

All Non-Isomorphic Residual (16, 24, 9, 6, 3)-Designs

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Abstract. We show that there are 1281 non-isomorphic residual (16, 24, 9, 6, 3)-designs.

1. Introduction.

A balanced incomplete block design (BIBD) is a pair (V, B) where V is a v -set and B is a collection of b k -subsets of V called blocks such that each element of V is contained in exactly r blocks and any 2-subset of V is contained in exactly λ blocks. The numbers v, b, r, k, λ are parameters of the BIBD. Trivial necessary conditions for the existence of a BIBD (v, b, r, k, λ) are

- (1) $vr = bk$,
- (2) $r(k - 1) = \lambda(v - 1)$.

A parameter set that satisfies (1) and (2) is said to be admissible.

Two BIBD's (V_1, B_1) and (V_2, B_2) are isomorphic if there exists a bijection $\alpha : V_1 \rightarrow V_2$ such that $B_1\alpha = B_2$. Given a symmetric BIBD (one with $v = b, r = k$), one obtains from it the residual design by deleting all elements of one block, and the derived design by deleting all elements of the complement of one block. The parameters of a derived design are $(k, v - 1, k - 1, \lambda, \lambda - 1)$, whereas the parameters of a residual design are $(v - k, v - 1, k, k - \lambda, \lambda)$.

Mathon and Rosa [3] have a very useful list of known bounds on the number of non-isomorphic BIBD's for admissible parameter sets. This paper will deal with the BIBD's with parameter sets (16, 24, 9, 6, 3), (9, 24, 8, 3, 2) and (25, 25, 9, 9, 3).

2. Residuals.

Denniston [1] has listed all pairwise non-isomorphic symmetric designs (25, 25, 9, 9, 3). There are 78 such designs. His listing was used to produce a listing of all pairwise non-isomorphic residual designs (16, 24, 9, 6, 3).

The 78 designs were used as input to a program that deleted the blocks, one by one, from each design to produce $78 \times 25 = 1,950$ residual designs.

Professor Wm. Kocay's very fast graph isomorphism program was then used to produce a canonical representation for each design. It could do about three a second on an Amdahl 580. These representations were then sorted and duplicates eliminated. Hence our theorem.

Theorem 1. The number of pairwise non-isomorphic residual (16, 24, 9, 6, 3)-BIBD's is 1281.

Professor Kocay's program also gave the order of the automorphism group of the design. I have summarized these results in Table 1.

**Number of Non-isomorphic Derived Designs (16, 24, 9, 6, 3)
Whose Automorphism Group Equals |G|**

G	1	2	3	4	6	9	12	18	72
# of designs	1112	115	18	29	1	2	1	2	1

TABLE 1

Rather than writing down all 1281 x 24 blocks, I will show in the next table which blocks need deleting from Denniston's list of symmetric designs to produce a representative of each isomorphism class for the 1281 designs.

The Non-isomorphic Residual Designs

THE DESIGNS WHOSE GROUP ORDER EQUALS 1

DESIGN	DELETED BLOCK(S)
3	10 22
4	1 5 9 25
7	5 13
8	9 11 14 25
11	1 2 3 4 5 6 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
12	8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
13	7 8 12 13 15 16 20 25
14	15 16 18 19 20 23
15	25
17	4 6 14 15 16 18 20 25
18	2 8 10 11 12 13 20 24
19	1 2 3 4 5 6 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
20	8 9 10 11 12 14 16 17 18 19 20 22 23 24 25
21	3 5 7 10 11 15 17 22
22	14 16 19 21 23 24
23	15
25	3 7 8 16 17 20 24 25
26	2 3 15 18 20 22 23 25
27	5 6 9 14 15 16 19 25
28	1 7 10 14 15 17 21 25
29	13 15 17 22 23 24
30	1 2 3 4 5 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
31	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
32	1 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
33	1 2 3 4 5 6 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
34	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
35	1 2 3 4 5 6 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
36	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 17 18 19 20 21 22 23 24 25
37	1 2 4 10 13 18 19 21
38	8 9 10 15 16 21
39	1 2 3 4 5 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
40	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
41	1 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
42	1 2 3 4 5 6 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
43	1 5 13 23
44	1 2 3 4 5 6 7 8 9 10 12 13 14 15 16 17 18 19 20 21 22 23 24 25
45	1 2 3 4 5 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
46	1 5 9 10 16 19 20 24
47	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
48	1 2 3 4 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
49	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
50	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
51	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
52	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
53	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
54	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
55	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
56	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 17 18 19 20 21 22 23 24 25
57	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
58	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
59	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
60	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
61	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
62	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
63	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
64	1 9 12 17 19 21 22 23
65	1 4 5 6 16 17 21 23
66	1 4 15 17 20 21 24 25
67	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
68	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
69	1 3 8 10 18 19 24 25
70	4 6 7 9 13 17 18 21
71	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25
72	3 6 12 16 20 22 24 25
73	2 4 8 17 19 20 21 23
74	6 7 9 10 13 14 22 24
75	2 4 9 15 21 23 24 25
76	2 3 9 11 17 19 21 23
77	3 5 9 15 20 22 23 24
78	1 8 9 16 17 18 19 23

The Non-isomorphic Residual Designs Continued

THE DESIGNS WHOSE GROUP ORDER EQUALS 2

DESIGN	DELETED BLOCK(S)
1	25
2	25
3	14 18
5	4 11 12 13 16 17 18 23 24 25
6	1 14 19 24
7	4 19
9	8 12 13 16 17 18 23 24 25
10	5 16 25
13	1 4 6 9 18 19 23
14	5 8 9 10 11 12 13
16	12 23 24
17	2 3 8 12 13 17 22
18	1 4 9 14 18 19 23
21	1 4 6 9 14 18 19 23
22	4 7 8 9 10 11 12 13
24	7 11 23 25
25	1 4 9 14 18 19 23
26	6 7 8 9 10 12 13
27	2 3 7 8 12 13 17 22
36	16
43	2
46	3 7 8 12 17 21 22
48	5

THE DESIGNS WHOSE GROUP ORDER EQUALS 3

DESIGN	DELETED BLOCK(S)
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15	1
19	7
23	1
28	6
37	6
38	5
64	3
65	3
66	22
69	2
70	24
72	17
73	9
74	17
75	14
76	12
77	10
78	7

THE DESIGNS WHOSE GROUP ORDER EQUALS 4

DESIGN	DELETED BLOCK(S)
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1	17
2	3
4	2 3 8 13 17 22
6	2 3 13
8	2 7 8 12 13 17 21 22
10	2 12 22
15	2 7 21
17	21
22	6
23	8 12

The Non-isomorphic Residual Designs Continued

THE DESIGNS WHOSE GROUP ORDER EQUALS 6	
DESIGN	DELETED BLOCK(S)
7	16
THE DESIGNS WHOSE GROUP ORDER EQUALS 9	
DESIGN	DELETED BLOCK(S)
7	2
29	7
THE DESIGNS WHOSE GROUP ORDER EQUALS 12	
DESIGN	DELETED BLOCK(S)
2	2
THE DESIGNS WHOSE GROUP ORDER EQUALS 18	
DESIGN	DELETED BLOCK(S)
3	1
7	1
THE DESIGNS WHOSE GROUP ORDER EQUALS 72	
DESIGN	DELETED BLOCK(S)
6	21

Also, since many people are interested in how residual designs are combined to produce symmetric designs, the next table shows into which class each of the 1,950 designs fall. The symbol n stands for the order of the group while class indicates into which isomorphism class each design falls. The design can be produced by finding the design listed in Denniston's list and deleting the block listed on the left-hand side under BLK.

The Residual Designs

BLK	DESIGN 1		DESIGN 2		DESIGN 3		DESIGN 4		DESIGN 5		DESIGN 6		DESIGN 7	
	CLASS	N	CLASS	N	CLASS	N	CLASS	N	CLASS	N	CLASS	N	CLASS	N
1	1128	6	1128	6	1247	18	1065	1	1006	2	1069	2	1246	18
2	1253	12	1253	12	1252	9	645	4	1006	2	1263	4	1252	9
3	1241	4	1243	4	1140	1	682	4	1114	2	1265	4	1134	1
4	1123	2	1120	2	1178	2	1000	1	1114	2	1124	2	1185	2
5	1123	2	1120	2	1131	1	1144	1	1264	72	1008	2	1139	1
6	1128	6	1128	6	1246	18	1065	1	1118	2	1069	2	1247	18
7	1253	12	1253	12	1252	9	1245	4	1118	2	1263	4	1252	9
8	1241	4	1243	4	1140	1	613	4	1113	2	982	4	1134	1
9	1123	2	1120	2	1182	2	999	1	1125	2	1127	2	1175	2
10	1123	2	1120	2	1131	1	1000	1	1075	2	1124	2	1139	1
11	1128	6	1128	6	1128	6	1065	1	1113	2	1069	2	1128	6
12	1241	4	1243	4	1140	1	593	4	1075	2	1265	4	1134	1
13	1241	4	1243	4	1140	1	1255	4	1125	2	1242	4	1134	1
14	1123	2	1120	2	1182	2	1144	1	1099	2	1008	2	1175	2
15	1123	2	1120	2	1131	1	999	1	1116	2	1127	2	1139	1
16	1128	6	1128	6	1128	6	1065	1	1101	2	1069	2	1128	6
17	1241	4	1243	4	1140	1	1261	4	1099	2	1242	4	1134	1
18	1123	2	1120	2	1178	2	999	1	1116	2	1127	2	1185	2
19	1123	2	1120	2	1182	2	1000	1	1101	2	1124	2	1175	2
20	1123	2	1120	2	1131	1	1144	1	1092	2	1008	2	1139	1
21	1264	72	1264	72	1253	12	589	4	1122	2	1264	72	1253	12
22	1241	4	1243	4	1140	1	1257	4	1089	2	982	4	1134	1
23	1123	2	1120	2	1178	2	1144	1	1092	2	1008	2	1185	2
24	1123	2	1120	2	1131	1	999	1	1122	2	1127	2	1139	1
25	1123	2	1120	2	1131	1	1000	1	1089	2	1124	2	1139	1

BLK	DESIGN 8		DESIGN 9		DESIGN 10		DESIGN 11		DESIGN 12		DESIGN 13		DESIGN 14	
	CLASS	N	CLASS	N	CLASS	N	CLASS	N	CLASS	N	CLASS	N	CLASS	N
1	1064	1	1008	2	1073	2	1079	1	1238	1	1067	2	1238	1
2	615	4	1008	2	1266	4	598	1	1006	2	649	1	1008	2
3	589	4	1114	2	1262	4	287	1	1238	1	884	1	1238	1
4	1007	1	1114	2	1081	2	1012	1	1114	2	1179	2	1008	2
5	1001	1	1264	72	1126	2	923	1	1248	2	964	1	1248	2
6	1064	1	1118	2	1073	2	627	1	1118	2	633	2	982	4
7	1245	4	1118	2	1266	4	1259	3	1231	2	649	1	1231	2
8	1256	4	1115	2	1244	4	614	1	1108	1	283	1	1177	2
9	1145	1	1070	2	1006	2	667	1	1104	1	1161	2	1202	2
10	1007	1	1094	2	1081	2	899	1	1077	1	882	1	1196	2
11	1064	1	1115	2	1073	2	935	1	1112	1	630	1	1183	2
12	650	4	1094	2	1262	4	594	1	1076	1	884	1	1194	2
13	1258	4	1070	2	982	4	674	1	1171	1	683	1	1200	2
14	1001	1	1106	2	1126	2	1051	1	1083	1	1021	2	1037	1
15	1145	1	1098	2	1006	2	321	1	1110	1	308	1	1040	1
16	1064	1	1103	2	1073	2	628	1	1167	1	630	1	1102	1
17	983	4	1106	2	982	4	263	1	1174	1	683	1	1042	1
18	1145	1	1098	2	1006	2	978	1	1107	1	977	2	1045	1
19	1007	1	1103	2	1081	2	671	1	1084	1	1063	2	1159	1
20	1001	1	1093	2	1126	2	967	1	1091	1	964	1	1037	1
21	646	4	1121	2	1264	72	590	1	1138	1	1245	4	1040	1
22	1239	4	1088	2	1244	4	579	1	1086	1	283	1	1102	1
23	1001	1	1093	2	1126	2	552	1	1129	1	556	2	1042	1
24	1145	1	1121	2	1006	2	290	1	1132	1	308	1	1045	1
25	1007	1	1088	2	1081	2	576	1	1135	1	882	1	1159	1

BLK	DESIGN 15		DESIGN 16		DESIGN 17		DESIGN 18		DESIGN 19		DESIGN 20		DESIGN 21	
	CLASS	N	CLASS	N	CLASS	N	CLASS	N	CLASS	N	CLASS	N	CLASS	N
1	1066	3	973	2	946	1	1009	2	1078	1	1238	1	1068	2
2	1260	4	973	2	652	2	1142	1	616	1	1008	2	597	1
3	644	4	973	2	1164	2	1035	1	288	1	1238	1	692	1
4	998	1	973	2	694	1	1188	2	1010	1	1114	2	979	2
5	998	1	1264	72	937	1	618	1	965	1	1248	2	880	1
6	1066	3	973	2	946	1	1022	2	631	1	1118	2	629	2
7	644	4	973	2	1207	2	1142	1	1259	3	1231	2	597	1
8	593	4	809	2	985	2	1002	1	596	1	1117	1	885	1
9	998	1	1119	2	929	1	561	2	1053	1	1071	1	1029	2
10	998	1	821	2	1004	1	1033	1	672	1	1172	1	309	1
11	1066	3	809	2	306	1	1229	1	934	1	1111	1	626	1
12	593	4	821	2	1154	2	1035	1	651	1	1096	1	692	1
13	1260	4	1119	2	1180	2	1041	1	580	1	1072	1	282	1
14	998	1	821	2	1226	1	1268	2	673	1	1105	1	1160	2
15	998	1	809	2	997	1	281	1	289	1	1173	1	922	1
16	1066	3	1119	2	306	1	1229	1	632	1	1085	1	626	1
17	1260	4	821	2	1205	2	1041	1	264	1	1109	1	282	1
18	998	1	809	2	929	1	1190	2	669	1	1082	1	562	2
19	998	1	1119	2	694	1	1197	2	1013	1	1168	1	1022	2
20	998	1	809	2	937	1	618	1	966	1	1090	1	880	1
21	593	4	1119	2	589	4	593	4	647	1	1137	1	1245	4
22	644	4	821	2	198	2	1002	1	900	1	1087	1	885	1
23	998	1	809	2	1226	1	1250	2	553	1	1136	1	1186	2
24	998	1	1119	2	997	1	281	1	320	1	1133	1	922	1
25	998	1	821	2	1004	1	1033	1	575	1	1130	1	309	1

BLK	DESIGN 22		DESIGN 23		DESIGN 24		DESIGN 25		DESIGN 26		DESIGN 27		DESIGN 28	
	CLASS	N	CLASS	N	CLASS	N	CLASS	N	CLASS	N	CLASS	N	CLASS	N
1	1238	1	1074	3	973	2	653	2	781	1	947	1	718	1
2	1006	2	1240	4	973	2	955	1	806	1	1032	2	322	1
3	1238	1	646	4	973	2	307	1	781	1	625	2	327	1
4	1006	2	1005	1	973	2	1189	2	806	1	601	1	1014	1
5	1248	2	1005	1	1264	72	926	1	118	2	1034	1	1014	1
6	982	4	1074	3	973	2	1021	2	988	2	947	1	717	3
7	1231	2	984	4	973	2	955	1	813	2	1207	2	327	1
8	1203	2	1240	4	819	2	996	1	555	2	980	2	328	1
9	1184	2	1005	1	811	2	1153	2	1269	2	902	1	664	1
10	1195	2	1005	1	1095	2	989	1	199	2	624	1	970	1
11	1201	2	1074	3	819	2	1225	1	589	4	1036	1	718	1
12	1193	2	984	4	1095	2	307	1	1187	2	1152	2	328	1
13	1176	2	984	4	811	2	1003	1	1198	2	187	2	322	1
14	1158	1	1005	1	819	2	1204	2	773	1	608	1	664	1
15	1043	1	1005	1	811	2	698	1	958	1	903	1	1014	1
16	1044	1	1074	3	1095	2	1225	1	280	1	1036	1	718	1
17	1080	1	646	4	819	2	1003	1	1141	1	1206	2	322	1
18	1038	1	1005	1	811	2	1249	2	522	1	902	1	970	1
19	1039	1	1005	1	1095	2	986	2	948	1	601	1	664	1
20	1158	1	1005	1	1095	2	926	1	773	1	1034	1	970	1
21	1043	1	646	4	819	2	593	4	958	1	646	4	328	1
22	1044	1	1240	4	811	2	996	1	280	1	1192	2	327	1
23	1080	1	1005	1	1095	2	1181	2	1141	1	608	1	574	1
24	1038	1	1005	1	819	2	698	1	522	1	903	1	574	1
25	1039	1	1005	1	811	2	989	1	948	1	624	1	574	1

	DESIGN	DESIGN	DESIGN	DESIGN	DESIGN	DESIGN	DESIGN	DESIGN	DESIGN	DESIGN			
	29	30	31	32	33	34	35						
BLK CLASS	N	CLASS	N	CLASS	N	CLASS	N	CLASS	N	CLASS			
1	1236	3	932	1	969	1	695	1	312	1	343	1	339
2	973	2	477	1	51	1	1207	2	622	1	20	1	910
3	1236	3	304	1	787	1	940	1	840	1	396	1	829
4	973	2	209	1	50	1	862	1	11	1	334	1	22
5	1236	3	933	1	201	1	684	1	569	1	581	1	14
6	973	2	1022	2	139	1	586	1	635	1	548	1	600
7	1267	9	839	1	1281	1	1031	1	1237	1	831	1	1237
8	807	1	994	1	568	1	826	1	420	1	25	1	592
9	1100	1	486	1	837	1	1025	1	700	1	428	1	1
10	822	1	876	1	84	1	883	1	169	1	266	1	611
11	810	1	1155	1	400	1	654	1	405	1	1237	1	277
12	818	1	764	1	1271	1	330	1	702	1	870	1	619
13	1169	1	1227	1	660	1	487	1	427	1	930	1	846
14	818	1	1221	1	801	1	892	1	917	1	657	1	804
15	810	1	893	1	272	1	464	1	543	1	234	1	8
16	1169	1	875	1	446	1	639	1	1016	1	638	1	28
17	822	1	1046	1	202	1	1214	1	668	1	10	1	426
18	807	1	1208	1	1052	1	348	1	1156	1	296	1	31
19	1100	1	1215	1	802	1	865	1	1018	1	315	1	404
20	810	1	953	1	231	1	310	1	32	1	920	1	401
21	1169	1	676	1	505	1	247	1	501	1	805	1	390
22	818	1	927	1	273	1	496	1	302	1	409	1	394
23	807	1	1219	1	373	1	921	1	294	1	962	1	301
24	1100	1	158	1	507	1	286	1	270	1	588	1	961
25	822	1	945	1	116	1	881	1	6	1	291	1	578

	DESIGN	DESIGN	DESIGN	DESIGN	DESIGN	DESIGN	DESIGN	DESIGN	DESIGN	DESIGN			
	36	37	38	39	40	41	42						
BLK CLASS	N	CLASS	N	CLASS	N	CLASS	N	CLASS	N	CLASS			
1	237	1	716	1	1236	3	690	1	771	1	599	1	909
2	465	1	326	1	973	2	479	1	59	1	1207	2	620
3	918	1	323	1	1236	3	172	1	912	1	331	1	13
4	1143	1	1011	1	973	2	1210	1	56	1	558	1	621
5	497	1	666	1	1236	3	847	1	119	1	925	1	570
6	337	1	719	3	973	2	1021	2	959	1	679	1	913
7	1232	1	329	1	1267	9	1058	1	777	1	828	1	1237
8	878	1	326	1	820	1	856	1	111	1	1151	1	438
9	60	1	1015	1	808	1	1222	1	1279	1	1151	1	438
10	1163	1	573	1	1170	1	952	1	444	1	898	1	156
11	386	1	716	1	817	1	1251	1	565	1	591	1	411
12	537	1	329	1	1097	1	305	1	508	1	64	1	703
13	557	1	329	1	812	1	928	1	789	1	495	1	421
14	736	1	1015	1	817	1	1213	1	503	1	127	1	336
15	55	1	573	1	812	1	659	1	147	1	462	1	540
16	1021	2	716	1	1097	1	689	1	271	1	338	1	1019
17	957	1	323	1	820	1	995	1	836	1	827	1	1049
18	784	1	666	1	808	1	1220	1	380	1	397	1	1049
19	173	1	1015	1	1170	1	489	1	114	1	864	1	845
20	1062	1	1011	1	1097	1	852	1	276	1	285	1	293
21	1234	1	323	1	817	1	697	1	976	1	637	1	279
22	854	1	326	1	812	1	763	1	504	1	488	1	7
23	851	1	573	1	1170	1	485	1	915	1	924	1	33
24	914	1	1011	1	820	1	661	1	357	1	311	1	482
25	494	1	666	1	808	1	894	1	82	1	62	1	303

BLK CLASS	DESIGN	N	CLASS	DESIGN	N	CLASS	DESIGN	N	CLASS	DESIGN	N	CLASS	DESIGN	N	CLASS	DESIGN	N	CLASS	DESIGN	N
43	DESIGN		44	DESIGN		45	DESIGN		46	DESIGN		47	DESIGN		48	DESIGN		49	DESIGN	
1238	1		340	1		956	1		603	1		398	1		476	1		750	1	
1118	2		23	1		148	1		646	4		109	1		770	1		472	1	
1111	1		610	1		200	1		188	2		521	1		782	1		66	1	
1085	4		314	1		49	1		816	1		643	1		463	1		74	1	
1137	1		577	1		919	1		949	1		726	1		117	2		747	1	
1008	2		585	1		1022	2		520	1		324	1		492	1		688	1	
1231	2		830	1		1233	1		824	2		526	1		814	1		680	1	
1096	1		9	1		1148	1		1191	2		724	1		1030	1		44	1	
1109	1		963	1		1162	1		905	1		435	1		1217	1		456	1	
1087	1		292	1		52	1		816	1		524	1		101	1		1149	1	
1238	1		1237	1		249	1		603	1		502	1		236	1		85	1	
1117	1		395	1		141	1		981	2		313	1		1216	1		61	1	
1072	1		408	1		872	1		118	2		189	1		491	1		268	1	
1082	1		297	1		498	1		972	1		559	1		774	1		1273	1	
1136	1		253	1		745	1		905	1		39	1		863	1		541	1	
1114	2		243	1		344	1		520	1		269	1		388	1		537	1	
1071	1		29	1		874	1		987	2		1017	1		607	1		1157	1	
1105	1		871	1		210	1		779	1		412	1		240	1		178	1	
1168	1		430	1		785	1		775	1		556	1		602	1		723	1	
1133	1		677	1		944	1		972	1		640	1		772	1		723	1	
1248	2		641	1		1235	1		623	2		572	1		584	1		416	1	
1172	1		1150	1		159	1		1199	2		665	1		466	1		216	1	
1173	1		429	1		1028	1		949	1		260	1		908	1		675	1	
1090	1		849	1		284	1		779	1		241	1		904	1		155	1	
1130	1		265	1		843	1		775	1		712	1		478	1		954	1	

BLK CLASS	DESIGN	N	CLASS	DESIGN	N	CLASS	DESIGN	N	CLASS	DESIGN	N	CLASS	DESIGN	N	CLASS	DESIGN	N	CLASS	DESIGN	N
50	DESIGN		51	DESIGN		52	DESIGN		53	DESIGN		54	DESIGN		55	DESIGN		56	DESIGN	
360	1		546	1		842	1		402	1		356	1		163	1		636	1	
19	1		211	1		943	1		3	1		887	1		886	1		149	1	
391	1		218	1		612	1		34	1		1224	1		246	1		475	1	
206	1		79	1		208	1		441	1		877	1		855	1		769	1	
67	1		1024	1		184	1		733	1		890	1		860	1		48	1	
549	1		98	1		566	1		422	1		123	1		167	1		481	1	
662	1		1280	1		453	1		1055	1		122	1		554	1		823	1	
24	1		183	1		500	1		389	1		89	1		1023	1		381	1	
361	1		125	1		721	1		1054	1		1050	1		170	1		990	1	
191	1		230	1		532	1		445	1		349	1		458	1		493	1	
706	1		399	1		1027	1		161	1		783	1		938	1		595	1	
77	1		1270	1		203	1		393	1		551	1		131	1		780	1	
737	1		873	1		1277	1		300	1		993	1		896	1		776	1	
658	1		800	1		563	1		16	1		796	1		1223	1		906	1	
254	1		275	1		232	1		437	1		239	1		372	1		490	1	
207	1		351	1		448	1		406	1		741	1		1026	1		118	2	
18	1		233	1		744	1		318	1		370	1		916	1		708	1	
298	1		869	1		866	1		678	1		765	1		799	1		815	1	
19	1		96	1		509	1		235	1		794	1		365	1		461	1	
317	1		730	1		70	1		759	1		346	1		788	1		792	1	
950	1		471	1		46	1		197	1		374	1		379	1		605	1	
358	1		473	1		105	1		12	1		1048	1		850	1		143	1	
415	1		42	1		834	1		26	1		128	1		132	1		47	1	
757	1		511	1		342	1		261	1		1230	1		1061	1		102	1	
190	1		112	1		433	1		117	1		212	1		145	1		825	1	
528	1																			

	DESIGN	DESIGN	DESIGN	DESIGN	DESIGN	DESIGN	DESIGN	DESIGN
	71	72	73	74	75	76	77	
BLK CLASS	N	CLASS	CLASS	CLASS	CLASS	CLASS	CLASS	CLASS
1	65	835	451	71	213	480	5	N
2	663	69	467	71	731	151	439	1
3	267	135	176	858	137	195	832	1
4	786	752	142	425	137	659	767	1
5	452	734	142	530	352	480	439	1
6	41	752	417	417	80	659	354	1
7	519	734	760	530	793	363	121	1
8	382	835	760	256	80	518	121	1
9	656	69	975	71	974	518	121	1
10	384	538	142	768	137	518	1275	3
11	63	514	467	425	793	582	30	1
12	484	538	176	768	352	516	832	1
13	228	514	368	425	974	151	439	1
14	844	135	368	858	992	363	767	1
15	457	259	467	768	80	195	5	1
16	95	69	451	256	731	582	2	1
17	710	911	368	705	974	363	2	1
18	704	538	545	37	731	165	30	1
19	251	259	545	37	213	480	5	1
20	244	734	740	858	732	151	30	1
21	604	752	451	417	213	165	354	1
22	144	259	740	37	732	582	2	1
23	221	135	176	530	352	659	767	1
24	99	835	760	256	793	195	354	1
25	529	514	740	417	732	165	832	1

DESIGN

78

BLK CLASS	N
1	686
2	655
3	136
4	449
5	136
6	655
7	711
8	449
9	470
10	383
11	686
12	470
13	383
14	75
15	345
16	655
17	75
18	136
19	345
20	470
21	686
22	345
23	383
24	449
25	75

3. Derived Designs.

Denniston [1] states that there are six non-isomorphic (9, 24, 8, 3, 2)-BIBD's which are not derived from the (25, 25, 9, 9, 3)-BIBD. However, he didn't indicate which ones they were. Mathon [3] indicates it would be of interest to know which six of the (9, 24, 8, 3, 2)-designs, listed in Mathon and Rosa [4], are not derived. We refer to the numbering system used by Morgan [5] and extended by Mathon and Rosa [4].

Theorem 2. Designs 13, 21, 22, 34, 35 and 36 are not derived.

Proof. Each of these designs contain four blocks isomorphic to $B_1 = \{1, 2, 3\}$
 $B_2 = \{1, 2, 4\}$
 $B_3 = \{1, 3, 4\}$
 $B_4 = \{2, 3, 4\}$.

If a (9, 24, 8, 3, 2)-BIBD containing B_1, B_2, B_3 and B_4 based on $V = \{1, 2, \dots, 9\}$ is imbedded in a (25, 25, 9, 9, 3)-BIBD then there must be at least 18 other varieties in the extensions of B_1, B_2, B_3 and B_4 to ensure that the extensions of those blocks intersect in exactly three varieties. This is a contradiction as $9 + 18$ is not less than or equal to 25. \square

REFERENCES

- [1] R.H.F. Denniston, **Enumeration of Symmetric Designs (25, 9, 3)**, Annals of Discrete Math. 15 (1982), 111-127.
- [2] R. Mathon, private communication.
- [3] R. Mathon and A. Rosa, **Tables of Parameters of BIBD's with $r \leq 41$ Including Existence, Enumeration and Resolvability Results**, Annals of Discrete Math. 265 (1985), 275-308.
- [4] R. Mathon and A. Rosa, **A Census of Mendelsohn Triple Systems of Order 9**, Ars Combinatoria 4 (1977), 309-315.
- [5] E.J. Morgan, **Some Small Quasi-Multiple Designs**, Ars Combinatoria 3 (1977), 233-250.