On a Conjecture Concerning Self-conjugate Self-orthogonal Diagonal Latin Squares

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ABSTRACT. We exhibit a self-conjugate self-orthogonal diagonal Latin square of order 25.

A Latin square of order n is an $n \times n$ array such that every row and every column is a permutation of an n-set N. A transversal in a Latin square is a set of positions, one per row and one per column, among which the symbols occur precisely once each. A diagonal Latin square is a Latin square whose main diagonal and back diagonal are both transversals.

Two Latin squares of order n are orthogonal if each symbol in the first square meets each symbol in the second square exactly once when they are superposed. A Latin square is self-orthogonal if it is orthogonal to its transpose.

In an earlier paper [1] Danhof, Phillips and Wallis considered the question of the existence of self-orthogonal diagonal Latin square of order 10 and introduced a special type of self-orthogonal diagonal Latin square, "self-conjugate squares".

Given an orthogonal pair A, B of Latin squares of order n, we define the conjugate pair A^*, B^* as follow:

$$\text{for } i,j \in N, A^*(A(i,j),B(i,j)) = i \text{ and } B^*(A(i,j),B(i,j)) = j.$$

A^*, B^* is again an orthogonal pair and its conjugate pair is A, B. Thus forming the conjugate pair is an involuntary operation. If A is self-orthogonal, we define the conjugate of A to be A^* where A^*, $(A^T)^*$ is the conjugate pair of A, A^T. We call A self-conjugate if $A = A^*$.

In paper [1], Danhof, Phillips and Wallis conjectured that there is no self-conjugate self-orthogonal diagonal Latin square of order n for any odd n.

This conjecture is false. Here is a self-conjugate self-orthogonal diagonal Latin square of order 25:

1	Q	H	I	J	M	G	3	N	L	A	7	5	6	4	C	8	F	D	9	B	E	K	2	P
P	2	B	L	3	D	6	N	M	J	5	8	7	K	A	E	4	9	F	G	H	I	C	Q	1
8	5	3	N	B	Q	K	H	D	6	2	9	C	P	7	L	1	G	E	F	I	4	M	J	A
F	H	M	4	E	J	P	5	6	7	G	Q	B	8	I	D	L	1	9	C	2	N	3	A	K
D	B	2	8	5	L	N	E	Q	I	3	A	J	4	9	P	7	C	1	6	K	G	H	M	F
E	7	A	9	K	6	D	G	J	Q	N	P	2	M	8	F	C	B	4	L	5	1	I	H	3
C	D	F	A	H	2	7	J	L	P	Q	G	6	9	K	1	N	8	I	3	E	B	5	4	M
H	C	1	E	4	F	I	8	P	M	9	N	Q	5	G	6	3	J	7	A	D	K	2	B	L
K	I	C	J	F	4	E	B	9	H	P	D	3	L	Q	A	G	M	6	N	7	2	1	8	5
B	F	Q	P	C	3	4	K	G	A	L	I	N	D	J	H	9	5	2	M	1	8	E	7	6
L	3	5	D	2	I	M	P	8	1	B	F	G	C	E	4	K	N	H	J	A	Q	6	9	7
G	N	D	K	I	H	1	2	3	5	E	C	9	F	B	7	P	A	M	Q	J	L	8	6	4
J	6	9	G	1	7	2	L	C	E	4	3	D	N	M	B	F	P	5	I	Q	H	A	K	8
M	G	J	5	8	1	L	4	7	N	F	B	A	E	C	K	Q	H	P	2	9	6	D	3	I
I	J	K	1	Q	G	3	6	5	2	C	E	H	B	F	8	M	4	A	7	P	D	L	N	9
7	K	I	B	M	8	C	F	H	9	D	1	4	2	6	G	A	L	Q	E	3	P	J	5	N
3	L	8	2	N	P	5	1	A	G	J	6	M	I	D	9	H	Q	K	B	4	F	7	C	E
4	M	L	F	A	N	J	7	2	C	6	5	K	Q	1	3	E	I	8	P	G	9	B	D	H
5	A	P	6	D	9	8	I	4	F	K	L	1	3	2	N	B	7	J	H	M	C	Q	E	G
N	E	7	Q	L	5	F	M	1	8	H	4	P	G	3	2	J	D	B	K	6	A	9	I	C
A	4	G	H	6	K	9	Q	E	B	1	M	8	7	N	I	2	3	C	5	L	J	P	F	D
6	9	N	3	P	E	Q	A	I	K	7	J	F	1	H	5	D	2	L	8	C	M	4	G	B
9	8	4	M	7	B	H	C	K	D	I	2	E	A	P	Q	5	6	G	1	F	3	N	L	J
Q	1	E	7	G	C	A	9	B	4	8	H	I	J	L	M	6	K	3	D	N	5	F	P	2
2	P	6	C	9	A	B	D	F	3	M	K	L	H	5	J	I	E	N	4	8	7	G	1	Q

Reference

[1] K.J. Danhof, N.C.K. Phillips and W.D. Wallis, On self-orthogonal diagonal Latin squares, *JCMCC* 8 (1990), 3–8.