Baer partitions of small order projective planes

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Dedicated to Professor Anne Penfold Street on the occasion of her retirement.

ABSTRACT. The partitions into baer subplanes of the Desarguesian projective planes of order 9, 16 and 25 are classified by computer. It is also shown that the non-Desarguesian projective planes of order 9 and the non-Desarguesian translation planes of order 16 and 25 do not admit such a partition.

1 Introduction

In a projective plane π of order q^2 a subplane of order q is called a *baer subplane*. A *baer partition* of π is a collection of baer subplanes such that every point of π is contained within a unique element of the collection. The size of such a partition in necessarily $(q^4+q^2+1)/(q^2+q+1) = q^2-q+1$.

Those lines of a projective plane of order q^2 that meet a baer subplane in $q+1$ points form the points of a baer subplane in the dual plane. Given a baer partition we can then construct the *dual partition* in the dual plane by taking the dual of each of the baer subplanes of the partition. In the case that a projective plane is isomorphic to its dual (as are the Desarguesian

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planes) it may or may not be that a baer partition is isomorphic to its dual partition.

It is well known that every Desarguesian projective plane of square order admits a baer partition. This follows from the fact that every Desarguesian projective plane of order q^2 admits a cyclic group of order $q^4 + q^2 + 1$ acting regularly on the points and the lines of the plane. The orbits of the group generated by taking the $(q^2 - q + 1)$-st power of a generator of the cyclic group are then baer subplanes, and they partition the plane. For the purposes of this paper we will call such a partition the *classical* partition. See [7, Chapter 4] for more on the classical partition.

In [1], Peter Yff gave an example of a non-classical baer partition of $PG(2, 9)$, the Desarguesian projective plane of order 9. It was stated that this partition together with the classical partition are the only baer partitions of $PG(2, 9)$. As far as the authors are aware the classical partition and Yffs result are the only known baer partitions of the Desarguesian projective planes.

In the following section we classify, by computer, the baer partitions of $PG(2, q^2)$ for $q = 3, 4$ and 5. It is shown that in $PG(2, 9)$ there are (up to isomorphism) two such partitions, confirming Yffs result. For $q = 4$ and $q = 5$ several new baer partitions are shown to exist. Similar computer searches in non-Desarguesian planes of order 9, translation planes of order 16 and 25 show that such planes (and hence their duals) do not admit baer partitions.

2 Method

The idea for constructing all baer partitions of Desarguesian projective planes up to isomorphism is as follows. A partition of the points $\mathcal{P}$ of $PG(2, q^2)$ into baer subplanes can be interpreted as a spread of subplanes from a collection $\mathcal{B}$. The algorithm is then an exhaustive search for spreads of subplanes in $PG(2, q^2)$ [9]. A spread $\mathcal{S}$ is constructed one subplane at a time. If $\mathcal{S}_i$ is a partial spread of i disjoint subplanes and $\mathcal{P}_i \subset \mathcal{P}$ is the set of points not covered by $\mathcal{S}_i$ a new subplane b must lie in the set $\mathcal{B}_i \in \mathcal{B}$ of subplanes disjoint from all members of $\mathcal{S}_i$. For $\mathcal{S}_i$ to have a completion, every point in $\mathcal{P}_i$ must belong to some subplane in $\mathcal{B}_i$. This is a strong condition which allows an early detection of bad partial spreads (a look ahead) and motivates the following powerful heuristic for subplane selection. Choose a point $p \in \mathcal{P}_i$ which is incident with a minimum number (say t) of subplanes in $\mathcal{B}_i$ and choose the next subplane from this set. If $t = 0$ then a backtrack step is necessary, if $t = 1$ the unique subplane through p is forced; in general, the fewest number of subplanes is being examined. Knowledge of the collineation group of the plane can significantly reduce the size of the search.

The collineation group of $PG(2,q^2)$ is the projective semi-linear group $P\Gamma L(3,q^2)$ and this group is transitive on ordered quadrangles [7]. It is well known that on any quadrangle in $PG(2,q^2)$ there exists a unique baer subplane [6, p. 401]. It follows that up to isomorphism any baer subplane, B_1 say, may be chosen to be the initial one.

The collineation stabiliser of a baer subplane of $PG(2,q^2)$ is isomorphic to $P\Gamma L(3,q)$. Nauty ([8]) was used to find the generators of the stabilizer of a baer subplane. A fast algorithm based on hashing was used to calculate the orbits of baer subplanes disjoint from B_1. A representative of each orbit was then chosen. For each orbit representative all baer subplanes disjoint from the representative and B_1 were then be found. Each of these pairs of baer subplanes were then used as "starter sets" in the algorithm described in the first paragraph.

Once all such partitions were found the isomorphism problem between partitions was solved using Nauty.

Using the above method it was possible to perform complete searches for all (up to isomorphism) baer partitions of $PG(2,9)$, $PG(2,16)$ and $PG(2,25)$. The rest of this paper is devoted to listing the partitions found.

3 Results

In the following the Desarguesian plane $PG(2,q^2)$ is represented via homogeneous coordinates over the Galois field $GF(q^2)$. I.e. represent the points of $PG(2,q^2)$ by $\langle(x,y,z)\rangle$, $x,y,z \in GF(q^2)$ and $(x,y,z) \neq (0,0,0)$, and similarly lines by $\langle[a,b,c]\rangle$ $a,b,c \in GF(q^2)$ and $[a,b,c] \neq [0,0,0]$. Incidence is given by the dot product $\langle(x,y,x)\rangle \ \mathrm{I} \ \langle[a,b,c]\rangle \Leftrightarrow ax+by+cz=0$. In this notation the the subset of points $\langle(x,y,z)\rangle$, $x,y,z \in GF(q)$ and $(x,y,z) \neq (0,0,0)$ form a baer subplane of $PG(2,q^2)$ which we shall call the *real* baer subplane.

We describe the baer partitions found in the following manner. For each partition a collection of q^2-q three by three matrices over $GF(q^2)$ is given. The real baer subplane is in every partition. Each other baer subplane in the partition is obtained by applying one of the matrices to the points of the real baer subplane.

In the following we do not list the classical partitions.

3.1 Partitions of $PG(2,9)$

In $PG(2,9)$ there is up to isomorphism one non-classical partition. It is self dual and has collineation stabiliser of order 21. The collineation stabiliser has a subgroup of order 7 that acts regularly on the elements of the partition. The classical partition in $PG(2,9)$ is self dual and has collineation stabiliser of order 546.

Let ω be a primitive element of $GF(9)$ satisfying $\omega^2 - \omega = 1$, then the non-classical partition is described by the matrices:

$$\begin{pmatrix} \omega^3 & \omega^1 & 0 \\ \omega^4 & 0 & \omega^6 \\ 1 & \omega^6 & \omega^3 \end{pmatrix} \begin{pmatrix} \omega^7 & \omega^6 & 0 \\ \omega^2 & 0 & \omega^1 \\ \omega^4 & \omega^4 & \omega^7 \end{pmatrix} \begin{pmatrix} \omega^1 & \omega^3 & 0 \\ \omega^4 & 0 & \omega^5 \\ \omega^4 & \omega^2 & \omega^4 \end{pmatrix}$$

$$\begin{pmatrix} \omega^7 & \omega^6 & 0 \\ \omega^2 & 0 & \omega^7 \\ 0 & \omega^7 & 1 \end{pmatrix} \begin{pmatrix} \omega^1 & \omega^3 & 0 \\ \omega^5 & 0 & \omega^6 \\ 1 & \omega^5 & 1 \end{pmatrix} \begin{pmatrix} \omega^3 & \omega^1 & 0 \\ \omega^5 & 0 & \omega^7 \\ \omega^7 & \omega^4 & \omega^2 \end{pmatrix}$$

3.2 Partitions of $PG(2, 16)$

In $PG(2, 16)$ there are up to isomorphism three non-classical partitions. Partition 1 (below) is self dual and has collineation stabiliser of order 39. Its collineation stabiliser has a subgroup of order 13 that acts regularly on the elements of the partition. Partition 2 has collineation stabiliser of order 12. Its collineation stabiliser has two orbits on the subplanes of the partition, one of length one and the other of length 12. Partition 2 is not isomorphic to its dual partition. The classical partition in $PG(2, 16)$ is self dual and has collineation stabiliser of order 3276.

Let ω be a primitive element $GF(16)$ satisfying $\omega^4 + \omega = 1$ then the non-classical partitions are as follows:

Partition 1:

$$\begin{pmatrix} \omega^{13} & \omega^6 & 0 \\ \omega^1 & 0 & \omega^9 \\ \omega^{14} & \omega^7 & \omega^{10} \end{pmatrix} \begin{pmatrix} \omega^{14} & \omega^3 & 0 \\ 1 & 0 & \omega^{14} \\ \omega^6 & \omega^5 & \omega^1 \end{pmatrix} \begin{pmatrix} \omega^4 & \omega^1 & 0 \\ \omega^7 & 0 & \omega^{10} \\ 0 & \omega^5 & \omega^{13} \end{pmatrix} \begin{pmatrix} \omega^{12} & \omega^{11} & 0 \\ \omega^{14} & 0 & \omega^{13} \\ \omega^3 & \omega^7 & \omega^2 \end{pmatrix}$$

$$\begin{pmatrix} \omega^{13} & \omega^6 & 0 \\ \omega^1 & 0 & 1 \\ \omega^2 & 1 & \omega^6 \end{pmatrix} \begin{pmatrix} \omega^{11} & \omega^{12} & 0 \\ \omega^{14} & 0 & \omega^8 \\ \omega^8 & \omega^{10} & 1 \end{pmatrix} \begin{pmatrix} \omega^4 & \omega^1 & 0 \\ \omega^7 & 0 & \omega^{13} \\ \omega^6 & \omega^4 & \omega^6 \end{pmatrix} \begin{pmatrix} \omega^8 & \omega^2 & 0 \\ \omega^{11} & 0 & \omega^3 \\ \omega^4 & \omega^1 & \omega^{12} \end{pmatrix}$$

$$\begin{pmatrix} \omega^9 & \omega^7 & 0 \\ \omega^{12} & 0 & \omega^{14} \\ \omega^{14} & 1 & \omega^{10} \end{pmatrix} \begin{pmatrix} \omega^4 & \omega^1 & 0 \\ \omega^8 & 0 & \omega^{14} \\ \omega^{12} & \omega^8 & \omega^{11} \end{pmatrix} \begin{pmatrix} \omega^9 & \omega^7 & 0 \\ \omega^{13} & 0 & 1 \\ \omega^4 & \omega^4 & \omega^{13} \end{pmatrix} \begin{pmatrix} \omega^{11} & \omega^{12} & 0 \\ \omega^{14} & 0 & \omega^1 \\ \omega^{10} & \omega^3 & 1 \end{pmatrix}$$

Partition 2:

$$\begin{pmatrix} \omega^{13} & \omega^6 & 0 \\ \omega^1 & 0 & \omega^9 \\ \omega^{14} & \omega^7 & \omega^{10} \end{pmatrix} \begin{pmatrix} \omega^9 & \omega^7 & 0 \\ \omega^{12} & 0 & \omega^6 \\ \omega^{11} & \omega^9 & \omega^8 \end{pmatrix} \begin{pmatrix} \omega^4 & \omega^1 & 0 \\ \omega^7 & 0 & \omega^{10} \\ \omega^9 & \omega^{12} & \omega^{13} \end{pmatrix} \begin{pmatrix} \omega^{11} & \omega^{12} & 0 \\ \omega^{13} & 0 & \omega^{14} \\ \omega^4 & \omega^6 & \omega^3 \end{pmatrix}$$

$$\begin{pmatrix} \omega^9 & \omega^7 & 0 \\ \omega^{12} & 0 & \omega^{14} \\ \omega^{13} & 1 & \omega^5 \end{pmatrix} \begin{pmatrix} \omega^8 & \omega^2 & 0 \\ \omega^{11} & 0 & \omega^3 \\ 1 & \omega^9 & \omega^{10} \end{pmatrix} \begin{pmatrix} \omega^{11} & \omega^{12} & 0 \\ \omega^{14} & 0 & \omega^8 \\ \omega^9 & \omega^3 & \omega^1 \end{pmatrix} \begin{pmatrix} \omega^3 & \omega^{14} & 0 \\ \omega^4 & 0 & 1 \\ \omega^{12} & \omega^2 & \omega^9 \end{pmatrix}$$

$$\begin{pmatrix} \omega^4 & \omega^1 & 0 \\ \omega^7 & 0 & \omega^{13} \\ \omega^1 & \omega^{13} & \omega^9 \end{pmatrix} \begin{pmatrix} \omega^4 & \omega^1 & 0 \\ \omega^8 & 0 & \omega^{14} \\ \omega^{13} & 1 & \omega^{11} \end{pmatrix} \begin{pmatrix} \omega^{11} & \omega^{12} & 0 \\ \omega^{14} & 0 & \omega^1 \\ \omega^7 & \omega^1 & \omega^{14} \end{pmatrix} \begin{pmatrix} \omega^9 & \omega^7 & 0 \\ \omega^{13} & 0 & 1 \\ \omega^6 & \omega^5 & \omega^{14} \end{pmatrix}$$

Partition 1 can be described in the following way. We assume that q is even. For definitions the reader is referred to [7].

Let σ be a generator for a cyclic group of order $q^4 + q^2 + 1$ in $PG(2, q^2)$. Then the orbits of the group generated by σ^{q^2-q+1} are a collection of baer subplanes $\mathcal{B}_i$, $i = 1 \dots q^2 - q + 1$, and give rise to the classical baer partition. The orbits of the group generated by σ^{q^2+q+1} are a collection $\mathcal{C}_i$, $i = 0 \dots q^2+q$ of complete arcs. Choose any point P_0. Then P_0 is contained within a unique baer subplane $\mathcal{B}_0$ and a unique complete arc $\mathcal{C}_0$, say. Since q is even, the set of lines that meet $\mathcal{C}_0$ in a unique point form a classical unital $\mathcal{U}$ in the dual plane (see [10]). The unital U induces a unique polarity σ of the projective plane that maps a point of the unital to the (unique) tangent line to the unital on that point. The line $P_0{}^{\sigma}$ does not contain P_0, but does contain $q + 1$ points $P_1, \dots, P_{q+1}$ of the baer subplane $\mathcal{B}_0$. There are then $q + 1$ baer subplanes of $PG(2, q^2)$ that contain the points $P_0, P_1, \dots, P_{q+1}$, exactly one of which, $\mathcal{B}_0$, is in the classical partition.

By computer we have shown that for $q = 4$, choosing any one of these $q + 1$ baer subplanes and taking its images under the group generated by σ^{q^2+q+1} gives rise to a baer partition of $PG(2, q^2)$. If $\mathcal{B}_0$ is chosen then by definition the partition is the classical one. Any other choice gives rise to a partition isomorphic to partition 1 in $PG(2, 16)$. We have also verified that the above construction for $q = 8$ gives rise to baer partitions of $PG(2, 64)$. A general proof that the construction gives a baer partition in even (square) order Desarguesian projective planes has so far been elusive.

Conjecture. There exist non-classical baer partitions of $PG(2, 2^{2e})$, $e > 1$, whose collineation stabiliser contains a cyclic group of order $2^{2e} - 2^e + 1$.

3.3 Partitions of $PG(2, 25)$

In $PG(2, 25)$ there are up to isomorphism four non-classical partitions. Partition 1 (below) is not self dual and has collineation stabiliser of order 63 which is transitive on the subplanes of the partition. Partition 2 has collineation stabiliser of order 18 and is not self dual. Its collineation stabiliser has two orbits on the subplanes of the partition, one of length 3 the other of length 18. The classical partition in $PG(2, 25)$ is self dual and has collineation stabiliser of order 3906.

Let ω be a primitive element $GF(25)$ satisfying $\omega^2 + 3\omega + 3 = 0$

Partition 1:

$$\begin{pmatrix}\omega^{14} & \omega^{21} & 0\\ \omega^{16} & 0 & \omega^{23}\\ \omega^{21} & \omega^{22} & 1\end{pmatrix} \begin{pmatrix}\omega^{17} & \omega^{16} & 0\\ \omega^{20} & 0 & \omega^{6}\\ \omega^{11} & \omega^{18} & \omega^{8}\end{pmatrix} \begin{pmatrix}\omega^{10} & \omega^{19} & 0\\ \omega^{13} & 0 & \omega^{17}\\ \omega^{12} & \omega^{23} & \omega^{20}\end{pmatrix} \begin{pmatrix}\omega^{13} & \omega^{8} & 0\\ \omega^{16} & 0 & \omega^{21}\\ \omega^{14} & \omega^{19} & \omega^{1}\end{pmatrix}$$

$$\begin{pmatrix}\omega^{13} & \omega^{8} & 0\\ \omega^{16} & 0 & \omega^{1}\\ \omega^{16} & 1 & \omega^{6}\end{pmatrix} \begin{pmatrix}\omega^{11} & \omega^{7} & 0\\ \omega^{15} & 0 & \omega^{4}\\ \omega^{20} & \omega^{10} & \omega^{11}\end{pmatrix} \begin{pmatrix}\omega^{3} & \omega^{5} & 0\\ \omega^{6} & 0 & \omega^{3}\\ 0 & \omega^{10} & \omega^{11}\end{pmatrix} \begin{pmatrix}\omega^{4} & \omega^{20} & 0\\ \omega^{8} & 0 & \omega^{5}\\ 0 & \omega^{10} & \omega^{14}\end{pmatrix}$$

$$\begin{pmatrix}\omega^{17} & \omega^{16} & 0\\ \omega^{21} & 0 & \omega^{7}\\ \omega^{1} & \omega^{2} & \omega^{17}\end{pmatrix} \begin{pmatrix}\omega^{5} & \omega^{3} & 0\\ \omega^{8} & 0 & \omega^{10}\\ \omega^{1} & \omega^{2} & \omega^{21}\end{pmatrix} \begin{pmatrix}\omega^{22} & \omega^{9} & 0\\ \omega^{1} & 0 & \omega^{12}\\ \omega^{3} & \omega^{7} & \omega^{1}\end{pmatrix} \begin{pmatrix}\omega^{23} & \omega^{2} & 0\\ \omega^{2} & 0 & \omega^{19}\\ \omega^{22} & \omega^{10} & \omega^{9}\end{pmatrix}$$

$$\begin{pmatrix}\omega^{13} & \omega^{8} & 0\\ \omega^{16} & 0 & \omega^{12}\\ \omega^{13} & \omega^{3} & \omega^{3}\end{pmatrix} \begin{pmatrix}\omega^{4} & \omega^{20} & 0\\ \omega^{7} & 0 & \omega^{4}\\ \omega^{20} & \omega^{13} & \omega^{20}\end{pmatrix} \begin{pmatrix}\omega^{11} & \omega^{7} & 0\\ \omega^{14} & 0 & \omega^{18}\\ \omega^{21} & \omega^{4} & \omega^{11}\end{pmatrix} \begin{pmatrix}\omega^{1} & \omega^{15} & 0\\ \omega^{2} & 0 & \omega^{16}\\ \omega^{13} & \omega^{4} & \omega^{11}\end{pmatrix}$$

$$\begin{pmatrix}\omega^{17} & \omega^{16} & 0\\ \omega^{20} & 0 & \omega^{3}\\ \omega^{1} & \omega^{12} & \omega^{23}\end{pmatrix} \begin{pmatrix}\omega^{19} & \omega^{10} & 0\\ \omega^{22} & 0 & \omega^{23}\\ \omega^{2} & \omega^{1} & \omega^{20}\end{pmatrix} \begin{pmatrix}\omega^{4} & \omega^{20} & 0\\ \omega^{7} & 0 & \omega^{18}\\ \omega^{1} & \omega^{5} & \omega^{16}\end{pmatrix} \begin{pmatrix}\omega^{11} & \omega^{7} & 0\\ \omega^{14} & 0 & \omega^{3}\\ \omega^{6} & \omega^{14} & \omega^{2}\end{pmatrix}$$

Partition 2:

$$\begin{pmatrix}\omega^{14} & \omega^{21} & 0\\ \omega^{16} & 0 & \omega^{23}\\ \omega^{21} & \omega^{22} & 1\end{pmatrix} \begin{pmatrix}\omega^{13} & \omega^{8} & 0\\ \omega^{16} & 0 & \omega^{1}\\ \omega^{9} & \omega^{10} & \omega^{3}\end{pmatrix} \begin{pmatrix}\omega^{13} & \omega^{8} & 0\\ \omega^{16} & 0 & \omega^{12}\\ \omega^{14} & \omega^{1} & \omega^{15}\end{pmatrix} \begin{pmatrix}\omega^{5} & \omega^{3} & 0\\ \omega^{8} & 0 & \omega^{10}\\ \omega^{14} & \omega^{22} & \omega^{14}\end{pmatrix}$$

$$\begin{pmatrix}\omega^{4} & \omega^{20} & 0\\ \omega^{7} & 0 & \omega^{4}\\ \omega^{15} & \omega^{11} & \omega^{9}\end{pmatrix} \begin{pmatrix}\omega^{4} & \omega^{20} & 0\\ \omega^{8} & 0 & \omega^{5}\\ \omega^{1} & \omega^{23} & \omega^{12}\end{pmatrix} \begin{pmatrix}\omega^{19} & \omega^{10} & 0\\ \omega^{22} & 0 & \omega^{21}\\ \omega^{12} & 1 & \omega^{5}\end{pmatrix} \begin{pmatrix}\omega^{17} & \omega^{16} & 0\\ \omega^{20} & 0 & \omega^{3}\\ \omega^{17} & \omega^{21} & \omega^{12}\end{pmatrix}$$

$$\begin{pmatrix}\omega^{17} & \omega^{16} & 0\\ \omega^{21} & 0 & \omega^{7}\\ \omega^{10} & \omega^{5} & \omega^{17}\end{pmatrix} \begin{pmatrix}\omega^{3} & \omega^{5} & 0\\ \omega^{4} & 0 & \omega^{6}\\ \omega^{6} & \omega^{2} & \omega^{17}\end{pmatrix} \begin{pmatrix}\omega^{13} & \omega^{8} & 0\\ \omega^{16} & 0 & \omega^{21}\\ \omega^{12} & \omega^{12} & \omega^{10}\end{pmatrix} \begin{pmatrix}\omega^{4} & \omega^{20} & 0\\ \omega^{7} & 0 & \omega^{18}\\ \omega^{7} & \omega^{6} & \omega^{8}\end{pmatrix}$$

$$\begin{pmatrix}\omega^{11} & \omega^{7} & 0\\ \omega^{14} & 0 & \omega^{3}\\ 0 & \omega^{23} & \omega^{18}\end{pmatrix} \begin{pmatrix}\omega^{17} & \omega^{16} & 0\\ \omega^{20} & 0 & \omega^{6}\\ \omega^{12} & 1 & \omega^{22}\end{pmatrix} \begin{pmatrix}\omega^{3} & \omega^{5} & 0\\ \omega^{6} & 0 & \omega^{3}\\ \omega^{11} & \omega^{4} & \omega^{20}\end{pmatrix} \begin{pmatrix}\omega^{11} & \omega^{7} & 0\\ \omega^{14} & 0 & \omega^{18}\\ \omega^{8} & \omega^{18} & \omega^{13}\end{pmatrix}$$

$$\begin{pmatrix}\omega^{16} & \omega^{17} & 0\\ \omega^{19} & 0 & \omega^{20}\\ \omega^{18} & \omega^{15} & \omega^{16}\end{pmatrix} \begin{pmatrix}\omega^{19} & \omega^{10} & 0\\ \omega^{22} & 0 & \omega^{23}\\ 1 & \omega^{19} & \omega^{20}\end{pmatrix} \begin{pmatrix}\omega^{11} & \omega^{7} & 0\\ \omega^{15} & 0 & \omega^{4}\\ \omega^{14} & \omega^{3} & \omega^{2}\end{pmatrix} \begin{pmatrix}\omega^{10} & \omega^{19} & 0\\ \omega^{13} & 0 & \omega^{17}\\ \omega^{2} & \omega^{2} & \omega^{16}\end{pmatrix}$$

3.4 Baer partitions of non-Desarguesian projective planes

The projective planes of order 9 were given in [5]. They are a Desarguesian plane, a Hall plane and its dual, and a Hughes plane.

The translation planes of order 16 were classified by Dempwolff and Reifart in [3]. They are: a Desarguesian plane; a semifield plane with kernel

$GF(4)$; a semifield plane with kernel $GF(2)$; a Hall plane; the Lorimer-Rahilly plane; the Johnson-Walker plane; a derived semifield plane; and the Dempwolff plane. The Desarguesian plane and the two semifield planes are self dual, while all the others are not.

The translation planes of order 25 were classified by Czerwinski and Oakden in [2]. There are 21 such planes.

Using techniques similar to those above it was shown by computer that no non-Desarguesian plane of order 9 admits a baer partition. It was also shown that no non-Desarguesian translation plane of order 16 or 25 admits a baer partition. It follows that the duals of such planes also admit no baer partition.

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