On a problem of Hartman and Heinrich concerning pairwise balanced designs with holes*

Yeow Meng Chee
Department of Computer Science
University of Waterloo
Waterloo, Ontario
Canada N2L 3G1
ymchee@neumann.uwaterloo.ca

Charles J. Colbourn, Robert P. Gallant and Alan C. H. Ling
Department of Combinatorics and Optimization
University of Waterloo
Waterloo, Ontario
Canada N2L 3G1
cjcolbourn@math.uwaterloo.ca
rpgallant@barrow.uwaterloo.ca
chaling@barrow.uwaterloo.ca

ABSTRACT. We consider the problem of constructing pairwise balanced designs of order v with a hole of size k. This problem was addressed by Hartman and Heinrich who gave an almost complete solution. To date, there remain fifteen unresolved cases. In this paper, we construct designs settling all of these.

1 Introduction

Let $\mathcal{K}$ be a set of positive integers. A *pairwise balanced design* (PBD) of *order* v with *block sizes* from $\mathcal{K}$, denoted PBD$(v, \mathcal{K})$, is a pair $(\mathcal{X}, \mathcal{B})$, where $\mathcal{X}$ is a finite set of v *points* and $\mathcal{B}$ is a set of subsets of $\mathcal{X}$, called *blocks*, with the property that $|B| \in \mathcal{K}$ for all $B \in \mathcal{B}$, and every 2-subset of $\mathcal{X}$ appears in precisely one block. PBD$(v, \mathcal{K} \cup \{k^*\})$ is a notation for a PBD

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of order v with one block of size k and all other blocks having sizes in $\mathcal{K}$. A PBD$(v, \mathcal{K} \cup \{k^*\})$ is also known as a PBD$(v, \mathcal{K})$ with a *hole* of size k.

Let $\mathbf{Z}_{\geq 3}$ be the set of all integers that are at least three. The problem of constructing designs PBD$(v, \mathbf{Z}_{\geq 3} \cup \{k^*\})$ was considered by Hartman and Heinrich in [2], where the following result is established.

Theorem 1.1. *A PBD$(v, \mathbf{Z}_{\geq 3} \cup \{k^*\})$ exists if and only if $v \geq 2k+1$ except when*

1. $v = 2k+1$ *and* $k \equiv 0$ *(mod 2);*
2. $v = 2k+2$ *and* $k \not\equiv 4$ *(mod 6),* $k > 1$*;*
3. $v = 2k+3$ *and* $k \equiv 0$ *(mod 2),* $k > 6$*;*
4. $(v,k) \in \{(7,2),(8,2),(9,2),(10,2),(11,4),(12,2),(13,2)\}$, *and possibly when* $(v,k) \in \mathcal{P} = \{(17,6),\ (21,8),\ (26,9),\ (28,11),\ (29,10),\ (29,12),\ (30,11),\ (33,14),\ (35,12),\ (37,14),\ (38,13),\ (39,14),\ (42,17),\ (47,18),\ (49,20),\ (55,20)\}$.

The possible exception $(v,k) = (17,6)$ in Theorem 1.1 was subsequently removed by Heathcote [3] who remarked that no PBD$(17, \mathbf{Z}_{\geq 3} \cup \{6^*\})$ exists. Fifteen pairs $(v,k) \in \mathcal{P}$ then remained for which the existence of a PBD$(v, \mathbf{Z}_{\geq 3} \cup \{k^*\})$ was undetermined. In this note, we construct PBDs settling the problem for all of these pairs.

The strategy we used in constructing a PBD$(v, \mathbf{Z}_{\geq 3} \cup \{k^*\})$ $(\mathcal{X}, \mathcal{B})$ is to completely specify the set of blocks $\mathcal{A} \subseteq \mathcal{B}$ with sizes greater than three, that is, $\mathcal{A} = \{B \in \mathcal{B} \mid |B| \geq 4\}$. Following [1], we call the partial design $(\mathcal{X}, \mathcal{A})$ the *prestructure* of the PBD. The remaining blocks of size three (*triples*) are then filled in by a variant of Stinson's hillclimbing algorithm [4] similar to the one described in [1].

2 Prestructures

The most difficult task in the construction of PBD$(v, \mathbf{Z}_{\geq 3} \cup \{k^*\})$ is the determination of suitable prestructures. The prestructures $(\mathcal{X}, \mathcal{A})$ used in this paper are constructed manually, taking into account the following elementary conditions that must be satisfied:

1. $\sum_{A \in \mathcal{A}} \binom{|A|}{2} \equiv \binom{v}{2}$ (mod 3);
2. for every $x \in \mathcal{X}$, $\sum_{A \in \mathcal{A} | x \in A} (|A| - 1) \equiv v - 1$ (mod 2).

In Table 2.1, we give prestructures of designs PBD$(v, \mathbf{Z}_{\geq 3} \cup \{k^*\})$ for which the hillclimbing algorithm succeeds in completing them to PBDs. In each case, the prestructure consists of only one block of size k, and the remaining blocks have sizes four and five. The point-set of a PBD of order v is taken to be the set consisting of the first v elements of $P = \{a, b, \ldots, z, A, B, \ldots, Z, 1, 2, 3\}$. The block of size k in each prestructure is the set consisting of the first k elements of P, and we omit it from the listing in Table 2.1.

(v, k)	(21, 8)	(26, 9)	(28, 11)	(29, 10)	(29, 12)	(30, 11)	(33, 14)	(35, 12)
Blocks in pre-structure	aijkl bimno ampq anrs aotu bjpr bkqt blsu cnpu doqr emst fkps gjqu hlrt	amouz ajkl bjmn cjop dkqs emqt fquv gkrw hmrx iryz	amxyz ilvAB alot blps clqu dmpv emqs fmrw gnpx hnqy inrz jorA kosB	alszC akpu bksv clpw dlqu emrv fmsw gnqx hnty iory jotx	erstv amqx bmry cmsz dmtA enqy fnrx gosB hotC ipvz jpwA kuvB luwC	anuvw klmzA alot blpu clqv dmrw empx fmqy gnrz hnpA inqB jorC kosD	auvwx bAEFG aotA bouC covE dowG epxB fpyD gpzF hptE iquG jqvB krwD lrxF msyA nszC	aopqr amsy bmtA cnuC dnvz eowB foxD gpsA hptC iquz jqvB krwD lrxy

(v, k)	(37, 14)	(38, 13)	(39, 14)	(42, 17)	(47, 18)	(49, 20)	(55, 20)	
Blocks in pre-structure	auvwx bAEFG aotA bouC covE dowG epxB fpyD gpzF hptE iquG jqvB krwD lrxF msyA nszC	zABCD anrz bnsA cntB dorC eosD fotE gpuF hpsG iptH jqvI kqwJ lqxK mryL	auvwx bAEFG aotA bouC covE dowG epxB fpyD gpzF hptE iquG jqvB krwD lrxF msyA nszC	rstuv LMNOP arwF brxC cryB dszC esxB fsyG gtBH htxI ityC juzA kuDJ luEK mvzL nvAM ovDN pwDO qwAP	KLMNO asBK bsCM ctDO dtEQ euFS fuGL gvHN hvIP iwJR jwBM kxCO lxDQ myES nyFL ozGN pzHP qAIR rAJK	DEFGH STUVW auDN buEP cuFR duGT evHV fvIO gwJQ hwKS ixLU jxMW kyDP lyER mzFT nzGV oAHO pAIQ qBJS rBKU sCLW tCMN	DEFGH STUVW auDN buEP cuFR duGT evHV fvIO gwJQ hwKS ixLU jxMW kyDP lyER mzFT nzGV oAHO pAIQ qBJS rBKU sCLW tCMN	

Table 2.1. Prestructures for PBD$(v, \mathbf{Z}_{\geq 3} \cup \{k^*\})$

Given these prestructures, it is easy to complete them with triples to PBDs using hillclimbing. Our program, running on a DEC 2000 4/200 Alpha system, took less than two seconds on the largest design. For the sake of completeness, we include in the Appendix the triples required to complete each prestructure to the desired PBD.

References

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Appendix: Listing of Triples

We exhibit here the set of triples required to complete each of the prestructures in Table 2.1 to the desired PBD.

$(v, k) = (21, 8)$:

cit cjo ckr clm cqs dis djm dku dlp dnt eip ejn eko elq eru fiu fjt flo
fmr fnq gir gkm gln gos gpt hiq hjs hkn hmu hop

$(v, k) = (26, 9)$:

dlr itu apt ckn gtx boy kmv anr fkp duy blt erv aqw fnw iko gms dwx fjr
bqx bkz buw hpw asy lnx imw pxz gno dov eny hjt hlq avx hnz gpy cvw jxy
elo ewz otw dnt lwy pru glu gqz eju clm fls npq gjv fox stv cux hvy oqr
eps ilp djz hos ijq jsw isx kty ekx bpv hku brs ftz dmp lvz cqy fmy crt
inv csz nsu

$(v, k) = (28, 11)$:

bno jlm fst dwz isw hpt fpq krt elz bmA kmu jsz dlx hmo env rsx dns juv
fox byB goz kln anB hlr btz twA ery kyA fly eow nuw aqw cop gsA auA kqz
etu fnA epA fvz apr bwx hxA jqx exB asv dty jwy crv ipy kvx bqv bru dqA
qtB imt hzB guy dou jnt iux gtv kpw gmB jpB ioq cwB csy glw cmn ctx fuB
hsu hvw ovy puz czA gqr drB

$(v, k) = (29, 10)$:

brw iBC gvA dpv tvw csx kln axA flr krA hkz fyA hmC bxy mux gpt hqw buA
hlv iuz rst gkm ctu jly cko jvz cnr dwy imn jqs moB ekq grC kyB guw jmp
anv esy boq cmq iqv jkC ikw qzB bmz isA osu aqr fov cvB dns nwC wxz fxB
gsB ipx fkt jru uvy jnB fqC atB lmA jwA hoA amy eop ewB czA hrx dmt drz
euC pqy aow blB vxC dkx doC elx cyC enA noz btC etz fpz gyz bnp fnu glo
prB dAB hps huB pAC ilt qtA

$(v, k) = (29, 12)$:

asA gmw kmn gnt guy hvy anC dyC hqA ewz ftw fvC btz coy lxB inA dsw jrC
bps fou grA itu imo awy eAB pyB cqC fzA iqr dru eux hnw iwB gzC lns hpr
hsu dqv jnv lqz cnp isC gpq dpx kpC fmp cuA aov bno doz krw apu lpt jox
ctx kyz nuz jzB jty koA arz bvA jqs cvw ksx gvx ixy fqB jmu emC oqw crB
atB lyA xAC fsy bwx lor eop bqu kqt hxz bBC dnB lmv hmB

$(v, k) = (30, 11)$:

bwD fow gou pvC bsA fvB gqA dpD ewz hrv jsw fuD ktu jvA etB crt jpz uxA
dyz ayA cwA dlx noy fpt ioA kqx qtC fAC bnt frx aqz euy ipy bvx jty enD
axC kwB hqu elC tAD erA eqs dAB mst oxB ryD amD wxy gtw iwC eov iru dtv

duC jqD gsv csu doq hmo isx hBD bqr dns arB vzD imv knC pqw boz gxD fln
hlw kvy fsz gly cmn cCD kpr hsy ilD bmB jmu lrs aps cop hzC sBC htx byC
itz jlB gpB jnx gmC cyB cxz uzB

$(v,k) = (33,14)$:

fuE lop hsB kst goq aqF ctw cpq dAD drs esF byB azD ioz moF jpu mqC eDG
cxD npG jxC cCF bqw nox cyG koB isv jzA huF aps mxG fCG gry crA fos kvF
nuD iBE hxz dpC ixy ayC csu brz fwF kqx fAB kuy gBD dqt ipr dvy jtG mpw
mzE kCE eAC hrG lDE muB mvD frv ewz kzG gsE itF dBF nvA duz gtu hvC czB
lvz nyF dxE iwA jDF eqE tyz aBG gwC hoD gvG kpA jyE bpv arE hqA etv ftx
lqy mrt ltC bsx lwB btD nwE hwy ntB jsw nqr fqz qsD eoy luA eru jor gxA
lsG iCD rBC

$(v,k) = (35,12)$:

puy gno nty gyE eqE wxG jnG qxF tzB jAC hFI ixA gzH cqD rEG lns boG jru
qtH hnw jDH vAI fzC knE lqI bBI dyD crA oCF hrH fwI avx epF rsF ezD gvD
fGH tEF ctw jyI ist cpv emC aCD brv etv awz kux muv cHI euH doE auE sEI
mwH fnr hyB gmx dmI jxE nAF bsu fqA duB dpG hAE bwy fmp fvE dtx xBH hov
drC erI huG irB wCE lvC guF kpH fsB csx bxC ozI kvF kmB yzG qCG kqy luw
dFH mrz esG ltD kCI ioH bpz ouA fyF anH hsD BDE inI dqs bEH jpw gGI eyA
uDI hmq enx iyC cBF dwA lzF lpE cmG aAB bnq lBG lmo ipD ftu imE coy kos
ivG npB gqw pxI atI jot jsz ADG svw ktG vyH sCH bDF aFG gBC hxz grt mnD
czE jmF iwF kzA lAH

$(v,k) = (37,14)$:

jEH fvK qtK lCK eqF iAC BCF cFI jxy yIK bsx yEJ ntD nwE iwy eor kpC evH
ayG cCH gqA oBJ grG lwI åCE los eCI gxC kvy lpv cpq nHJ bBH koz euy dAI
ewJ gsB qxD wAK kGH hqw btw qsI nuI gwH asD iEI isJ itv mxG foq nox ezA
aqH txH bpr est hsF lyH fuz bqy svG dtu jDJ crt dBE cyz eDG jsK fsE fAH
jwC gvD hxK cxA gty kFK csw jrI ksu nGK cGJ kxE guE hCD frJ cuK aFJ nvF
lAD dps kqJ wzB fCG mtF iFH dvC aBI jpG npA rvA eEK dqz moD ftB gJK nyB
oHK hzH irB dyF mHI ryC fwF ktI hAJ iop mpw hvI bvJ tCJ arz mvz hru jtz
iDK zDE hBG nqr hoy lqE luB rsH mrE pIJ mBK drK bzK ixz joF goI apK lzJ
puH bDI dxJ mqC dDH muJ fxI kAB cBD juA uDF ltG zGI

$(v,k) = (38,13)$:

jpr iBE hwC rsH uvH dyE avD bxI jCF tvx yIJ fxD AGL joL jnE iyK mCH dsJ
goA kuA gDK tCI gEI vGJ aCG fny bqC uwG oyF gnG knC gzJ lvC apK oBJ hIK
irw mDE lyz fvw pvL htu cuz wDI equ dxL grv iCJ cpC jxJ jtw dpB stz nwK
dqz bvy CEL aox suK hJL mpx iov aHI iux noI lBL evz inL muB gqH bBH lwF
hnv qrB gtL qyA dvF EHK koH lou mvA hrD kzL DHL hxA boK fFI asL nxH lpA
uEJ hFH gxB isI mFJ zFK ltJ krK iAF dnu qsF enF xyC lnD mnq iqD aEF lsE
pyD msw lGH uIL btG mzI moG crA jAK cxE gwy kvE fqG fsB bzE cqL etA cDF
npJ jsy erx bru gsC hoz epE fKL auy rtF dtD juD ksx kty kpI kDG ewL wAE
opq eBI fuC aAJ kBF eyG hqE rEG xFG dAI bDJ fAH izG eCK vBK dGK hyB frJ
dwH jBG wxz fpz csv cow cyH awB mtK aqt jzH eHJ cGI cJK lrI bpw bFL

$(v,k) = (39,14)$:

fzI cBG eqt ksB hBM iDE lyE gLM aqC gEH HIL bpv wyL drz kuF dvM jyI nBE
cJM koJ sGM tuz aBD jtH hCL aEL kyG gyB cyz erv uEM ctw fAJ nuK dFH nIJ
fsx vHJ jwJ pwI eyC cFL ktx cAK kEK dsE eos cCH mxE kHM fBL gDI cxI huy
arG fvF mqH dxJ ioI GHK frE xAC eGL uAB euJ gCK svI bBI moL fCG nwA iFK
joF mBF dqy zBH gsw cqD dAD mrJ bst xDL azJ hqx jpA zKL nry iAM dtI kzA
hJK gox lwz iwH apH loM opq zDG kqI bwM luI juD psJ bDH vyK wBK ixz hrH
lsH ayF byJ nxH gGJ csu aIM qsF ipL rtK gqA jsL cpr dpK jrC eAH CEI jxG
jzE hwF mvz noD nvG brL ewE ntF gru ltB bxK hsD irs mDM orB fuH npM dBC
iBJ hoz lpG mtG CFM lAL bqz ftM tCD lvD qEJ kpC eDK mpu qrM hvA hGI lCJ
nqL tJL foK fqw mwC rAI ity duL gtv jKM ivC kvL xyM eFI mIK DFJ ezM oyH
asK lqK

$(v,k) = (42,17)$:

bFN wKN jIN euN dFL ewL eDF kzP nFP qyM pyK eEM iwH fFI dwI aDG cxF gsD
owJ gzJ irD evy hHO ouL pEJ orP dyP hGK CGJ atO eJP BJM ivJ hJL cJO drE
grK qKO muO nsE AEH fEN yHL fwB jxJ duG gAC nCN kvE psI ksw guw GIO quF
gyN ctz hBP ixN cDK nuH hrz lzF oxz dtM otE erH nGL iAK nrO kBK ezK zEI
cCL bAG qzD fKL qrN ktN ADI oyA dBD jyF gEO fDP nIJ gvP xDH pzH nyD cHN
aKM hAF nzB lrA lwC huy ftA kxM izG pAB mCD ptP oFO lGM avC hDE aHP qBC
hCM qHI osM dvK mwE iIM mtF eCI BFG oGH qxE jsO jvB aAJ lIP byE ntK iBO
mxA lvx etG pxL cuM nwx eAO aBE jDM bDL jtw bzO kyI lBL fzM hvw krG qvG
mGP fux xKP ltD cvI jHK jCP bwM gIL bsP gFM jrL pGN kFH kAL dHJ csA buB
cEP iuP prM isF FJK qsJ oBI azN dxO msK bIK qtL puC mrI mBN wyz oCK gxG
jEG asL myJ cwG mHM fCH CEF bvH frJ btJ kCO pvF iEL lsH axy auI hsN lJN
dAN fvO lyO

$(v,k) = (47,18)$:

iFU fEK quv uyQ ptu dvG juH fxF jtU hOR dMS CFK bHR nAQ gwK lLR nIT bxI
fyI cuE gRS lsS dCD jCQ lFG ayO mvA dwA mwP AFO rFQ ksu kLQ gBG ouO btG
dsJ aDM kwN kEP aHU nsw CGI hMT ktH gJM dyR jEJ cGM iGS mFM hFH fBR fsz
gAT mIN bAD bvE hwQ mtJ nux lCJ pOS byK nSU rCS qKP rzR HJS tBC qCN lHI
GKU vwL oKS cJL vyJ kGR pyN qMQ lOP fwH vzO rsL pCE rDG aEG oIJ gtI rxP
muD nCH hzC pwF bNS uzJ kyA pxJ qLT ENR pAB nzM qwO isP lAM eBH eIO xBS
ltK dzB rEI hBE hJU wyU mxH jvD uCR aJN nDP eyD psQ iHO evR aAS gxL buB
wIS msT huA bwT avF iuT azI iKQ oyH otF owD jIL bzL pMR eMU cAU pvU BIQ
fAN jFR ftM gDF kST kDK eCL lwE ivB mGO oAC iAL guU kBJ jNT dFI GPQ kIU
ovQ cHK yMP cvS fQS aCT EFT qFJ tAP nEO lvT mzQ pDL mBL cxy qxG nvK qsH
ixz HQT awx rNU rHM oxM rBO aQR AGH zDS sDR svx ruw gzE ntR lzU eJQ iyC
hDN eKT oBP DEH rtv txT jzK cBF oRT mKR kzF atL jyG twz dHL est eNP dOU
oEL bFP iEM cNQ csI cPR gsy dxN fOT mCU dPT hxK jPS LPU qEU ewG bJO hty
czT osU cwC ryT gOQ kvM nBN gCP exE jsO xRU qtS bQU fDU fvC hsG qBD itN
DJT pIK hLS ezA iDI jEH fvK qtK lCK eqF iAC BCF cFI jxy yIK bsx yEJ ntD
nwE iwy eor kpC evH ayG cCH gqA oBJ grG lwI aCE los eCI gxC kvy lpv cpq
nHJ bBH koz euy dAI ewJ gsB qxD wAK kGH hqw btw qsI nuI gwH asD iEI isJ
itv mxG foq nox ezA aqH txH bpr est hsF lyH fuz bqy svG dtu jDJ crt dBE
cyz eDG jsK fsE fAH jwC gvD hxK cxA gty kFK csw jrI ksu nGK cGJ kxE guE
hCD frJ cuK aFJ nvF lAD dps kq

$(v,k) = (49,20)$:

fGW puL cKL dxH fxE rwM hOU izE cHQ gAG lHJ fwF dJW iMR txO sDS tEV sxG
pMU lzN oFI pJT ozW dvN aFJ rLN ALM sKP iHS mxD qzR bIR fzD jQU kBH eAJ
pCG lFO nFP guy bAN rHP quV NOP ruJ jwy eFK qFN tDQ lGP iPV iKN kuW bHU
pvy ezU bzM fAT kvK bxC oEQ kGO tBF aKO szA yFL dAK cwD fRV azQ byS nDI
iDJ fyB iCT dzL sHR hGN iOW ovR ivB gDM lAS eDW rzC oNU pxF EOS qIU gxB
euO fPQ cOT kIM eCP fuS jJO dDO iwG tyA mOQ cxy bFW zIP twH fCU nRT ewT
nMO cAV nwU hFM lvx bvJ suM dMV czB sJU hxI oDV nCQ xNQ tRS kNS mwE lCV
myK jKR lDU iAF jzH hyz oGK tJK mGJ bDK nuK sBE QRW syQ cPU eLR pDR dyU
jEN bBO mRU eGQ gNT vwz eBI eyN qLO qKQ mIS IJV jCF swO nEL lMT fJN hAE
pzS JLP jvA rvF tGU gHW tuz qDT cEJ xzK pOV oxP nAW yCO uxA rAD nxJ nvS
qHM oyT cCI pBP rOR kzJ hvD lLQ avC juI svT fKM nBN tIW oBM yJM huC BGR
gKV bwV kET muH mMP aAR dFS aEU cMS gIL jGS qCE IKT aGM mAC rQS dBQ aPS
ouw luB uvU pHN jDL dEI cvG nyH hPW axV kAU gvE bGL sIN oLS kwC wNR hJR
dCR wAB CHK dwP lKW jPT rxT hHT jBV ayW qyG qAP awL mvL tLT lwI mBW bQT
fHL kxR kLV hQV qwx aBT pEK oCJ gFU pwW iuQ sFV ryV kFQ qvW gzO cNW mNV
vMQ iyI hBL eEM rEW tvP gPR exS BCD gCS aHI rGI

$(v,k) = (55,20)$:

LR2 lvG ayA ALM oLZ fx1 rDZ pTZ axB jHQ hXZ pRY bCH uvz gO3 fwG dEO iyI
sKZ guY oRW aVZ av3 hT3 kuB rxO eC3 aPT yzJ eZ1 lN1 oDQ uxA lDW kKL hEM

gAP jET 1MX qAG pHS sP1 wAY S12 fAE 1AS oCP bDT bK2 hy1 nuL iMS nR1 INX
dJL cTX nQX dDV sDI iRX bxS txZ sHR DM3 cJP nBT NPV tzW dKN qv1 LP3 tV1
rNR 1HP puw eOU rHL nPS J01 KPQ zRS cQ2 fCR tuK iOV jvS wNO ezQ qPU dy3
iwF sBM mxy kJU oxF pvC tIT bI3 bJR ACU JV2 gFK fuX jOY hxP qN3 jAB cLV
IPR dIS jDJ 1JT CEI vPX nFW ow3 dB1 k13 MQ1 uSZ tX2 lwz CJK cMZ hvQ sOS
fB2 fKV xGI QY3 cBC oEK bGQ mOQ hAN mIJ tDS jI1 rzC pz1 eBN eDK pEV yQU
fzD jCF qFO nMO swE nx3 fHT aKW quQ eFJ vxD luV qyW DX1 rvY kxE wxT yCY
hGU qDL iG3 rA2 BGR KR3 ew2 cxK sAV tvF aXY FM2 hu2 BWY ART jyL nAZ mvL
EL1 gDR cGO bLX hJW yHK kwW xQV hCV tEQ aGJ twL dFP 1xC cIU BX3 ou1 bvM
cW1 tyO nwC zKY cwD kOX oyB bzN kzH dAX cyN exR hzB sTY dzM yFZ fNS zE3
nvy bA1 uCO rWX LOT aFI oGS tRU nUY dwU jUX pOP aEU hHI BDO gU1 iCD kMT
sxz bwB nEN jwV py2 hDY aLS eyT tA3 bYZ hOR eEY iPY gEW aw1 aH2 oJN vW2
vwZ azO GY1 gBI dxY ozX mP2 tGP nD2 hFL oIY rwP qCX pKM HNU dHW qKT xHX
tBH cvE fU3 aMR czA izZ dvR IWZ fyM oMV gCZ gzL euI BFV ovU kVY mwR wIM
eAW kI2 iuH ju3 qwH sFX gVX mV3 sNQ syG ruy byV suU fJZ mCG wyX EJX LNY
s23 eSX kCS svJ qMY nHJ kQR bOW aCQ 1UZ eGL gHM pDU fLQ iK1 FQS cHY dC2
ivB rGM 1IL 1KO 1Y2 uJM mNZ kGZ nIK 1F3 1BQ rIV BPZ qRV vAK muW tJY rF1
pBL rQT jGK OZ2 fFY kAF dQZ pFN iE2 iAJ gyS mH1 rJ3 mSY jRZ rES jzP mKX
qEZ jN2 mAD zU2 mMU kvN eMP cS3 mBE pGX gG2 bFU qx2 qzI iNT pW3 HZ3 oT2
pxJ gxN fPW CT1 gvT iQW GNW