On $(n-2)$-Extendable Graphs - II

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ABSTRACT. A simple graph G with a perfect matching is said to be *k-extendable* if for every set M of k independent edges, there exists a perfect matching in G containing all the edges of M. In an earlier paper, we characterized $(n-2)$-extendable graphs on $2n \geq 10$ vertices. In this paper we complete the characterization by resolving the remaining small cases of $2n = 6$ and 8. In addition, the subclass of k-extendable graphs that are "critical" and "minimal" are determined.

1 Introduction

This paper is a continuation of Ananchuen and Caccetta [3]; we assume familiarity with that paper. Throughout this paper G is a simple graph on $2n$ vertices having a perfect matching. For $1 \leq k \leq n-1$, G is *k-extendable* if for every matching M in G of size k there exists a perfect matching in G containing all the edges of M. We say that G is *minimally (critically) k-extendable* or simply *k-minimal (k-critical)* if it is k-extendable but $G-uv$ $(G+uv)$ is not k-extendable for every edge uv of G $(uv \notin E(G))$.

Observe that a cycle C_{2n} of order $2n \geq 6$ is 1-minimal but not 1-critical. The complete graph K_{2n} and the complete bipartite graph $K_{n,n}$ with bipartitioning sets of order n are each k-extendable for $1 \leq k \leq n-1$. Further, these graphs are k-critical. However, K_{2n} and $K_{n,n}$ are k-minimal if and only if $k = n-1$ (see Ananchuen and Caccetta [2]). In fact, $K_{n,n}$ and K_{2n} are the only $(n-1)$-extendable graphs for $n \geq 2$. The situation is not so simple for other values of k. In [3], we characterized $(n-2)$-extendable graphs for $n \geq 5$. Our result is:

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Theorem 1.1. *Let G be a graph on $2n \geq 10$ vertices. Then G is $(n-2)$-extendable if and only if G:*

(i) is $K_{n,n}$ or K_{2n}, or

(ii) is a bipartite graph with a perfect matching and minimum degree $n-1$, or

(iii) has minimum degree $2n-3$ and independence number at most 2, or

(iv) has minimum degree $2n-2$.

□

In addition, for $(n-2)$-minimally extendable graphs we proved:

Theorem 1.2. *Let G be an $(n-2)$-extendable graph on $2n \geq 10$ vertices. Then G is minimal if and only if G:*

(i) is an $(n-1)$-regular bipartite graph, or

(ii) is a $(2n-3)$-regular graph, or

(iii) contains one vertex of degree $2n-1$ and $2n-1$ vertices of degree $2n-3$, or

(iv) contains $2n-2$ vertices of degree $2n-3$ and 2 vertices of degree $2n-2$, u and v say, such that $N_G(u) \setminus \{v\} = N_G(v) \setminus \{u\}$.

□

In this paper we complete the characterization of $(n-2)$-extendable graphs by resolving the only outstanding cases $2n = 6$ and 8. This is done in Section 3. The minimality is considered in Section 4.

For k-critical graphs we proved, in [1], the following result:

Theorem 1.3.

(a) For $2n \geq 4$, G is $(n-1)$-critical if and only if $G \cong K_{n,n}$ or K_{2n}.

(b) For $2n \geq 10$, G is $(n-2)$-critical if and only if $G \cong K_{n,n}$ or K_{2n}.

□

In Section 4, we use our characterization of $(n-2)$-extendable graphs to complete the characterization of $(n-2)$-critical graphs. For completeness, in the next section, we state a number of results which are needed in our work.

2 Preliminaries

Theorem 2.1 [6]. *Let G be a graph on $2n$ vertices, $1 \leq k \leq n-1$. Then*

(a) G is $(k-1)$-extendable;

(b) G is $(k+1)$-connected.

□

Theorem 2.2 [6]. *Let G be a graph on $2n$ vertices and $1 \leq k \leq n-1$. If $\delta(G) \geq n+k$, then G is k-extendable.* □

Theorem 2.3 [7]. *Suppose G is a k-extendable bipartite graph. Let e be an edge of $\overline{G}$ such that $G+e$ is still bipartite. Then $G+e$ is also k-extendable.* □

Theorem 2.4 [1]. *Let G be a k-extendable graph on $2n$ vertices with $\delta(G) = k+t$, $1 \leq t \leq k \leq n-1$. If $d_G(u) = \delta(G)$, then the subgraph $G[N_G(u)]$ has at most $t-1$ independent edges.* □

Theorem 2.5 [4]. *If G is a k-extendable graph on $2n$ vertices, $1 \leq k \leq n-1$, then $k+1 \leq \delta(G) \leq n$ or $\delta(G) \geq 2k+1$.* □

Lemma 2.6 [3]. *Let G be a graph on $2n \geq 8$ vertices with a perfect matching and $\delta(G) = n-1$. Then G is $(n-2)$-extendable if and only if G is bipartite.* □

Lemma 2.7 [3]. *Let G be a graph on $2n \geq 8$ vertices with $\delta(G) = 2n-3$. Then G is $(n-2)$-extendable if and only if independence number of G is at most 2.* □

Theorem 2.8 [2].

(a) K_{2n} is k-minimal, $1 \leq k \leq n-1$, if and only if $k = n-1$.

(b) $K_{n,n}$ is k-minimal, $1 \leq k \leq n-1$, if and only if $k = n-1$.

□

Lemma 2.9 [3]. *If G is an $(n-2)$-minimal graph on $2n \geq 6$ vertices, then $\delta(G) = n-1$, n or $2n-3$. Furthermore, for $2n \geq 10$, $\delta(G) \neq n$.* □

Lemma 2.10 [3]. *If G is a $(2n-3)$-regular $(n-2)$-extendable graph on $2n \geq 8$ vertices, then G is minimal.* □

Theorem 2.11 [3]. *G is an $(n-2)$-minimal graph on $2n \geq 8$ vertices with $\delta(G) = n-1$ if and only if G is an $(n-1)$-regular bipartite graph.* □

Theorem 2.12 [3]. *Let G be an $(n-2)$-extendable graph on $2n \geq 8$ vertices with $\delta(G) = 2n-3$ and $\Delta(G) = 2n-1$. Then G is minimal if and only if G has only one vertex of degree $2n-1$ and $2n-1$ vertices of degree $2n-3$.* □

Theorem 2.13 [3]. *Let G be an $(n-2)$-extendable graph on $2n \geq 8$ vertices with $\delta(G) = 2n-3$ and $\Delta(G) = 2n-2$. Then G is minimal if and only if G has $2n-2$ vertices of degree $2n-3$ and 2 vertices, u and v say, of degree $2n-2$ such that $N_G(u) \setminus \{v\} = N_G(v) \setminus \{u\}$.* □

Theorem 2.14 [1]. *If $G \neq K_{2n}$ is a k-critical graph on $2n$ vertices, $1 \leq k \leq n-1$, then*

$$\delta(G) \leq \begin{cases} n, & \text{for } n < 2k \\ n + 2\lfloor \frac{k-1}{2} \rfloor, & \text{for } n \geq 2k. \end{cases}$$

□

Theorem 2.15 [1]. *A graph G on $2n$ vertices is 1-critical if and only if $G \cong K_{n,n}$ or K_{2n}.* □

3 Characterization of $(n-2)$-extendable graphs on $2n$ vertices

Let $\mathcal{G}(2n, k, \delta)$ denote the class of k-extendable graphs on $2n$ vertices with minimum degree δ. Theorem 1.1 gives $\mathcal{G}(2n, n-2, \delta)$ for $2n \geq 10$. In this section, we consider the classes $\mathcal{G}(8, 2, \delta)$ and $\mathcal{G}(6, 1, \delta)$. We begin with $\mathcal{G}(8, 2, \delta)$.

Let $G \in \mathcal{G}(8, 2, \delta)$. Then, by Theorem 2.5, $\delta \in \{3, 4, 5, 6, 7\}$. According to Lemma 2.6, the only members of $\mathcal{G}(8, 2, 3)$ are bipartite graphs with a perfect matching and minimum degree 3. In fact,

$\mathcal{G}(8, 2, 3) = \{K_{4,4} - M_t | M_t$ is a matching of size t, $1 \leq t \leq 4\}$.

Also, by Lemma 2.7, all members of $\mathcal{G}(8, 2, 5)$ have independence number at most 2. There are 30 non-isomorphic graphs in $\mathcal{G}(8, 2, 5)$ as listed in Table 3.1. We obtained this list by considering the degree sequence of $G \in \mathcal{G}(8, 2, 5)$; it is convenient to consider the complement $\overline{G}$ which, by Lemma 2.7, is triangle free. Note that P_t, C_t and W_t in the Table 3.1 denote the path, cycle and wheel of order t, respectively.

degree sequence of G	$\overline{G}$	G
5,5,5,5,5,5,5,5	C_8 $2C_4$	K_8 - {a hamiltonian cycle} $2K_2 \vee 2K_2$
5,5,6,6,6,6,6,6	$2P_3 \cup K_2$ $P_4 \cup 2K_2$	$[(K_1 \cup K_2) \vee (K_1 \cup K_2)] \vee 2K_1$ $P_4 \vee C_4$
5,5,5,5,6,6,6,6	$P_3 \cup P_5$ $2P_4$ $C_4 \cup 2K_2$ $P_6 \cup K_2$	$(K_1 \cup K_2) \vee (K_5$ - {a hamiltonian path}) $P_4 \vee P_4$ $2K_2 \vee C_4$ $(K_6$ -{a hamiltonian path}) $\vee 2K_1$
5,5,5,5,5,5,6,6	$P_3 \cup C_5$ $P_4 \cup C_4$ $C_6 \cup K_2$ P_8	$(K_1 \cup K_2) \vee C_5$ $P_4 \vee 2K_2$ $(K_6$ -{a hamiltonian cycle})$\vee 2K_1$ K_8 - {a hamiltonian path}
5,6,6,6,6,6,6,7	$P_3 \cup 2K_2 \cup K_1$	$(K_1 \cup K_2) \vee W_5$
5,6,6,6,6,7,7,7	$P_3 \cup K_2 \cup 3K_1$	$(K_1 \cup K_2) \vee (K_5$ - an edge $e)$
5,5,6,6,6,6,7,7	$2P_3 \cup 2K_1$ $P_4 \cup K_2 \cup 2K_1$	$[(K_1 \cup K_2) \vee (K_1 \cup K_2)] \vee K_2$ $P_4 \vee (K_4$ - an edge $e)$
5,5,5,6,6,6,6,7	$P_3 \cup P_4 \cup K_1$ $P_5 \cup K_2 \cup K_1$	$(K_1 \cup K_2) \vee (P_4 \vee K_1)$ $(K_5$ - {a hamiltonian path})$\vee P_3$
5,6,6,7,7,7,7,7	$P_3 \cup 5K_1$	$(K_1 \cup K_2) \vee K_5$
5,5,6,6,7,7,7,7	$P_4 \cup 4K_1$	$P_4 \vee K_4$
5,5,5,6,6,7,7,7	$P_5 \cup 3K_1$	$(K_5$ - {a hamiltonian path})$\vee K_3$
5,5,5,5,6,6,7,7	$C_4 \cup K_2 \cup 2K_1$ $P_6 \cup 2K_1$	$2K_2 \vee (K_4$ - an edge $e)$ $(K_6$ - {a hamiltonian path})$\vee K_2$
5,5,5,5,5,6,6,7	$P_3 \cup C_4 \cup K_1$ $C_5 \cup K_2 \cup K_1$ $P_7 \cup K_1$	$(K_1 \cup K_2) \vee (2K_2 \vee K_1)$ $C_5 \vee P_3$ $(K_7$ - {a hamiltonian path})$\vee K_1$
5,5,5,5,7,7,7,7	$C_4 \cup 4K_1$	$2K_2 \vee K_4$
5,5,5,5,5,7,7,7	$C_5 \cup 3K_1$	$C_5 \vee K_3$
5,5,5,5,5,5,7,7	$C_6 \cup 2K_1$	$(K_6$ - {a hamiltonian cycle})$\vee K_2$
5,5,5,5,5,5,5,7	$C_7 \cup K_1$	$(K_7$ -{a hamiltonian cycle})$\vee K_1$

Table 3.1

As (Theorem 2.2) every graph G with $\delta(G) \geq 6$ on 8 vertices is 2-extendable, we need only consider the class $\mathcal{G}(8,2,4)$. We now establish that $\mathcal{G}(8,2,4)$ contains exactly 7 non-isomorphic graphs. We begin with the following lemma.

Lemma 3.1. *Let $G \in \mathcal{G}(8,2,4) \setminus K_{4,4}$ and let u be a vertex of G with degree 4. Then $G[N_G(u)] \cong K_1 \cup K_3$.*

Proof: Let $H = G[N_G(u)]$. By Theorem 2.4, H contains at most one independent edge. First we suppose that $E(H) = \phi$. If $v_1v_2 \in G[\overline{N}_G(u)]$, then $G - v_1 - v_2$ is a graph on 6 vertices containing an independent set of order 4 and thus G cannot have a perfect matching containing the edge v_1v_2. This contradicts the fact that G is 2-extendable. Hence, $G[\overline{N}_G(u)]$ has no edges. But then $G \cong K_{4,}$, a contradiction. Consequently, $E(H) \neq \phi$.

Let $V(H) = N_G(u) = \{x,y,z,v\}$, $\overline{N}_G(u) = \{a,b,c\}$ and suppose without any loss of generality that $xy \in E(H)$. Then, since H cannot have two independent edges, $zv \notin E(G)$. Since G is 2-extendable, the edge xy is contained in a perfect matching F in G. Clearly F must contain an edge of $G[\overline{N}_G(u)]$, ab say. Now if $\{x,z,v\}$ is an independent set of vertices of G, then G cannot have a perfect matching containing the edges uy and ab, contradicting the extendability of G. Therefore, x must be joined to at least one of z or v. Similarly, $\{y,z,v\}$ cannot be an independent set of vertices of G and thus y must be joined to at least one of z or v. Since H contains at most one independent edge, the only possibility is for $H \cong K_1 \cup K_3$. This completes the proof of the lemma. □

Remark 3.1: Consider the proof of Lemma 3.1 above. It follows that if $G[\{x,y,z\}] \cong K_3$, then $d_G(v) \leq 4$. Since $\delta(G) = 4$, $d_G(v) = 4$. Further, $N_G(v) = \{u\} \cup \overline{N}_G(u)$. Thus G contains the graph G^* displayed in Figure 3.1 as a spanning subgraph. Moreover, if $xa \in E(G)$ with $d_G(x) = 4$, then $d_G(a) = 4$.

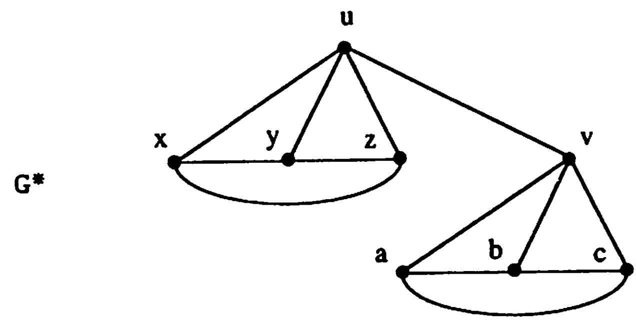

Figure 3.1

In the following we often refer to the graph G^*.

Corollary 3.2. *Let $G \in \mathcal{G}(8,2,4) \setminus K_{4,4}$ be a 4-regular graph. Then G is the graph displayed in Figure 3.2.*

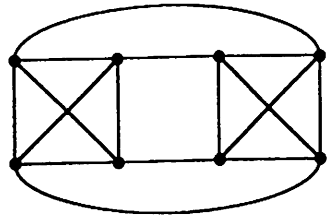

Figure 3.2

Proof: Since G^* is a spanning subgraph of G and G is 4-regular, the only possibility is that G is obtained from G^* by joining the vertices of the $\{x, y, z\}$ and $\{a, b, c\}$ with a perfect matching. Hence, G is the graph of Figure 3.2 as required. □

Corollary 3.3. *Let $G \in \mathcal{G}(8, 2, 4) \setminus K_{4,4}$. Then $\Delta(G) \leq 6$. Further, if $\Delta(G) = 6$, then there are exactly two vertices of degree 4.*

Proof: Since G^* is a spanning subgraph of G and $d_G(u) = d_G(v) = 4$, $\Delta(G) \leq 6$. Suppose G contains at least three vertices of degree 4. Without any loss of generality we may suppose that $d_G(x) = 4$ and $xa \in E(G)$. Then, by Remark 3.1, $d_G(a) = 4$ and thus G cannot contain a vertex of degree 6. Hence, if $\Delta(G) = 6$, then G has exactly two vertices of degree 4, as required. □

Lemma 3.4.

(i) $G \in \mathcal{G}(8, 2, 4)$ with $\Delta(G) = 5$ if and only if G is one of the graphs (up to isomorphism) displayed in Figure 3.3.

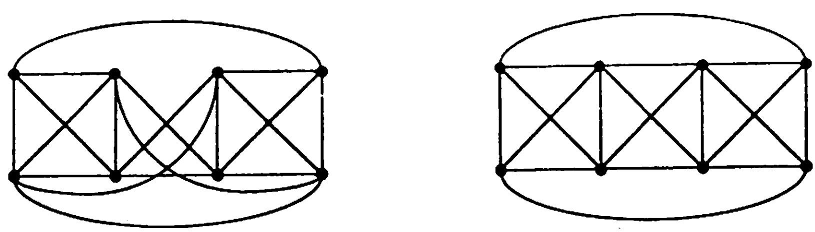

Figure 3.3

(ii) $G \in \mathcal{G}(8, 2, 4)$ with $\Delta(G) = 6$ if and only if G is one of the graphs (up to isomorphism) displayed in Figure 3.4.

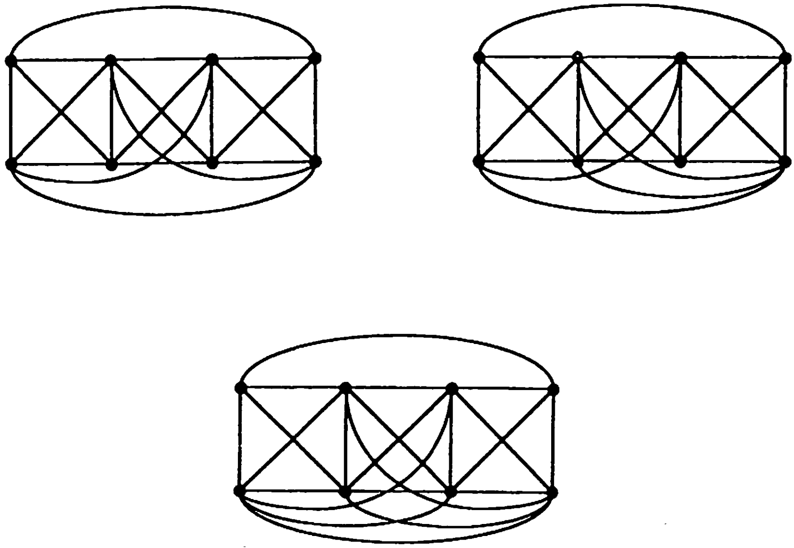

Figure 3.4

Proof: It is not too difficult to verify that the graphs in figures 3.3 and 3.4 are 2-extendable. Now let $G \in \mathcal{G}(8,2,4)$. It is sufficient to consider the bipartite subgraph G^{**} of G^* with bipartitioning sets $\{x,y,z\}$ and $\{a,b,c\}$. Using Lemma 3.1, Remark 3.1 and the minimum degree of G, it is not too difficult to show that $\{x,y,z\}$ and $\{a,b,c\}$ must have the same degree sequence in G^{**}.

Suppose $\Delta(G) = 6$. By Corollary 3.3, u and v are only two vertices of degree 4. Hence, each vertex of $\{x,y,z,a,b,c\}$ must have degree at least 2 in G^{**}. It easily follows that the only non-isomorphic graphs in $\mathcal{G}(8,2,4)$ with $\Delta(G) = 6$ are the graphs displayed in Figure 3.4.

Next, we suppose that $\Delta(G) = 5$. Then each vertex of $\{x,y,z,a,b,c\}$ must have degree at least 1 in G^{**}. Without any loss of generality, we may assume that $d_G(x) = 5$ and $\{a,b\} \subseteq N_G(x)$. Lemma 3.1 together with the fact that $\Delta(G) = 5$ implies that $d_G(a) = d_G(b) = 5$. Hence, G must have at least 4 vertices of degree 5. Therefore, G must be one of the graphs displayed in Figure 3.3, as required. □

Lemma 3.4 and Corollary 3.2 together yield the following theorem:

Theorem 3.5. *The class $\mathcal{G}(8,2,4)$ consists of $K_{4,4}$ and the six graphs in figures 3.2, 3.3 and 3.4.* □

Now we turn our attention to a characterization of $(n-2)$-extendable graphs on $2n = 6$ vertices. Theorem 2.5 ensures that a 1-extendable graph

G on 6 vertices has minimum degree 2, 3, 4, or 5. It turns out that the class $\mathcal{G}(6,1) = \cup_{\delta=2}^{5}\mathcal{G}(6,1,\delta)$ has 24 members. This can be established directly through a tedious and detailed case analysis. A simpler alternative is to take advantage of the complete catalogue of graphs on 6 vertices (see Harary [5] pp 218-224). Of the 60 graphs that satisfy the degree requirement, only 24 of them are 1-extendable; this can be established by routine checking. We summarize the result in the following theorem.

Theorem 3.6. *There are exactly 24 non-isomorphic 1-extendable graphs on 6 vertices, namely the graphs displayed in Figure 3.5.*

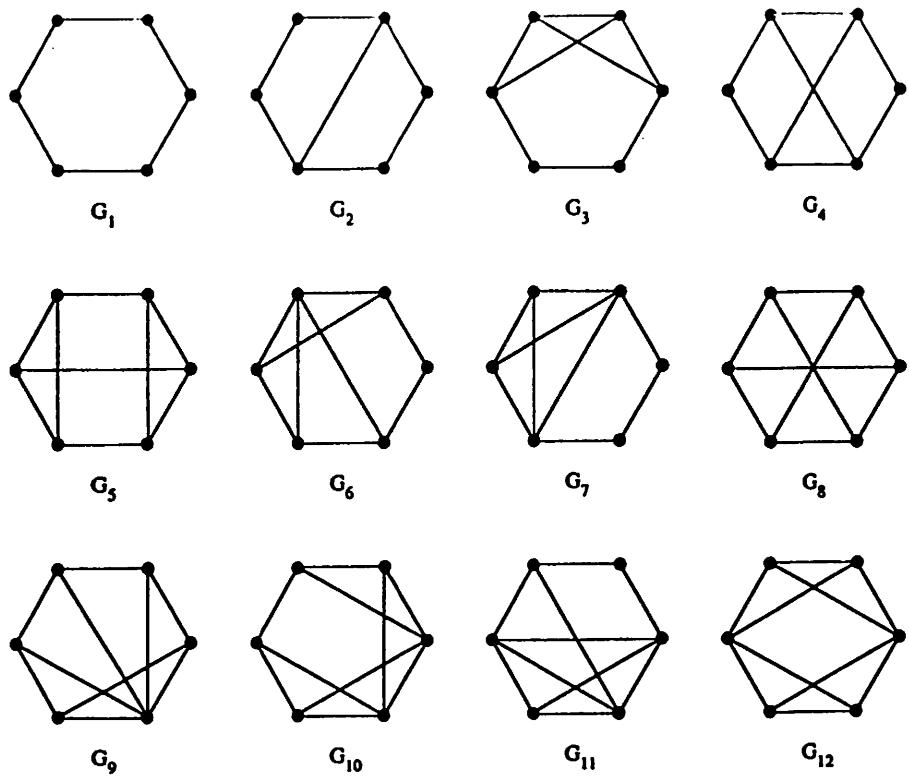

Figure 3.5

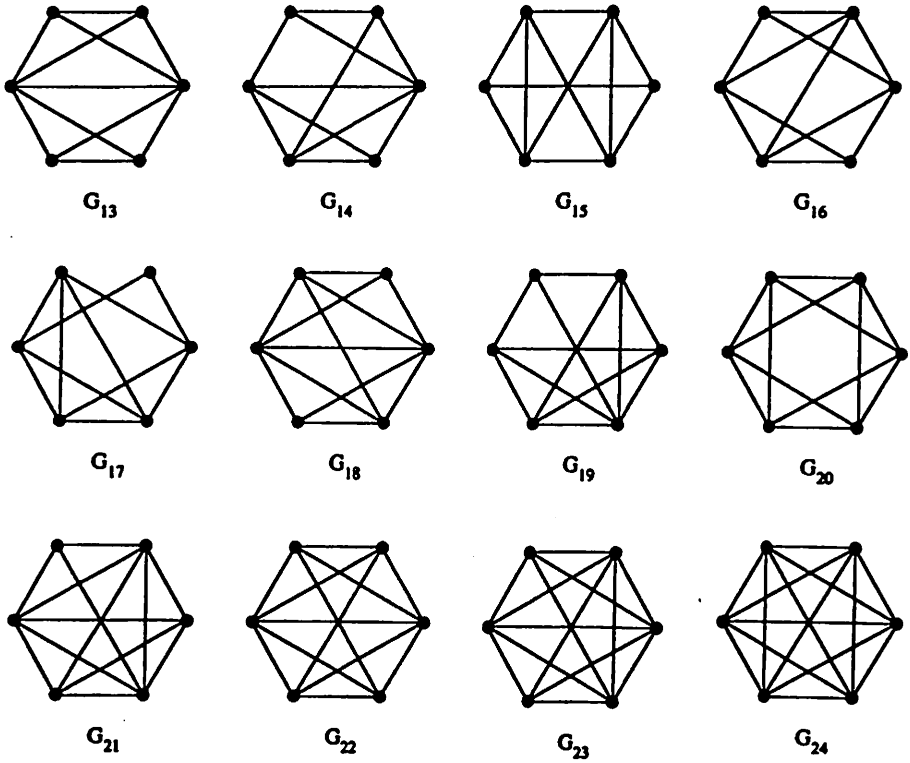

Figure 3.5 (Cont)

4 Minimal and critical graphs

We begin with $(n-2)$-critical graphs. We have observed that an $(n-2)$-extendable graph has order at least 6. Theorem 1.3(b) characterizes $(n-2)$-critical graphs of order $2n \geq 10$. Theorem 2.15 ensures that the only $(n-2)$-critical graphs on $2n = 6$ vertices are $K_{4,4}$ and K_{2n}. The remaining case is when $2n = 8$. Consider the graphs displayed in Figure 4.1. It is not too difficult to show that each G_1 is 2-critical.

We will show that besides $K_{4,4}$ and K_8 the above are the only 2-critical graphs on 8 vertices.

Theorem 4.1. *G is a 2-critical graph on 8 vertices if and only if G is $K_{4,4}$ or K_8 or one of the graphs (up to isomorphism) displayed in Figure 4.1.*

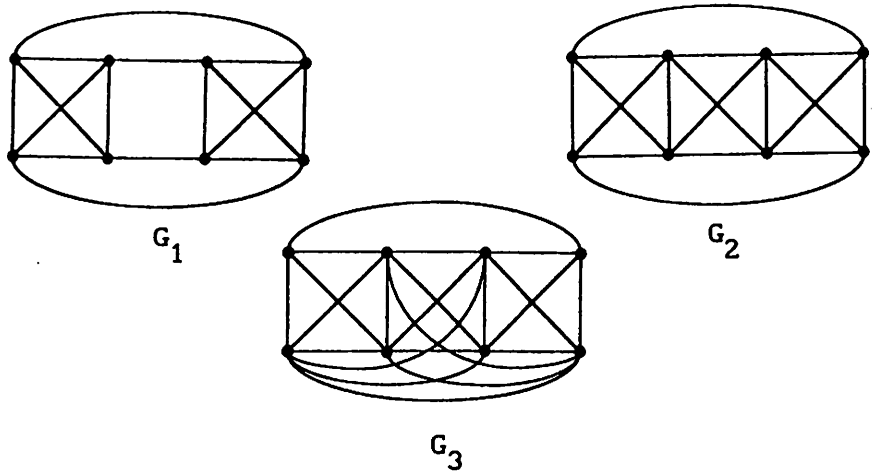

Figure 4.1

Proof: The sufficiency is obvious. For the necessity part, suppose G is a 2-critical graph on 8 vertices. By Lemma 2.14, $\delta(G) = 3$, 4 or 7.

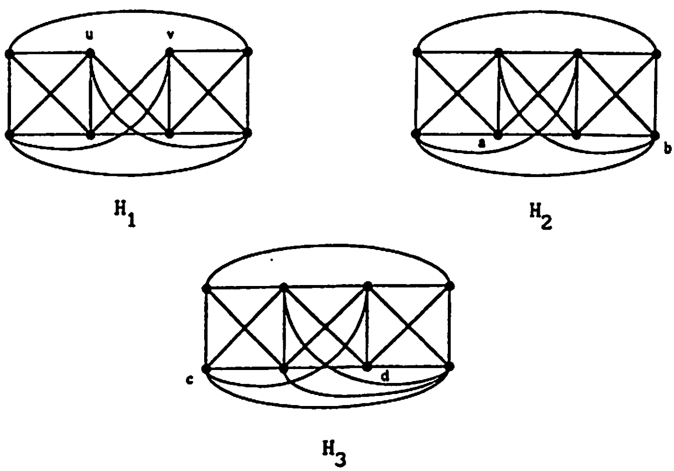

Figure 4.2

Clearly, if $\delta(G) = 7$, then $G \cong K_8$. Suppose $\delta(G) = 3$. Then, by Lemma 2.6, G is bipartite and thus, by Theorem 2.3, G is not critical. So the only remaining case is $\delta(G) = 4$. By Theorem 3.5, there are exactly seven 2-extendable graphs on 8 vertices with $\delta(G) = 4$. One of them is $K_{4,4}$ and the rest are the graphs displayed in figures 4.1 and 4.2. Notice that $H_1 + uv \cong H_2$, $H_2 + ab \cong H_3$ and $H_3 + cd \cong G_3$. Since H_2, H_3 and G_3 are 2-extendable, H_1, H_2 and H_3 are not 2-critical. This completes the proof of our theorem. □

We conclude the paper by a discussion of $(n-2)$-minimal graphs, $n = 3$ and 4. Consider the graphs displayed in Figure 4.3.

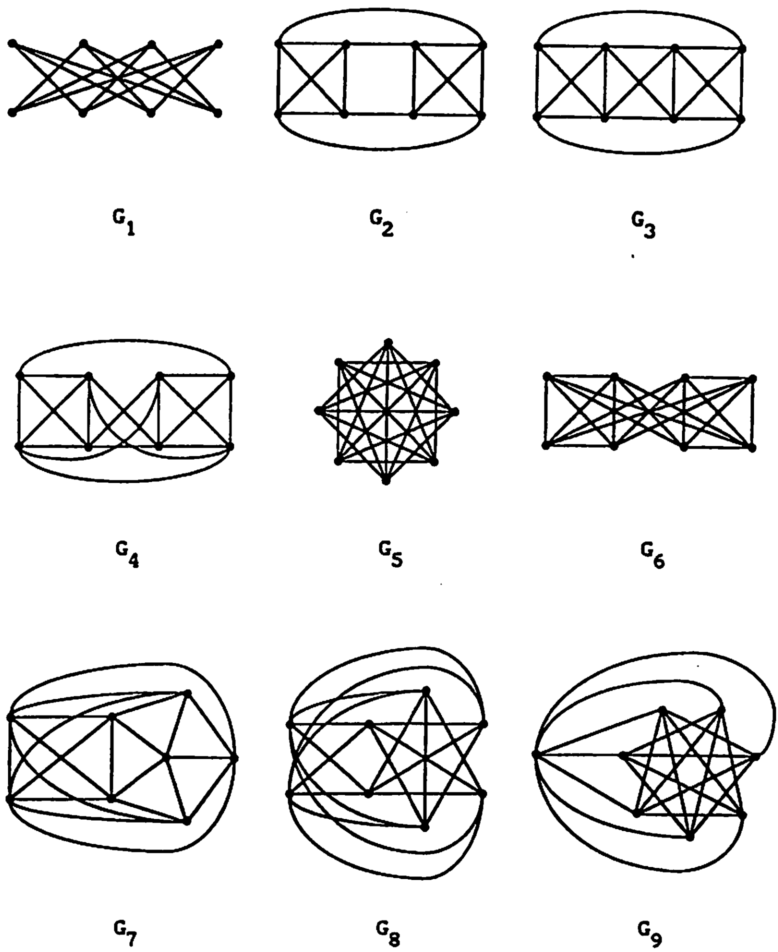

Figure 4.3

It is not too difficult to verify that each G_i is 2-minimal. We will first prove that these graphs are the only 2-minimal graphs on 8 vertices

Theorem 4.2. *Let G be a 2-minimal graph on 8 vertices. Then G is one of the graphs (up to isomorphism) displayed in Figure 4.3.*

Proof: By Lemma 2.9, $\delta(G) = 3$, 4 or 5. If $\delta(G) = 3$, then, by Theorem 2.11, G is 3-regular bipartite graph. Hence $G = G_1$. Suppose $\delta(G) = 4$. Then, by Theorem 3.5, there are exactly seven members in $\mathcal{G}(8, 2, 4)$. Three of them are G_2, G_3 and G_4. The rest are $K_{4,4}$ and the graphs H_1, H_2 and H_3 in Figure 4.4.

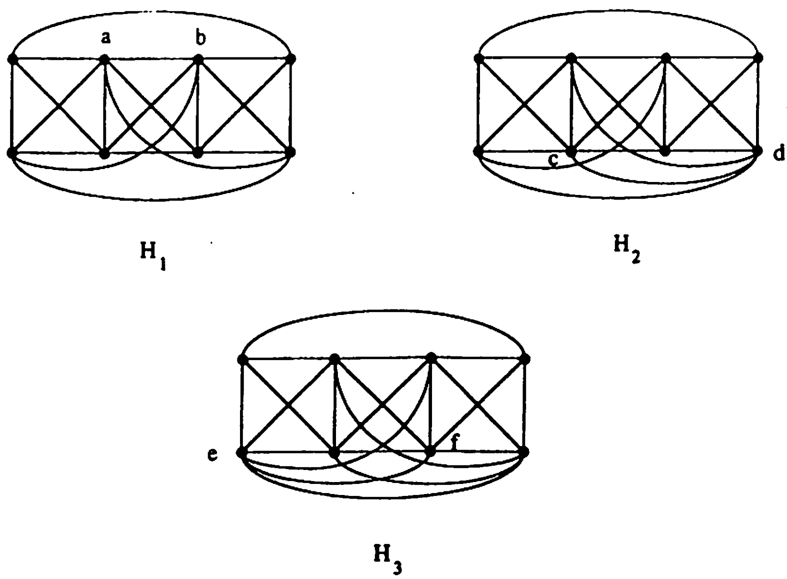

Figure 4.4

By Theorem 2.8(b), $K_{4,4}$ is not 2-minimal. Consider the graphs H_1, H_2 and H_3. Observe that $H_1 - ab \cong G_4$, $H_2 - cd \cong H_1$ and $H_3 - ef \cong H_2$. Hence, H_1, H_2 and H_3 are not 2-minimal. This proves the theorem for the case $\delta(G) = 4$.

The only remaining case is $\delta(G) = 5$. If G is a 5-regular 2-extendable graph, then, by Lemma 2.10, G is 2-minimal. According to Table 3.1, there are exactly two 5-regular 2-extendable graphs on 8 vertices, namely G_5 and G_6. So we need to consider the case when G is non-regular. We proceed according to $\Delta(G)$ which is 6 or 7.

Suppose $\Delta(G) = 6$. Then, by Theorem 2.13, G contains exactly two vertices of degree 6, u and v say, such that $N_G(u) \setminus \{v\} = N_G(v) \setminus \{u\}$.

Table 3.1 shows that there are four 2-extendable graphs with degree sequence 5, 5, 5, 5, 5, 5, 6, 6. But only two of them, $G_7((K_1 \cup K_2) \vee C_5)$ and $G_8((K_6$ - {a hamiltonian cycle}$)\vee 2K_1)$ satisfy the above mentioned condition concerning neighbour sets. Finally, consider the case $\Delta(G) = 7$. By Theorem 2.12 and Table 3.1, there is exactly one such graph, namely G_9. This completes the proof of our theorem. □

Now we turn our attention to the case $2n = 6$. Consider the graphs displayed in Figure 4.5.

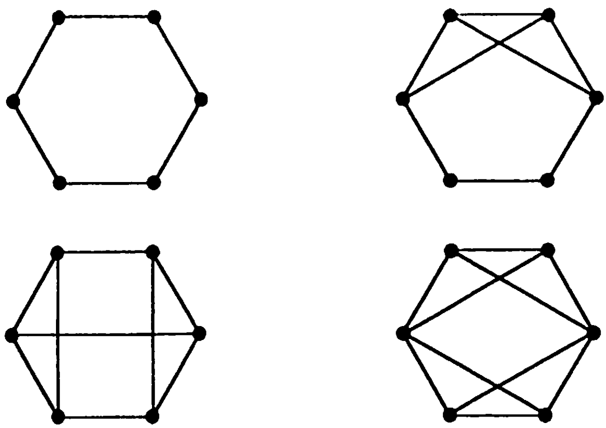

Figure 4.5

It is easy to check that each of these graphs is 1-minimal. We now prove that these are the only 1-minimal graphs on 6 vertices.

Theorem 4.3. *There are exactly four non-isomorphic 1-minimal graphs on 6 vertices, namely the graphs displayed in Figure 4.5.*

Proof: By Theorem 3.6, there are exactly 24 non-isomorphic 1-extendable graphs on 6 vertices. Four of them are the graphs in Figure 4.5, the other twenty are the graphs $H_1, \ldots, H_{20}$ in Figure 4.6. Notice that for each i, $i = 1, \ldots, 20$, $H_i - ab$ is 1-extendable. Hence, none of the graphs displayed in Figure 4.6 are 1-minimal. This completes the proof of our theorem. □

Remark 4.1: By Theorem 2.9, H_{16}, H_{18}, H_{19} and H_{20} are not 1-minimal since each of them has minimum degree at least 4.

Remark 4.2: Theorems 4.2 and 4.3 were stated without proof in [3]. The proof that was available at that time was very tedious as the characterization of $(n-2)$-extendable graphs on $2n = 6, 8$ was not available.

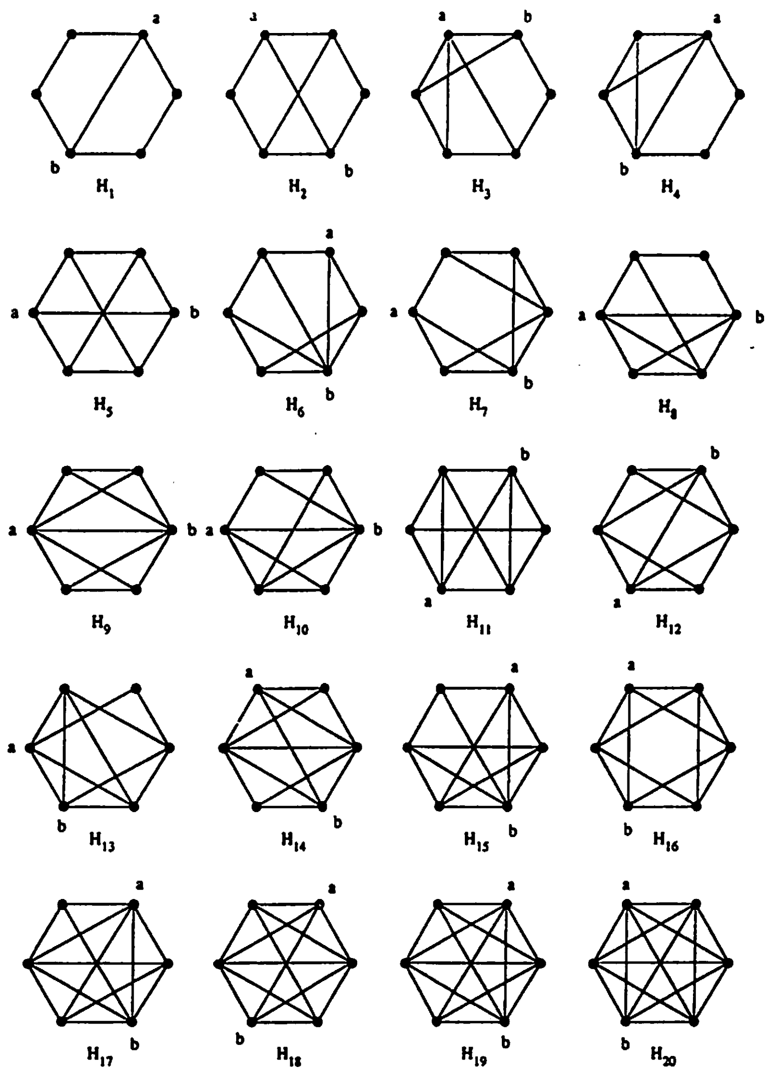
H_1
H_2
H_3
H_4
H_5
H_6
H_7
H_8
H_9
H_{10}
H_{11}
H_{12}
H_{13}
H_{14}
H_{15}
H_{16}
H_{17}
H_{18}
H_{19}
H_{20}

Figure 4.6

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