxis, and the time for both to reach a steady state is delayed; compared with the cooperative simulation results, the size of online taxis and taxis is somewhat constrained.

6. Conclusion

By constructing a Lotka-Volterra evolutionary model of online taxis and taxis in a competitive, cooperative and competitive environment, and using the qualitative discriminant method of differential equations to analyze their stability conditions, there is more room for both to develop in a cooperative environment, and the number of taxis upgraded online belongs to the overlapping part of online taxis and taxis, and the actual development scale is smaller than the simulated development scale. Therefore, the cooperation between net taxis and taxis should be promoted to complement each other's strengths and reduce the losses caused by excessive competition.

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Conflict of interest

The authors declare no conflict of interests.

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