# Domination in Graphs: A Brief Overview

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ABSTRACT. In a graph $G = (V, E)$, a set $S \subset V$ is a dominating set if each vertex of $V - S$ is adjacent to at least one vertex in $S$. Approximately 1000 papers have been written on domination related concepts with more than half of them appearing in the literature in the last five years. Obviously, a comprehensive survey is beyond the scope of this paper, so a brief overview is presented.

## 1 Introduction

Let $G = (V, E)$ be a graph with $|V| = n$. Each vertex $v \in V$ *dominates* every vertex in its closed neighborhood $N[v]$. A set $S \subset V$ is a *dominating set* if each vertex in $V$ is dominated by at least one vertex of $S$. The *domination number* $\gamma(G)$ is the minimum cardinality of a dominating set. We refer to a minimum dominating set as a $\gamma$-set.

The concept of domination has its origins in the game of chess, where the goal is to cover (or dominate) various squares of a chessboard by certain chess pieces. In 1862 de Jaenisch [12] considered dominating the squares with queens and posed the following problem: Determine the minimum number of queens that can be placed on a chessboard such that every square is either occupied by a queen or can be occupied by one of the queens in a single move. (On a single move a queen can move any number of squares in one direction along its row, column, or diagonal.) The minimum number of such queens is five. For one possible placement of five queens, see Figure 1.

*Research partially supported by the National Science Foundation under Grant CCR-9408167

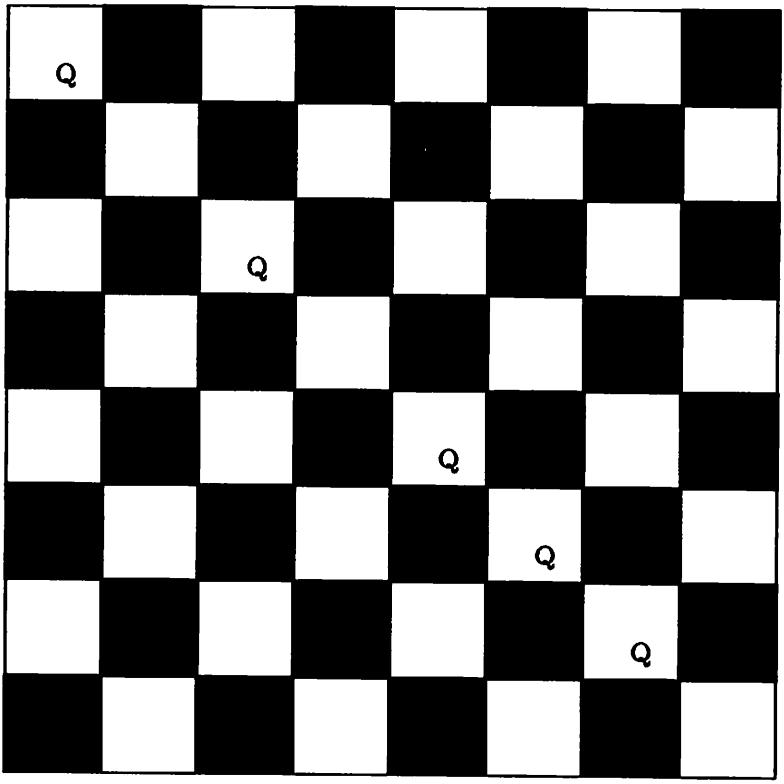

Figure 1: A Solution to the Five Queens Problem.

Using graph theory to model this problem, the Queen's Graph is formed by representing each of the 64 squares of the chessboard as a vertex of a graph $G$. Two vertices (squares) are adjacent in $G$ if each square can be reached by a queen on the other square in a single move. Obviously, to solve the queens problems we are looking for the minimum number of queens that dominate all the squares of the chessboard, i.e., $\gamma(G)$. (Note that many variations on this problem are formed by considering different chess pieces and/or different size chessboards.)

The next appearance of domination in the literature was also associated with game applications. In their book on game theory (1944), von Neumann and Morgenstern [40] considered domination in digraphs to find solutions (kernels) for cooperative $n$-person games.

In 1958, domination was formalized as a theoretical area in graph theory by Berge [5]. He referred to the domination number as the *coefficient of* *external stability* and denoted it $\beta(G)$. In 1962, Ore [31] was the first to use the term "domination" for undirected graphs and he denoted the domination number by $\delta(G)$.

However, it was not until Cockayne and Hedetniemi's [10] survey paper appeared in 1977 that the growing interest in domination unfolded. They introduced the accepted notation $\gamma(G)$ to denote the domination number. Much attention to domination followed and in 1990, Hedetniemi and Laskar [25] edited an issue of Discrete Mathematics devoted entirely to domination. The 1990 bibliography revealed an impressive increase in thirteen years from approximately 20 to 400 references. The explosive growth is further evidenced by the current bibliography [20] which has over 950 entries and is growing daily. Obviously, writing a comprehensive survey paper is unrealistic, so only a brief overview is presented here. However, a book on domination in graphs by Haynes, Hedetniemi and Slater [20] is in preparation. An edited book on selected research topics in domination [21] is also being prepared as a companion to the textbook. Chartrand and Lesniak [7] have included a chapter on domination in their revised book, *Graphs & Digraphs*.

Perhaps some of the widespread interest in domination stems from the many different perspectives from which it can be viewed. For example, we mention a few of the equivalent definitions of a dominating set:

**Vertex-Vertex Set Covering Problem.** Set $S \subset V$ is a dominating set of a graph $G$ if each vertex in $V - S$ has at least one neighbor (is covered by a vertex) in $S$.

**Set Intersection.** Set $S \subset V$ is a dominating set if for each vertex $x \in V - S$,

$$N(x) \cap S \neq \emptyset.$$

**Union of Neighborhoods.** Set $S \subset V$ is a dominating set if

$$\bigcup_{v \in S} N[v] = V.$$

**Dominating Function.** Let $f$ be the function $f : V \to \{0, 1\}$ such that for each $v \in V$,

$$\sum_{u \in N[v]} f(u) \geq 1.$$

Also, finding the order of a minimum dominating set may be expressed as a

**Linear Programming Problem.** The linear programming representation

$$\gamma(G) = \min \sum_{i=1}^{n} x_i$$

$$\text{subject to } \ N \cdot X \geq \overline{1}_n$$

$$\text{with } \ x_i \in \{0,1\}.$$

Another motivation for the study of domination may be the natural formation of domination parameters. For instance, varying the range in the functional domination definition yields *minus domination* [13] where $f$ has the range $\{-1,0,1\}$, *signed domination* [14] where $f$ has the range $\{-1,1\}$, and *fractional domination* [23] where $f$ has the range $[0,1]$. Hedetniemi and Laskar [25] suggested that over 60 types of domination parameters exist. A recent count suggests that over 100 have been defined and there are numerous possibilities for defining additional ones.

Many domination parameters are formed by combining domination with another graph theoretical property $P$. That is, parameters may be defined by imposing an additional constraint on the dominating set or the condition may also be placed on the dominated set or on the method of dominating. For example, imposing the condition that the subgraph induced by the dominating set $S$ be independent yields *independent domination*. *Total domination* is defined by the restriction on $S$ that the induced subgraph $< S >$ has no isolated vertices. Other properties imposed on the dominating set include that $< S >$ is connected, $< S >$ is a clique, and $< S >$ has a hamiltonian cycle. Also, new domination parameters may be defined by changing the method of dominating. For example, requiring that each vertex outside the dominating set have at least $k$ neighbors in the dominating set is $k$-multiple domination. The generic nature of this provides a method for defining many new invariants by considering different properties $P$.

Moreover, the applications of domination in a wide variety of fields have surely added to its escalating popularity. For a sample of its applications, consider communication networks, guard location problems, surveillance systems, and coding theory.

Hence domination has emerged as one of the most studied areas in graph theory. At the Carbondale conference corresponding to this proceedings, I presented only seven of the numerous topics in domination. The topic selection was made with the goal of presenting favorite open problems and hence, a strong author bias could not be avoided. A thorough discussion of even seven problems is beyond the space limitations of this paper, so four problems are selected.

## 2 Bounds on $\gamma(G)$

An obvious upper bound on the domination number is the number of vertices of the graph. This worst case situation is achieved if and only if the graph $G$ is a set of isolated vertices. Note that each isolated vertex must be in every dominating set. Considering graphs without isolated vertices, the upper bound is much improved in a classical result due to Ore [31].

**Theorem 1** (Ore) *If graph $G$ has no isolated vertices, then $\gamma(G) \leq n/2$.*

In general, the corona $G_1 \circ G_2$ is the graph $G = G_1 \circ G_2$ formed from one copy of $G_1$ and $|V(G_1)|$ copies of $G_2$ for which the *ith* vertex of $G_1$ is adjacent to every vertex in the *ith* copy of $G_2$. The corona $G = H \circ K_1$, in particular, is the graph constructed from a copy of $H$ and for each vertex $v \in V(H)$, a new vertex $v'$ and the pendant edge $vv'$ are added. Hence $G$ has even order and achieves the bound of Theorem 1. Graphs having no isolated vertices and domination number exactly half their order were characterized independently by Payan and Xuong [32] and Fink, Jacobson, Kinch, and Roberts [16].

**Theorem 2** *For a graph $G$ with even order $n$ and no isolated vertices, $\gamma(G) = n/2$ if and only if the components of $G$ are the cycle $C_4$ or the corona $H \circ K_1$ for some connected graph $H$.*

Cockayne, Haynes, and Hedetniemi [8] characterized the odd order graphs for which $\gamma(G) = \lfloor n/2 \rfloor$. The graphs of even order having $\gamma(G) = n/2$ are a special case of this result.

Ore's theorem applies to graphs having minimum degree $\delta(G)$ at least one. Restricting their attention to graphs $G$ having $\delta(G) \geq 2$, McCuaig and Shepherd [29] made another improvement on the upper bound. Let $\mathcal{B}$ be the collection of graphs in Figure 2.

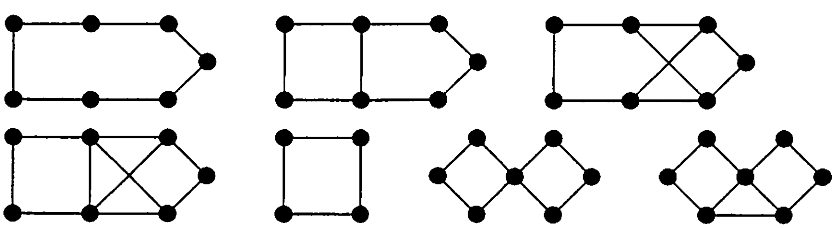

Figure 2: Graphs in family $\mathcal{B}$, "Bad Graphs"

**Theorem 3** *If $G$ is connected with $\delta(G) \geq 2$ and $G \notin \mathcal{B}$, then $\gamma(G) \leq 2n/5$.*

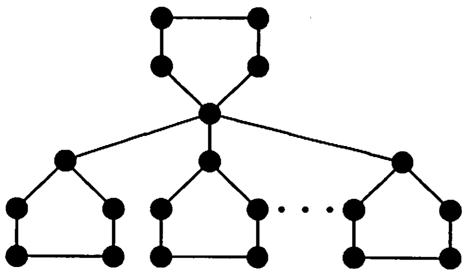

Figure 3: A family of graphs having $\gamma = 2n/5$.

We note that the bound of Theorem 3 is sharp and is achieved by the family of graphs illustrated in Figure 2. McCuaig and Shepherd [29] characterized the extremal (edge-minimal and edge-maximal) graphs which obtain this upper bound.

Reed [33] extended this type result to graphs having minimum degree at least three.

**Theorem 4** *If $G$ is connected and $\delta(G) \geq 3$, then $\gamma(G) \leq 3n/8$.*

These results are summarized in the following table.

| lower bound for $\delta(G)$ | upper bound for $\gamma(G)$ |
|---|---|
| 0 | $n$ |
| 1 | $n/2$ |
| 2, ($G$ connected, $G \notin \mathcal{B}$) | $2n/5$ |
| 3, ($G$ connected) | $3n/8$ |

Observing the decrease in the upper bound on the domination number each time the bound on the minimum degree is increased raises a very interesting open question (first noted by Stephen Hedetniemi). That is, in general, does an increase in the minimum degree requirement by one justify a decrease in the upper bound on the domination number and, if so, what are these upper bounds on $\gamma(G)$ for a given $\delta(G)$?

## 3 Nordhaus-Gaddum Type Results

In 1972 Jaegar and Payan [26] published the first Nordhaus-Gaddum type results involving domination.

**Theorem 5** *For any graph $G$,*

$$\gamma(G) + \gamma(\overline{G}) \leq n + 1.$$

Cockayne and Hedetniemi [10] improved the upper bound on the sum.

**Theorem 6** *For any graph $G$, $\gamma(G) + \gamma(\overline{G}) \le n + 1$ with equality if and only if $G = K_n$ or $\overline{K}_n$.*

A corollary to a result by Bollobás and Cockayne [6] made a significant improvement in the upper bound for the sum for the case when neither $G$ nor $\overline{G}$ has isolated vertices. Independently, Joseph and Arumugam [27] gave a simple, yet elegant, proof for this result.

**Theorem 7** *If $G$ and $\overline{G}$ have no isolates, then*

$$\gamma(G) + \gamma(\overline{G}) \le \lfloor n/2 \rfloor + 2.$$

Joseph and Arumugam [27] determined the graphs achieving the upper bound.

**Theorem 8** *For $G$ of order $n \ne 9$, such that $G$ and $\overline{G}$ have no isolated vertices, $\gamma(G)+\gamma(\overline{G}) = \lfloor n/2 \rfloor+2$ if and only if either $\gamma(G)$ or $\gamma(\overline{G}) = \lfloor n/2 \rfloor$.*

Payan and Xuong showed that [32] the self-complementary graph $K_3 \times K_3$ is the only graph for which $\gamma(G) = \gamma(\overline{G}) = 3$. Hence the above theorem can be restated as follows.

**Theorem 9** *For $G$ and $\overline{G}$ without isolates, $\gamma(G) + \gamma(\overline{G}) = \lfloor n/2 \rfloor + 2$ if and only if $G = K_3 \times K_3$, or $\gamma(G)$ or $\gamma(\overline{G}) = \lfloor n/2 \rfloor$.*

Recognizing that the upper bound of $n+1$, in a sense, corresponds to the upper bound of $n$ on $\gamma(G)$ and when neither $G$ nor $\overline{G}$ has isolated vertices, the upper bound of $\lfloor n/2 \rfloor + 2$ corresponds to the upper bound of $\lfloor n/2 \rfloor$ on $\gamma(G)$, it is natural to ask the following questions:

Is $\gamma(G) + \gamma(\overline{G}) \le 2n/5 + 3$ when both $G$ and $\overline{G}$ have minimum degree 2? And similarly, is $3n/8 + 4$ an upper bound on the sum when both $G$ and $\overline{G}$ have minimum degree 3? Affirmative answers to these questions are given in [9]. Note that this work is still in progress and there is a chance the bounds may be improved. The known results are summarized in the following table.

| lower bound for $\delta(G)$, $\delta(\overline{G})$ | upper bound for $\gamma(G)$ | Nordhaus-Gaddum bound |
|---|---|---|
| 0 | $n$ | $n+1$ |
| 1 | $n/2$ | $n/2+2$ |
| 2 ($G, \overline{G}$ connected, $G, \overline{G} \notin \mathcal{B}$) | $2n/5$ | $2n/5+3$ |
| 3, ($G, \overline{G}$ connected) | $3n/8$ | $3n/8+4$ |

Open Problem: Generalize the above table.

## 4 The Inequality Chain and Strong Equality

A set $S$ is *independent* if no two vertices in $S$ are adjacent. The *independent domination number* $i(G)$ is the cardinality of a minimum independent dominating set (or equivalently, the minimum order of a maximal independent set). The *vertex independence number* $\beta_0(G)$ is the cardinality of a maximum independent set of $G$.

Obviously, every graph has an independent dominating set and $\gamma(G) \leq i(G) \leq \beta_0(G)$. These bounds are sharp as can be seen with the corona $K_{1,t} \circ K_1$ which has $\gamma = i = \beta_0 = t+1$. On the other hand, the difference between each pair of these parameters can be made arbitrarily large. For example, the double star $S_{s,t}$ is the graph obtained by connecting the centers of two stars $K_{1,s}$ and $K_{1,t}$ with an edge. For $3 \leq s \leq t$, the double star has $\gamma = 2 < i = s+1 < \beta_0 = s+t$.

A set $S \subseteq V$ is *irredundant* if each $v \in S$ dominates at least one vertex of $G$ which is not dominated by any other vertex of $S$. Equivalently, $S$ is irredundant if for each $v \in S$, $v$ is isolated in $< S >$, the subgraph generated by $S$, or $v$ has at least one *private neighbor relative to* $S$ in $V - S$, i.e., a vertex which is adjacent to $v$ but not to any other vertex of $S$. For any graph $G$, $ir(G)$ and $IR(G)$ denote respectively the cardinality of a smallest and a largest maximal irredundant set of vertices. The *upper domination number* $\Gamma(G)$ is the cardinality of a largest minimal dominating set. For completeness, we list a well-known inequality chain which relates the upper and lower domination, independence, and irredundance parameters.

$$ir(G) \leq \gamma(G) \leq i(G) \leq \beta_0(G) \leq \Gamma(G) \leq IR(G)$$

Characterizing graphs which obtain sharpness in one or more of the inequalities has been the object of extensive investigation in more than 100 papers [24]. One of the most studied of these issues is characterizing graphs for which $\gamma(G) = i(G)$. Note that a forbidden subgraph characterization cannot be obtained since adding a vertex adjacent to all the vertices in any graph $H$ constructs a new graph, the join $G = K_1 + H$, with $i(G) = \gamma(G) = 1$. Graph $G$ is *claw-free* if it has no induced subgraphs isomorphic to $K_{1,3}$. Allan and Laskar [1] presented a sufficient condition for $\gamma(G) = i(G)$ in terms of this forbidden subgraph.

**Theorem 10** *If $G$ is claw-free, then $\gamma(G) = i(G)$.*

Topp and Volkmann [39] extended Theorem 10 by presenting 16 forbidden subgraphs as a sufficient condition for $\gamma(G) = i(G)$. Others have characterized such graphs for specific families:

- Harary and Livingston: caterpillars [17], trees [18]

- Topp and Volkmann: unicyclic graphs [37], bipartite and block graphs [38].

Harary and Livingston [19] applied results from coding theory to obtain information about $\gamma(G)$ and $i(G)$ when $G$ is the hypercube $Q_k$ of dimension $k$. They found that $\gamma(Q_k) = i(Q_k)$ for infinitely many values of $k$ and conjectured that $\gamma(Q_k)$ and $i(Q_k)$ differ only when $k = 5$ for which $\gamma(Q_5) = 7$ and $i(Q_5) = 8$.

On the other hand, Barefoot, Harary, and Jones [4] constructed an infinite family of 2-connected cubic graphs for which the difference between the domination and independent domination number may be arbitrarily large. They conjectured that a similar class exists for cubic graphs with connectivity 1. Mynhardt [30] proved this conjecture and described infinite families of 1-connected and 3-connected cubic graphs for which $i(G)-\gamma(G)$ becomes unbounded. Cockayne and Mynhardt [11] and Kostochka [28] independently found other infinite classes of cubic graphs for which the difference between $i(G)$ and $\gamma(G)$ may be arbitrarily large. Moreover, Seifter [34] showed that for every triple $(r, k, t)$, $r \geq 5$, $2 \leq k \leq r$, $t > 1$, there exist $r$-regular $k$-connected graphs $G$ having $i(G) - \gamma(G) > t$.

The problem of characterizing graphs for which $\gamma(G) = i(G)$ remains unsolved and seems to be extremely difficult. Here we consider a related subproblem introduced in [22]. Suppose that, in fact, $\gamma(G) = i(G)$. Then every $i(G)$-set is also a $\gamma(G)$-set, but not every $\gamma(G)$-set must be an $i(G)$-set. For example, the path $P_4$ has four $\gamma(P_4)$-sets, only three of which are $i(P_4)$-sets. On the other hand, $\gamma(C_5) = i(C_5) = 2$ and each of the five $\gamma(C_5)$-sets is also an $i(C_5)$-set. We say that $\gamma(C_5)$ and $i(C_5)$ are *strongly equal.*

Definition [22] Let $P_1$ and $P_2$ be properties of vertex subsets of a graph, and assume that every subset of $V(G)$ with property $P_2$ also has property $P_1$. Let $\psi_1(G)$ and $\psi_2(G)$, respectively, denote the minimum cardinalities of sets with properties $P_1$ and $P_2$, respectively. Then $\psi_1(G) \leq \psi_2(G)$. If $\psi_1(G) = \psi_2(G)$ and every $\psi_1(G)$-set is also a $\psi_2(G)$-set, then we say $\psi_1(G)$ *strongly equals* $\psi_2(G)$, written $\psi_1(G) \equiv \psi_2(G)$. (Note that one could also define strong equality for maximization properties such as $\beta_0(G)$ and $\Gamma(G)$.)

For example, we give the results for strong equality in paths and cycles.

**Proposition 1** [22] *For the path $P_n$ and cycle $C_n$,*

$$\gamma(P_{3k}) \equiv i(P_{3k}) = \gamma(C_{3k}) \equiv i(C_{3k}) = k,$$

$$\gamma(P_{3k+2}) \equiv i(P_{3k+2}) = \gamma(C_{3k+2}) \equiv i(C_{3k+2}) = k+1, \textit{ and}$$

$$\gamma(P_{3k+1}) = i(P_{3k+1}) = \gamma(C_{3k+1}) = i(C_{3k+1}) = k+1$$

$$\textit{but } \gamma(P_{3k+1}) \not\equiv i(P_{3k+1}) \textit{ and } \gamma(C_{3k+1}) \not\equiv i(C_{3k+1}).$$

We are currently trying to characterize the graphs $G$ for which $\gamma(G) \equiv i(G)$. Analogous problems remain open for other parameters in the inequality chain.

## 5 Edge-Domination-Critical Graphs

Sumner and Blitch [36] defined a graph $G$ to be *edge-domination-critical* if for any edge $e \in \overline{G}$, $\gamma(G+e) = \gamma(G) - 1$. That is, the addition of any edge to $G$ decreases the domination number. We consider one of the many open problems related to edge-domination-critical graphs.

It was conjectured in [36] that an edge-domination-critical graph has equal domination and independent domination numbers. However, the conjecture was proven false with a counterexample due to Ao [2] for the case when $\gamma(G) = 4$. Moreover, Ao, Cockayne, MacGillivray, and Mynhardt [3] constructed graphs having $\gamma(G) < i(G)$ for each $\gamma(G) > 3$. On the other hand, the conjecture is known [36] to be true for graphs having $\gamma(G) \leq 2$ and is still open for graphs with $\gamma(G) = 3$. Sumner [35] stated that based on a prolonged computer search, he believes the conjecture is true in the case of $\gamma(G) = 3$. Based on my work on the problem, I also believe it is true, but have not been able to prove it.

**Conjecture 1** [35] *If $G$ is an edge-domination-critical graph with $\gamma(G) = 3$, then $\gamma(G) = i(G)$.*

Note that the conjecture has been proven for several special cases in [36]. For example, if $G$ is edge-domination-critical with $\gamma(G) = 3$ and has diameter 3, or $\delta(G) \leq 2$, or a cut-vertex, then $\gamma(G) = i(G)$. Favaron, Tian, and Zhang [15] showed that every edge-domination-critical graph $G$ with $\gamma(G) = 3$ and $\delta(G) \geq 2$ has $\beta_0(G) \leq \delta(G) + 2$ and if $\beta_0(G) = \delta(G) + 2$, then $\gamma(G) = i(G)$.

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