On the Existence of Abelian Difference Sets with $100 < k \leq 150$

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ABSTRACT. Kibler, Baumert, Lander and Kopilovich (cf. [7], [1], [10] and [8] respectively), studied the existence of (v, k, λ)-abelian difference sets with $k \leq 100$. In Lander and Kopilovich's works, there were 13 and 8 (v, k, λ) tuples, respectively, in which the problem was open. Later, several authors have completed these studies and nowadays the problem is open for 6 and 7 tuples, respectively.

Jungnickel (cf. [9]) lists some unsolved problems on difference sets. One of them is to extend Lander's table somewhat. By following this idea, this paper deals with the existence or nonexistence of (v, k, λ)-abelian difference sets with $100 < k \leq 150$. There exist 277 tuples that satisfy the basic relationship between the parameters v, k and λ, $k \leq v/2$, Schutzenberger and Bruck-Chowla-Ryser's necessary conditions and $100 < k \leq 150$. In order to reduce this number, we have written in C several programs which implement some known criteria on nonexistence of difference sets. We conclude that the only (v, k, λ) tuples, with $100 < k \leq 150$, for which a difference set in some abelian group of order v can exist are $(10303, 102, 1)$, $(10713, 104, 1)$, $(211, 105, 52)$, $(11557, 108, 1)$, $(223, 111, 55)$, $(11991, 110, 1)$, $(227, 113, 56)$, $(12883, 114, 1)$, $(378, 117, 36)$, $(239, 119, 59)$, $(256, 120, 56)$, $(364, 121, 40)$, $(243, 121, 60)$, $(14763, 122, 1)$, $(251, 125, 62)$, $(15751, 126, 1)$, $(351, 126, 45)$, $(255, 127, 63)$, $(16257, 128, 1)$, $(16513, 129, 1)$, $(263, 131, 65)$, $(17293, 132, 1)$, $(1573, 132, 11)$, $(1464, 133, 12)$, $(271, 135, 67)$, $(18907, 138, 1)$, $(19461, 140, 1)$, $(283, 141, 70)$, $(22351, 150, 1)$, $(261, 105, 42)$, $(429, 108, 27)$, $(1200, 110, 10)$, $(768, 118, 18)$, $(841, 120, 17)$, $(715, 120, 20)$, $(5085, 124, 3)$, $(837, 133, 21)$, $(419, 133, 42)$, $(1225, 136, 15)$, $(361, 136, 51)$, $(1975, 141, 10)$, $(1161, 145, 18)$, $(465, 145, 45)$, $(5440, 148, 4)$, $(448, 150, 50)$. It is known that there exist difference sets for the first 29 tuples and the problem is open for the remaining 16. Besides, in table 1, we give the criterion that we have applied to determine the nonexistence of (v, k, λ)-difference sets for the rest of the tuples.

1 Introduction

Let $(G, \cdot)$ be an abelian group of order v. A subset D of G of size k is called a (v, k, λ)-difference set if every nonidentity element can be expressed in exactly λ ways as a "difference" $d_i \cdot d_j^{-1}$, $d_i, d_j \in D$.

A (v, k, λ)-difference set is said to be cyclic (resp. noncyclic) if the abelian group G is cyclic (resp. noncyclic).

In the group ring $\mathbb{Z}(G)$, the difference set condition for $D \subset G$ can be expressed by

$$DD^{-1} = n + \lambda G, \tag{1.1}$$

where $n = k - \lambda$, $D = \sum_{d \in D} d$, $D^{-1} = \sum_{d \in D} d^{-1}$ and $G = \sum_{g \in G} g$.

A simple count argument gives the following relationship between the parameters v, k and λ of a (v, k, λ)-difference set:

$$k(k-1) = \lambda(v-1). \tag{1.2}$$

Besides, it is easy to show that if D is a (v, k, λ)-difference set, then $D^c = \{g \in G \mid g \notin D\}$ is a $(v, v-k, v-2k+\lambda)$-difference set. Therefore, when we study the existence of (v, k, λ)-difference sets, we may assume that $k \leq \frac{v}{2}$, without loss of generality.

In addition, the following necessary conditions for a difference set to exist were proved in [16] and in [2] and [3], respectively:

Theorem 1.1. (Schutzenberger's necessary condition) *Let D be a (v, k, λ)-difference set. If v is even, then $n = k - \lambda$ is a square.*

Theorem 1.2. (Bruck-Chowla-Ryser's necessary condition) *Let D be a (v, k, λ)-difference set in a group G. If v is odd, then the equation $nx^2 + (-1)^{\frac{(v-1)}{2}} \lambda y^2 = z^2$ has a solution in integers x, y, z not all zero.*

On the other hand, to construct or prove the nonexistence of a putative (v, k, λ)-difference set, there are useful tools: multipliers and w-multipliers. Let us remember that an integer t with $(t, |G|) = 1$ is a multiplier of the (v, k, λ)-difference set D in G if $D^t = Dg$ for some $g \in G$ and an integer t with $(t, |G/H|) = 1$ is called a w-multipliplier of D if t is a multiplier of $\overline{D}$ in G/H. Their importance is based on the next theorem (cf. [15]).

Theorem 1.3. *If D is a (v, k, λ)-difference set and t is a multiplier of D, then there exists a translate of D, Dg, fixed by t.*

We refer the reader to [1], [9], [10] and [12] for other basic definitions and results on difference sets.

2 Work method

In order to find the (v, k, λ) tuples for which there exists a difference set with $100 < k \leq 150$, first of all we have looked for the tuples such that (1.2) is true. When $100 < k \leq 150$, there are 699 (v, k, λ) tuples which satisfy it. Among these 699 (v, k, λ) tuples, we have ruled out those with $k > (v/2)$ (144) and those that fail Schutzenberger's condition (126) or

Bruck-Chowla-Ryser's condition (155). This leads us to consider the 277 tuples that appear in Table 1.

However, it is not always possible to construct a difference set for each of these tuples. Because of this, we have applied several known results on nonexistence of difference sets which appear in [1], [10], [12], [19] and [20]. In particular, we have used the following results in the indicated order: Theorem 2 of [19], Theorem 7.2 of [12], Corollary 1 of Theorem 6 of [19], Theorems 4.18, 4.19, 4.20, 4.32, 4.33 of [10], two results of [20] (which are Theorems 4.38 and 4.39 in [10]) and Theorem 4.5 of [1]. Thus, we have concluded the nonexistence of (v, k, λ)-difference sets for 187 tuples and for another 10 in particular group configurations. In the fourth column of Table 1, we indicate the exact result, with its values, that we have applied for each tuple.

In addition to these nonexistence results, we have used multipliers and w-multipliers in order to know if a (v, k, λ)-difference set exists or not. By Theorem 1.3, we can suppose, without loss of generality, that we are looking for a (v, k, λ)-difference set D fixed by the multiplier t. Then, D is a union of orbits under the action of the multiplier t on G. By looking at the possible unions of orbits, we have been able to rule out 28 tuples and another 3 for some particular groups. These tuples are marked by a \21\ in the fourth column of Table 1.

Finally, we have examined the systems of equations that appear when we use multipliers and w-multiplier by (1.1). This has allowed us to conclude the nonexistence of difference sets for 14 tuples and for some groups for another 2 tuples. These tuples are marked in the fourth column of Table 1 by \22\ or \23\, depending on whether we have used multipliers or w-multipliers.

3 Main results

After this process, we can conclude the following result:

Theorem 3.4. *If there exists a (v, k, λ)-difference set in some abelian group of order v, with $100 < k \leq 150$, then (v, k, λ) must be one of the following tuples:* $(10303, 102, 1)$, $(10713, 104, 1)$, $(261, 105, 42)$, $(211, 105, 52)$, $(11557, 108, 1)$, $(429, 108, 27)$, $(11991, 110, 1)$, $(1200, 110, 10)$, $(223, 111, 55)$, $(227, 113, 56)$, $(12883, 114, 1)$, $(378, 117, 36)$, $(768, 118, 18)$, $(239, 119, 59)$, $(841, 120, 17)$, $(715, 120, 20)$, $(256, 120, 56)$,$(364, 121, 40)$, $(243, 121, 60)$, $(14763, 122, 1)$, $(5085, 124, 3)$, $(251, 125, 62)$, $(15751, 126, 1)$, $(351, 126, 45)$, $(255, 127, 63)$, $(16257, 128, 1)$, $(16513, 129, 1)$, $(263, 131, 65)$, $(17293, 132, 1)$, $(1573, 132, 11)$, $(1464, 133, 12)$, $(837, 133, 21)$, $(419, 133, 42)$, $(271, 135, 67)$, $(1225, 136, 15)$, $(361, 136, 51)$, $(18907, 138, 1)$, $(19461, 140, 1)$, $(1975, 141, 10)$, $(283, 141, 70)$, $(1161, 145, 18)$, $(465, 145, 45)$, $(5440, 148, 4)$, $(22351, 150, 1)$, $(448, 150, 50)$.

For the most of these last tuples, we can construct a (v, k, λ)-difference set, because the parametres v, k and λ and the group G we are working on satisfy the relations of one of the known families of difference sets. For instance, Lehmer (cf. [11]) proves that the quadratic residues are a $\left(p^{2k+1}, \frac{p^{2k+1}-1}{2}, \frac{p^{2k+1}-3}{4}\right)$-difference set in the additive group of $\mathbb{F}_{p^{2k+1}}$, for p a prime such that $p \equiv 3 mod\ 4$ and $k \geq 0$. Singer (cf. [18]) proves that there exist cyclic difference sets with parametres $\left(\frac{p^{m+1}-1}{p-1}, \frac{p^m-1}{p-1}, \frac{p^{m-1}-1}{p-1}\right)$, where p is a prime. McFarland (cf. [14]) gives a construction for $\left(q^{s+1}\left(\frac{q^{s+1}-1}{q-1}+1\right), q^s\left(\frac{q^{s+1}-1}{q-1}\right), q^s\left(\frac{q^s-1}{q-1}\right)\right)$-noncyclic difference sets, where q is a prime power. Later, Dillon and Spence (cf. [5] and [17], respectively) find new families of difference sets making some variations on McFarland's scheme. Thus, we obtain:

Theorem 3.5. *There exists a (v, k, λ)-difference set in some abelian group G of order v for the following tuples:* $(10303, 102, 1)$, $(10713, 104, 1)$, $(211, 105, 52)$, $(11557, 108, 1)$, $(11991, 110, 1)$, $(223, 111, 55)$, $(227, 113, 56)$, $(12883, 114, 1)$, $(378, 117, 36)$, $(239, 119, 59)$, $(364, 121, 40)$, $(256, 120, 56)$, $(243, 121, 60)$, $(14763, 122, 1)$, $(251, 125, 62)$, $(15751, 126, 1)$, $(351, 126, 45)$, $(255, 127, 63)$, $(16257, 128, 1)$, $(16513, 129, 1)$, $(263, 131, 65)$, $(17293, 132, 1)$, $(1573, 132, 11)$, $(1464, 133, 12)$, $(271, 135, 67)$, $(18907, 138, 1)$, $(19461, 140, 1)$, $(283, 141, 70)$, $(22351, 150, 1)$.

4 Table

There are four columns in Table 1. Under the first column, we list the (v, k, λ) tuples that satisfy the indicated necessary conditions. Under the second one, we give the group type we are working on. We have denoted the cyclic group of order m by C_m and the direct product of s cyclic group of order m by C_m^s. When a * appears after a particular group, it means that computational search has been necessary to get the conclusion in that group. Under the third one, we have used the symbols + or − depending on whether there exists or not a difference set. If this information is not known, a ? appears. Finally, under the fourth column, we indicate the reason for either the existence or nonexistence of a difference set. In this column, when we give the particular values we have used in a result, we have followed its original notation.

(v,k,λ)	GROUP	EXISTS	REASON
(10101, 101, 1)	C_{10101}	−	[10, Th.4.19] $(m, \exp G/H) = (2, 481)$
(5051, 101, 2)	C_{5051}	−	[10, Th.4.38] $q = 5051$
(1011, 101, 10)	C_{1011}	−	[19, Th.2] $(p,q) = (7, 337)$
(506, 101, 20)	C_{506}	−	\21\ $t = 3$
(405, 101, 25)	$K \times C_5$, with $\lvert K\rvert = 81$	−	[19, Th.2] $(p,q) = (19, 5)$
(203, 101, 50)	C_{203}	−	[19, Th.2] $(p,q) = (3, 7)$
(10303, 102, 1)	C_{10303}	+	[18]
(5152, 102, 2)	$C_2^5 \times C_7 \times C_{23}$	−	[12, Th.7.2] $(p, v^*) = (5, 322)$
	$C_2^3 \times C_4 \times C_7 \times C_{23}$	−	[10, Th.4.19] $(m, \exp G/H) = (5, 322)$
	$C_2 \times C_4^2 \times C_7 \times C_{23}$	−	[10, Th.4.19] $(m, \exp G/H) = (5, 322)$
	$C_2^2 \times C_8 \times C_7 \times C_{23}$	−	[10, Th.4.19] $(m, \exp G/H) = (5, 322)$
	$C_4 \times C_8 \times C_7 \times C_{23}$	−	[10, Th.4.19] $(m, \exp G/H) = (5, 322)$
	$C_2 \times C_{16} \times C_7 \times C_{23}$	−	[10, Th.4.19] $(m, \exp G/H) = (5, 322)$
	C_{5152}	−	[10, Th.4.19] $(m, \exp G/H) = (5, 322)$
(3503, 103, 3)	C_{3503}	−	[10, Th.4.19] $(m, \exp G/H) = (2, 113)$
(207, 103, 51)	$C_3^2 \times C_{23}, C_{207}$	−	\22\ $t = 4$
(10713, 104, 1)	C_{10713}	+	[18]
(2679, 104, 4)	C_{2679}	−	[10, Th.4.19] $(m, \exp G/H) = (5, 141)$
(10921, 105, 1)	C_{10921}	−	[19, Th.2] $(p,q) = (2, 67)$
(5461, 105, 2)	C_{5461}	−	\22\ $t = 103$
(2731, 105, 4)	$C_{2731}{}^*$	−	\21\ $t = 101$
(2185, 105, 5)	C_{2185}	−	\23\ $t = 4$
(1561, 105, 7)	C_{1561}	−	\23\ $t = 2$
(911, 105, 12)	C_{911}	−	\21\ $t = 31$
(841, 105, 13)	C_{841}	−	[12, Th.7.2] $(p, v^*) = (2, 841)$
	C_{29}^2	−	[12, Th.7.2] $(p, v^*) = (2, 29)$
(781, 105, 14)	C_{781}	−	[19, Th.2] $(p,q) = (7, 11)$
(729, 105, 15)	K, with $\lvert K\rvert = 729$	−	[19, Th.2] $(p,q) = (2, 3)$
(456, 105, 24)	$C_3 \times C_{19} \times K$, with $\lvert K\rvert = 8$	−	[19, Cor.1] $(m, w) = (9, 57)$
(421, 105, 26)	C_{421}	−	\21\ $t = 79$
(391, 105, 28)	C_{391}	−	[19, Th.2] $(p,q) = (7, 17)$
(261, 105, 42)	C_{261}	−	[19, Cor.1] $(m, w) = (3, 261)$
	$C_3^2 \times C_{29}$	?	
(211, 105, 52)	C_{211}	+	[11]
(3711, 106, 3)	C_{3711}	−	\21\ $t = 103$
(2227, 106, 5)	C_{2227}	−	[19, Th.2] $(p,q) = (101, 17)$

Table 1. Existence of abelian difference sets

(v,k,λ)	GROUP	EXISTS	REASON
(1856, 106, 6)	$K \times C_{29}$, $\|K\| = 64$	-	[10, Th.4.19] $(m, \exp G/H) = (5, 29)$
(1591, 106, 7)	C_{1591}	-	[19, Th.2] $(p,q) = (11, 37)$
(531, 106, 21)	$C_3^2 \times C_{59}$, C_{531}	-	[19, Th.2] $(p,q) = (5, 3)$
(319, 106, 35)	C_{319}	-	[19, Th.2] $(p,q) = (71, 29)$
(266, 106, 42)	C_{266}	-	[19, Cor.1] $(m, w) = (8, 38)$
(11343, 107, 1)	C_{11343}	-	[19, Th.2] $(p,q) = (2, 3)$
(215, 107, 53)	C_{215}	-	[19, Th.2] $(p,q) = (2, 5)$
(11557, 108, 1)	C_{11557}	+	[18]
(1285, 108, 9)	C_{1285}	-	[19, Th.2] $(p,q) = (11, 257)$
(429, 108, 27)	C_{429}	?	
(11773, 109, 1)	C_{11773}	-	[19, Th.2] $(p,q) = (3, 61)$
(5887, 109, 2)	C_{5887}, $C_7 \times C_{29}^2$	-	\22\ $t = 107$
(1963, 109, 6)	C_{1963}	-	[19, Th.2] $(p,q) = (103, 13)$
(1309, 109, 9)	C_{1309}	-	[10, Th.4.19] $(m, \exp G/H) = (10, 17)$
(219, 109, 54)	C_{219}	-	[19, Th.2] $(p,q) = (5, 3)$
(11991, 110, 1)	C_{11991}	+	[18]
(2399, 110, 5)	C_{2399}	-	[10, Th.4.38] $q = 2399$
(1200, 110, 10)	$C_2^4 \times C_3 \times C_5^2$, $C_2^4 \times C_3 \times C_{25}$	-	[10, Th.4.20] $(m, \exp G/H) = (5, 6)$
	$C_2^2 \times C_4 \times C_3 \times C_{25}$, $C_4^2 \times C_3 \times C_{25}$	-	[10, Th.4.32] $(m, \exp G/H) = (5, 150)$
	$C_2 \times C_8 \times C_3 \times C_{25}$, C_{1200}	-	[10, Th.4.32] $(m, \exp G/H) = (5, 150)$
	$C_2^2 \times C_4 \times C_3 \times C_5^2$, $C_4^2 \times C_3 \times C_5^2$	?	
	$C_2 \times C_8 \times C_3 \times C_5^2$	?	
(1091, 110, 11)	C_{1091}	-	\21\ $t = 3$
(4071, 111, 3)	C_{4071}	-	[10, Th.4.19] $(m, \exp G/H) = (2, 177)$
(1111, 111, 11)	C_{1111}*	-	\22\ $t = 5$
(815, 111, 15)	C_{815}	-	[19, Th.2] $(p,q) = (2, 5)$
(408, 111, 30)	$C_{17} \times C_3 \times K$, with $\|K\| = 8$	-	[19, Cor.1] $(m, w) = (9, 51)$
(223, 111, 55)	C_{223}	+	[11]
(12433, 112, 1)	C_{12433}	-	[19, Th.2] $(p,q) = (3, 12433)$
(4145, 112, 3)	C_{4145}	-	[19, Th.2] $(p,q) = (109, 5)$
(3109, 112, 4)	C_{3109}	-	[19, Th.2] $(p,q) = (3, 3109)$
(2073, 112, 6)	C_{2073}	-	[19, Th.2] $(p,q) = (2, 691)$
(1037, 112, 12)	C_{1037}	-	[10, Th.4.19] $(m, \exp G/H) = (5, 61)$
(889, 112, 14)	C_{889}	-	[19, Cor.1] $(m, w) = (7, 889)$
(519, 112, 24)	C_{519}	-	[19, Th.2] $(p,q) = (2, 3)$
(445, 112, 28)	C_{445}	-	[19, Th.2] $(p,q) = (3, 5)$

Table 1. Existence of abelian difference sets (Cont.)

(v,k,λ)	GROUP	EXISTS	REASON
(337, 112, 37)	C_{337}	–	[19, Th.2] $(p,q) = (3,337)$
(260, 112, 48)	$C_2^2 \times C_5 \times C_{13}$, C_{260}	–	[19, Cor.1] $(m,w) = (8,65)$
(12657, 113, 1)	C_{12657}	–	[12, Th.7.2] $(p,v^*) = (2,12657)$
(3165, 113, 4)	C_{3165}	–	[19, Th.2] $(p,q) = (109,5)$
(1809, 113, 7)	$K \times C_{67}$, with $\|K\| = 9$	–	[19, Th.2] $(p,q) = (2,3)$
(905, 113, 14)	C_{905}	–	[19, Th.2] $(p,q) = (11,181)$
(227, 113, 56)	C_{227}	+	[11]
(12883, 114, 1)	C_{12883}	+	[18]
(4295, 114, 3)	C_{4295}	–	[19, Th.2] $(p,q) = (3,5)$
(679, 114, 19)	C_{679}	–	[19, Th.2] $(p,q) = (5,7)$
(4371, 115, 3)	C_{4371}	–	\22\ $t = 7$
(875, 115, 15)	C_{875}	–	[19, Cor.1] $(m,w) = (10,125)$
	$C_5^3 \times C_7$	–	[10, Th.4.19] $(m, \exp G/H) = (2,5)$
	$C_5 \times C_{25} \times C_7$	–	[10, Th.4.19] $(m, \exp G/H) = (2,25)$
(571, 115, 23)	C_{571}	–	[12, Th.7.2] $(p,v^*) = (2,571)$
(231, 115, 57)	C_{231}	–	[19, Th.2] $(p,q) = (2,3)$
(13341, 116, 1)	C_{13341}	–	[19, Th.2] $(p,q) = (5,3)$
(6671, 116, 2)	C_{6671}	–	[19, Th.2] $(p,q) = (2,953)$
(1335, 116, 10)	C_{1335}	–	[19, Th.2] $(p,q) = (2,5)$
(13573, 117, 1)	$C_7^2 \times C_{277}$, C_{13573}	–	[19, Th.2] $(p,q) = (29,277)$
(1509, 117, 9)	C_{1509}	–	[10, Th.4.38] $q = 503$
(755, 117, 18)	C_{755}	–	[10, Th.4.20] $(m, \exp G/H) = (3,151)$
(523, 117, 26)	C_{523}	–	[10, Th.4.38] $q = 523$
(378, 117, 36)	C_{378}, $C_2 \times C_3 \times C_9 \times C_7$	–	[19, Cor.1] $(m,w) = (9,63)$
	$C_2 \times C_3^3 \times C_7$	+	[14]
(235, 117, 58)	C_{235}	–	[19, Th.2] $(p,q) = (59,5)$
(13807, 118, 1)	C_{13807}	–	[1, Th.4.5]
(1535, 118, 9)	C_{1535}	–	[19, Th.2] $(p,q) = (109,5)$
(768, 118, 18)	C_{768}	–	[19, Cor.1] $(m,w) = (2,768)$
	$K \times C_3$, $\|K\| = 256$ and $\exists H \leq K$, with $\|H\| = 32$ and $w^* = 6$	–	[10, Th.4.20] $(m, \exp G/H) = (5,6)$
	$C_2 \times C_3 \times C_{128}$	–	[10, Th.4.32] $(m, \exp G/H) = (2,384)$
	$C_4 \times C_{64} \times C_3$, $C_8 \times C_{32} \times C_3$	?	
	$C_{16}^2 \times C_3$	?	
(355, 118, 39)	C_{355}	–	[19, Th.2] $(p,q) = (79,5)$
(2007, 119, 7)	C_{2007}, $C_3^2 \times C_{223}$	–	\22\ $t = 4$
(239, 119, 59)	C_{239}	+	[11]

Table 1. Existence of abelian difference sets (Cont.)

(v,k,λ)	GROUP	EXISTS	REASON
$(14281,120,1)$	C_{14281}	-	[19, Th.2] $(p,q)=(7,14281)$
$(4761,120,3)$	$K\times K_1$, with $\|K\|=9$ and $\|K_1\|=529$	-	\21\ $t=13$
$(3571,120,4)$	C_{3571}	-	[12, Th.7.2] $(p,v^*)=(2,3571)$
$(2041,120,7)$	C_{2041}	-	\22\ $t=113$
$(1191,120,12)$	C_{1191}	-	[19, Th.2] $(p,q)=(3,397)$
$(841,120,17)$	C_{841}	-	\21\ $t=103$
	C_{29}^2	?	
$(715,120,20)$	C_{715}	?	
$(511,120,28)$	C_{511}	-	[19, Th.2] $(p,q)=(23,73)$
$(409,120,35)$	C_{409}^*	-	\21\ $t=5$
$(256,120,56)$	$C_2\times C_{128}$, C_{256}	-	[19, Cor.1] $(m,w)=(4,128)$
	$C_2^2\times C_{64}$, $C_4\times C_{64}$	-	[10, Th.4.33] $(m,\exp G/H)=(8,64)$
	$C_2^4\times K$, with $\|K\|=16$	+	[14]
	$C_2^3\times K$, with $\|K\|=32$	+	[6]
	$C_{32}\times C_8$, C_{16}^2	+	[6]
	Rest groups	+	[4]
$(14521,121,1)$	C_{14521}	-	[19, Th.2] $(p,q)=(2,13)$
$(7261,121,2)$	C_{7261}	-	[19, Th.2] $(p,q)=(7,53)$
$(4841,121,3)$	C_{4841}	-	[10, Th.4.38] $q=103$
$(3631,121,4)$	C_{3631}	-	[12, Th.7.2] $(p,v^*)=(3,3631)$
$(2905,121,5)$	C_{2905}	-	[19, Th.2] $(p,q)=(29,5)$
$(2421,121,6)$	C_{2421}	-	[19, Th.2] $(p,q)=(5,3)$
$(1453,121,10)$	C_{1453}	-	\21\ $t=37$
$(1321,121,11)$	C_{1321}	-	[19, Th.2] $(p,q)=(2,1321)$
$(1211,121,12)$	C_{1211}	-	\21\ $t=109$
$(969,121,15)$	C_{969}	-	[19, Th.2] $(p,q)=(2,3)$
$(727,121,20)$	C_{727}	-	\21\ $t=101$
$(661,121,22)$	C_{661}	-	[12, Th.7.2] $(p,v^*)=(3,661)$
$(485,121,30)$	C_{485}	-	[19, Th.2] $(p,q)=(7,5)$
$(364,121,40)$	C_{364}	+	[18]
	$C_2^2\times C_7\times C_{13}$	?	
$(265,121,55)$	C_{265}	-	[19, Th.2] $(p,q)=(2,5)$
$(243,121,60)$	C_3^5	+	[11]
	C_{243}	-	[10, Th.4.38] $q=3$
	$C_3^3\times C_9$, $C_3\times C_{81}$, $C_3^2\times C_{27}$	-	\21\ $t=61$
	$C_9\times C_{27}$, $C_3\times C_9^2$	?	

Table 1. Existence of abelian difference sets (Cont.)

(v,k,λ)	GROUP	EXISTS	REASON
(14763, 122, 1)	C_{14763}	+	[18]
(1343, 122, 11)	C_{1343}	−	[19, Th.2] $(p,q)=(3,17)$
(672, 122, 22)	$C_2^5 \times C_{21}$, $C_2^3 \times C_4 \times C_{21}$	−	[10, Th.4.18] $(m, \exp G/H) = (5,42)$
	$C_2 \times C_4^2 \times C_{21}$, $C_2^2 \times C_8 \times C_{21}$	−	[10, Th.4.18] $(m, \exp G/H) = (5,42)$
	$C_4 \times C_8 \times C_{21}$, $C_2 \times C_{16} \times C_{21}$, C_{672}	−	[10, Th.4.20] $(m, \exp G/H) = (5,42)$
(15007, 123, 1)	C_{15007}	−	[19, Th.2] $(p,q)=(2,43)$
(7504, 123, 2)	$K \times C_7 \times C_{67}$, with $\|K\| = 16$	−	[10, Th.4.19] $(m, \exp G/H) = (11,67)$
(367, 123, 41)	C_{367}	−	[10, Th.4.38] $q = 367$
(247, 123, 61)	C_{247}	−	[19, Th.2] $(p,q)=(2,13)$
(15253, 124, 1)	C_{15253}	−	[19, Th.2] $(p,q)=(3,7)$
(5085, 124, 3)	$C_{5085}{}^*$	−	\21\ $t = 11$
	$C_3^2 \times C_5 \times C_{113}$	?	
(2543, 124, 6)	C_{2543}	−	[10, Th.4.38] $q = 2543$
(493, 124, 31)	C_{493}	−	[19, Th.2] $(p,q)=(3,17)$
(373, 124, 41)	C_{373}	−	\21\ $t = 83$
(15501, 125, 1)	C_{15501}	−	[1, Th.4.5]
(7751, 125, 2)	C_{7751}	−	[19, Th.2] $(p,q)=(3,337)$
(3876, 125, 4)	C_{3876}, $C_2^2 \times C_3 \times C_{17} \times C_{19}$	−	[10, Th.4.19] $(m, \exp G/H) = (11,17)$
(1551, 125, 10)	C_{1551}	−	[19, Th.2] $(p,q)=(5,3)$
(621, 125, 25)	$C_3^3 \times C_{23}$	−	[12, Th.7.2] $(p, v^*) = (5,69)$
	$C_3 \times C_9 \times C_{23}$	−	[12, Th.7.2] $(p, v^*) = (5,207)$
	C_{621}	−	[12, Th.7.2] $(p, v^*) = (5,621)$
(311, 125, 50)	C_{311}	−	[10, Th.4.38] $q = 311$
(251, 125, 62)	C_{251}	+	[11]
(15751, 126, 1)	C_{15751}	+	[18]
(3151, 126, 5)	C_{3151}	−	[10, Th.4.19] $(m, \exp G/H) = (11,137)$
(1751, 126, 9)	C_{1751}	−	[19, Th.2] $(p,q)=(13,17)$
(631, 126, 25)	C_{631}	−	\21\ $t = 101$
(451, 126, 35)	C_{451}	−	[19, Th.2] $(p,q)=(7,11)$
(351, 126, 45)	$C_3^3 \times C_{13}$	+	[17]
	$C_3 \times C_9 \times C_{13}$, C_{351}	?	
(5335, 127, 3)	C_{5335}	−	[19, Th.2] $(p,q)=(31,97)$
(2668, 127, 6)	C_{2668}, $C_2^2 \times C_{23} \times C_{29}$	−	[10, Th.4.19] $(m, \exp G/H) = (11,23)$
(255, 127, 63)	C_{255}	+	[18]
(16257, 128, 1)	C_{16257}	+	[18]
(8129, 128, 2)	C_{8129}	−	[19, Th.2] $(p,q)=(2,11)$

Table 1. Existence of abelian difference sets (Cont.)

(v,k,λ)	GROUP	EXISTS	REASON
(4065, 128, 4)	C_{4065}	−	\21\ $t = 31$
(1017, 128, 16)	C_{1017}, $C_3^2 \times C_{113}$	−	[10, Th.4.20] $(m, \exp G/H) = (4, 113)$
(16513, 129, 1)	C_{16513}	+	[18]
	$C_7^2 \times C_{337}$	?	
(8257, 129, 2)	C_{8257}	−	[10, Th.4.38] $q = 23$
(4129, 129, 4)	C_{4129}	−	[19, Th.2] $(p,q) = (5, 4129)$
(2065, 129, 8)	C_{2065}	−	\22\ $t = 11$
(1377, 129, 12)	$K \times C_{17}$, $\lvert K\rvert = 81$	−	[19, Th.2] $(p,q) = (13, 17)$
(1033, 129, 16)	C_{1033}^*	−	\21\ $t = 113$
(517, 129, 32)	C_{517}	−	[10, Th.4.38] $q = 47$
(385, 129, 43)	C_{385}	−	[19, Th.2] $(p,q) = (2, 5)$
(345, 129, 48)	C_{345}	−	[10, Th.4.39] $(q,r) = (23, 5)$
(259, 129, 64)	C_{259}	−	[19, Th.2] $(p,q) = (5, 7)$
(5591, 130, 3)	C_{5591}	−	[10, Th.4.38] $q = 5591$
(3355, 130, 5)	C_{3355}	−	[19, Th.2] $(p,q) = (5, 61)$
(1291, 130, 13)	C_{1291}	−	[12, Th.7.2] $(p, v^*) = (3, 1291)$
(1119, 130, 15)	C_{1119}	−	[19, Th.2] $(p,q) = (5, 3)$
(560, 130, 30)	$C_2^4 \times C_{35}$, $C_2^2 \times C_4 \times C_{35}$	−	[10, Th.4.20] $(m, \exp G/H) = (5, 14)$
	$C_4^2 \times C_{35}$, $C_2 \times C_8 \times C_{35}$	−	[10, Th.4.33] $(m, \exp G/H) = (5, 70)$
	C_{560}	−	[19]
(17031, 131, 1)	C_{17031}	−	[19, Th.2] $(p,q) = (2, 3)$
(3407, 131, 5)	C_{3407}	−	[10, Th.4.38] $q = 3407$
(1704, 131, 10)	$C_2^3 \times C_3 \times C_{71}$	−	[12, Th.7.2] $(p, v^*) = (11, 426)$
	$C_2 \times C_4 \times C_3 \times C_{71}$	−	[12, Th.7.2] $(p, v^*) = (11, 852)$
	C_{1704}	−	[19, Cor.1] $(m, w) = (11, 213)$
(1311, 131, 13)	C_{1311}	−	[19, Th.2] $(p,q) = (2, 3)$
(263, 131, 65)	C_{263}	+	[11]
(17293, 132, 1)	C_{17293}	+	[18]
(1573, 132, 11)	C_{1573}	−	[19, Cor.1] $(m, w) = (11, 1573)$
	$C_{11}^2 \times C_{13}$	+	[14]
(17557, 133, 1)	C_{17557}	−	[19, Th.2] $(p,q) = (3, 97)$
(8779, 133, 2)	C_{8779}	−	\21\ $t = 131$
(2927, 133, 6)	C_{2927}	−	[10, Th.4.38] $q = 2927$
(1464, 133, 12)	$C_2^3 \times C_3 \times C_{61}$	−	[10, Th.4.19] $(m, \exp G/H) = (11, 6)$
	$C_2 \times C_4 \times C_{61}$	−	[10, Th.4.19] $(m, \exp G/H) = (11, 12)$
	C_{1464}	+	[18]

Table 1. Existence of abelian difference sets (Cont.)

(v,k,λ)	GROUP	EXISTS	REASON
$(988,141,20)$	$C_2^2 \times C_{13} \times C_{19}$	−	[10, Th.4.19] $(m, \exp G/H) = (11,26)$
	C_{988}	−	\22\ $t = 11$
$(659,141,30)$	C_{659}	−	[10, Th.4.38] $q = 659$
$(330,141,60)$	C_{330}	−	\23\ $t = 3$
$(283,141,70)$	C_{283}	+	[11]
$(6675,142,3)$	C_{6675}, $C_3 \times C_5^2 \times C_{89}$	−	[19, Th.2] $(p,q) = (139,5)$
$(1847,143,11)$	C_{1847}	−	\21\ $t = 2$
$(924,143,22)$	C_{924}	−	[19, Cor.1] $(m,w) = (11,132)$
	$C_2^2 \times C_3 \times C_7 \times C_{11}$	−	[10, Th.4.33] $(m, \exp G/H) = (11,66)$
$(287,143,71)$	C_{287}	−	[19, Th.2] $(p,q) = (2,41)$
$(20593,144,1)$	C_{20593}	−	[19, Th.2] $(p,q) = (11,20593)$
$(10297,144,2)$	C_{10297}	−	\21\ $t = 71$
$(6865,144,3)$	C_{6865}	−	[19, Th.2] $(p,q) = (3,5)$
$(5149,144,4)$	C_{5149}*	−	\21\ $t = 7$
$(3433,144,6)$	C_{3433}	−	[19, Th.2] $(p,q) = (2,3433)$
$(2575,144,8)$	C_{2575}, $C_5^2 \times C_{103}$	−	[19, Th.2] $(p,q) = (2,5)$
$(2289,144,9)$	C_{2289}	−	[19, Th.2] $(p,q) = (3,7)$
$(1873,144,11)$	C_{1873}	−	[19, Th.2] $(p,q) = (19,1873)$
$(1717,144,12)$	C_{1717}	−	[19, Th.2] $(p,q) = (3,17)$
$(1585,144,13)$	C_{1585}	−	\21\ $t = 131$
$(1145,144,18)$	C_{1145}	−	[19, Th.2] $(p,q) = (2,5)$
$(937,144,22)$	C_{937}	−	\21\ $t = 61$
$(793,144,26)$	C_{793}	−	[19, Th.2] $(p,q) = (2,13)$
$(625,144,33)$	K, with $\|K\| = 625$	−	[19, Th.2] $(p,q) = (3,5)$
$(573,144,36)$	C_{573}	−	[10, Th.4.20] $(m, \exp G/H) = (2,3)$
$(529,144,39)$	C_{529}, C_{23}^2	−	[19, Th.2] $(p,q) = (5,23)$
$(469,144,44)$	C_{469}	−	[12, Th.7.2] $(p,v^*) = (5,469)$
$(397,144,52)$	C_{397}	−	[19, Th.2] $(p,q) = (23,397)$
$(313,144,66)$	C_{313}	−	[19, Th.2] $(p,q) = (2,313)$
$(20881,145,1)$	C_{20881}	−	[12, Th.7.2] $(p,v^*) = (2,20881)$
$(6961,145,3)$	C_{6961}	−	[19, Th.2] $(p,q) = (2,6961)$
$(5221,145,4)$	C_{5221}	−	[10, Th.4.38] $q = 227$
$(3481,145,6)$	C_{3481}	−	[10, Th.4.38] $q = 59$, $l = 2$
	C_{59}^2	−	[13]]
$(2611,145,8)$	C_{2611}	−	[19, Th.2] $(p,q) = (137,373)$
$(2321,145,9)$	C_{2321}	−	[19, Th.2] $(p,q) = (2,11)$

Table 1. Existence of abelian difference sets (Cont.)

(v,k,λ)	GROUP	EXISTS	REASON
$(2089,145,10)$	C_{2089}	–	[19, Th.2] $(p,q)=(3,2089)$
$(1161,145,18)$	$K\times C_{43}$, with $\lvert K\rvert=27$	?	
$(1045,145,20)$	C_{1045}	–	\23\ $t=5$
$(871,145,24)$	C_{871}	–	\21\ $t=11$
$(721,145,29)$	C_{721}	–	[10, Th.4.38] $q=103$
$(581,145,36)$	C_{581}	–	[10, Th.4.38] $q=83$
$(465,145,45)$	C_{465}	?	
$(361,145,58)$	C_{361}, C_{19}^2	–	[19, Th.2] $(p,q)=(3,19)$
$(349,145,60)$	C_{349}	–	[19, Th.2] $(p,q)=(5,349)$
$(291,145,72)$	C_{291}	–	[19, Th.2] $(p,q)=(73,97)$
$(21171,146,1)$	C_{21171}	–	[19, Th.2] $(p,q)=(5,3)$
$(10586,146,2)$	C_{10586}	–	[12, Th.7.2] $(p,v^*)=(3,10586)$
$(4235,146,5)$	C_{4235}, $C_5\times C_7\times C_{11}^2$	–	[19, Th.2] $(p,q)=(3,5)$
$(731,146,29)$	C_{731}	–	[19, Th.2] $(p,q)=(13,17)$
$(21463,147,1)$	C_{21463}	–	[19, Th.2] $(p,q)=(2,13)$
$(7155,147,3)$	$C_3^3\times C_5\times C_{53}$, $C_3\times C_9\times C_5\times C_{53}$	–	[10, Th.4.20] $(m,\exp G/H)=(3,265)$
	C_{7155}	–	[19, Cor.1] $(m,w)=(3,7155)$
$(439,147,49)$	C_{439}	–	\21\ $t=2$
$(295,147,73)$	C_{295}	–	[19, Th.2] $(p,q)=(2,5)$
$(21757,148,1)$	C_{21757}	–	[12, Th.7.2] $(p,v^*)=(7,21757)$
$(10879,148,2)$	C_{10879}	–	[19, Th.2] $(p,q)=(2,11)$
$(5440,148,4)$	$C_2^6\times C_5\times C_{17}$, $C_2^4\times C_4\times C_5\times C_{17}$	–	[10, Th.4.19] $(m,\exp G/H)=(3,10)$
	$C_2^2\times C_4^2\times C_{85}$, $C_2^3\times C_8\times C_{85}$	–	[10, Th.4.19] $(m,\exp G/H)=(3,10)$
	$C_2\times C_{32}\times C_5\times C_{17}$, C_{5440}	–	[10, Th.4.32] $(m,\exp G/H)=(2,544)$
	$C_4\times C_{16}\times C_{85}$, $C_2^2\times C_{16}\times C_{85}$	–	[10, Th.4.32] $(m,\exp G/H)=(4,272)$
	$C_4^3\times C_5\times C_{17}$, $C_8^2\times C_5\times C_{17}$	?	
	$C_2\times C_4\times C_8\times C_5\times C_{17}$	?	
$(3109,148,7)$	C_{3109}	–	[19, Th.2] $(p,q)=(3,3109)$
$(1037,148,21)$	C_{1037}	–	[19, Th.2] $(p,q)=(127,17)$
$(589,148,37)$	C_{589}	–	[19, Th.2] $(p,q)=(3,19)$
$(519,148,42)$	C_{519}	–	[19, Th.2] $(p,q)=(2,3)$
$(445,148,49)$	C_{445}	–	[19, Th.2] $(p,q)=(11,89)$
$(22053,149,1)$	C_{22053}	–	[1, Th.4.5]
$(11027,149,2)$	C_{11027}	–	[12, Th.7.2] $(p,v^*)=(7,11027)$
$(597,149,37)$	C_{597}	–	[10, Th.4.38] $q=199$

Table 1. Existence of abelian difference sets (Cont.)

(v,k,λ)	GROUP	EXISTS	REASON
$(299,149,74)$	C_{299}	−	[10, Th.4.20] $(m, \exp G/H) = (5,23)$
$(22351,150,1)$	C_{22351}	+	[18]
$(7451,150,3)$	C_{7451}	−	\21\ $t = 7$
$(4471,150,5)$	C_{4471}	−	[19, Th.2] $(p,q) = (5,17)$
$(3726,150,6)$	C_{3726}	−	[10, Th.4.19] $(m, \exp G/H) = (4,81)$
	$C_2 \times C_3 \times C_{27} \times C_{23}$	−	[10, Th.4.19] $(m, \exp G/H) = (4,27)$
	$C_2 \times C_3^2 \times C_9 \times C_{23}$, $C_2 \times C_9^2 \times C_{23}$	−	[10, Th.4.19] $(m, \exp G/H) = (4,9)$
	$C_2 \times C_3^4 \times C_{23}$	−	[10, Th.4.19] $(m, \exp G/H) = (4,3)$
$(1491,150,15)$	C_{1491}	−	[19, Th.2] $(p,q) = (3,7)$
$(895,150,25)$	C_{895}	−	[10, Th.4.38] $q = 179$
$(448,150,50)$	$C_2^6 \times C_7$	−	[12, Th.7.2] $(p, v^*) = (5,14)$
	C_{448}	−	[19]
	Rest groups	?	

Table 1. Existence of abelian difference sets (Cont.)

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