The codomatic number of a cubic graph

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ABSTRACT. The maximum cardinality of a partition of the vertex set of a graph G into dominating sets is the *domatic number* of G, denoted $d(G)$. The *codomatic number* of G is the domatic number of its complement, written $d(\overline{G})$. We show that the codomatic number for any cubic graph G of order n is $n/2$, unless $G \in \{K_4, G_1\}$ where G_1 is obtained from $K_{2,3} \cup K_3$ by adding the edges of a 1-factor between K_3 and the larger partite set of $K_{2,3}$.

1 Introduction

In a graph $G = (V, E)$ the open neighborhood of a vertex $v \in V$ is $N(v) = \{x \in V \mid vx \in E\}$, the set of vertices adjacent to v. The closed neighborhood is $N[v] = N(v) \cup \{v\}$. A set $S \subset V$ is a *dominating set* if every vertex in V is either in S or is adjacent to a vertex in S, that is, $V = \bigcup_{s \in S} N[s]$.

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The *domination number* $\gamma(G)$ is the minimum cardinality of a dominating set. A *domatic partition* is a partition of V into dominating sets and the *domatic number* $d(G)$ is the largest number of sets in a domatic partition [3]. The *codomatic number* of G is the domatic number of its complement, written $d(\overline{G})$. It follows from the definition that $\gamma(G) \cdot d(G) \leq n$. For every graph G, it is evident that $d(G) \leq \delta(G) + 1$ as first observed by Cockayne and Hedetniemi [3]. The domatic number of a graph has been extensively studied, see for example [2, 3, 5, 10, 12].

Results on domination in cubic graphs have been presented in [1, 4, 7, 8, 9, 13, 14], and elsewhere. In this paper, we show that the codomatic number for any cubic graph G of order n is either $n/2 - 1$ or $n/2$. We then characterize those cubic graphs for which $d(\overline{G}) = n/2 - 1$.

2 Possible values for $d(\overline{G})$

In this section, we prove that the only possible values for $d(\overline{G})$, where G is a cubic graph of order n, are $n/2 - 1$ or $n/2$. A graph G is *F-free* if G contains no induced subgraph isomorphic to a graph F. We begin with the following observation from [5].

Observation 1 *If G is a cubic graph, then either $G \cong K_4$ or $\gamma(\overline{G}) = 2$.*

Theorem 2 *If G is a cubic graph of order n, then $d(\overline{G}) = n/2$ or $d(\overline{G}) = n/2 - 1$.*

Proof: Let G be a cubic graph of order n. Then n is even. If $G \cong K_4$, then its codomatic number is $1 = n/2 - 1$, so we may assume $n \geq 6$. We know that $d(\overline{G}) \leq n/\gamma(\overline{G}) = n/2$. We need only show that $d(\overline{G}) \geq n/2 - 1$. Let ℓ be the maximum number of disjoint 2-element dominating sets of $\overline{G}$. By Observation 1, we know that $\ell \geq 1$. Let $D = \{D_1, \dots, D_\ell\}$ be a set of ℓ disjoint 2-element dominating sets of $\overline{G}$. We will show that $\ell \geq n/2 - 1$.

Assume, to the contrary, that $\ell \leq n/2 - 2$. We now partition V into two sets S and T, such that $S = \cup_{i=1}^{\ell} D_i$ and $T = V - S$. We will refer to two vertices of S as *partners* if they belong to the same set D_i for some i, $1 \leq i \leq \ell$. Furthermore, for each vertex v in S, we will denote the partner of v by $\overline{v}$. Since $|S| = 2\ell \leq n - 4$, $|T| \geq 4$. Before proceeding further, we prove a series of claims. Unless otherwise stated, all adjacencies refer to adjacencies in G. Let $\langle S \rangle$ denote the subgraph induced by the set S.

Claim 1 *Every two vertices of T have a common neighbor in G.*

Proof: If two vertices x and y of T have no common neighbor, then $\{x, y\}$ is a dominating set of $\overline{G}$ and can therefore be added to D to produce $\ell + 1$ disjoint 2-element dominating sets of $\overline{G}$, a contradiction. Hence every two vertices of T have a common neighbor in G. □

Claim 2 *For any $u \in S$ and $x, y \in T$, $\{u, x\}$ and $\{\overline{u}, y\}$ do not both dominate $\overline{G}$.*

Proof: If both $\{u, x\}$ and $\{\overline{u}, y\}$ dominate $\overline{G}$, then $(D - \{u, \overline{u}\}) \cup \{\{u, x\}, \{\overline{u}, y\}\}$ is a set of $\ell+1$ disjoint 2-element dominating sets of $\overline{G}$, contradicting our choice of ℓ. □

Claim 3 *$\langle T \rangle$ is $(K_4 - e)$-free.*

Proof: Assume, to the contrary, that T contains four vertices x, y, w, z that induce a $K_4 - e$. We may assume that x and w are nonadjacent. Let $D_1 = \{u, \overline{u}\}$. If neither u nor $\overline{u}$ has a neighbor in $\{x, w\}$, then both $\{u, y\}$ and $\{\overline{u}, z\}$ dominate $\overline{G}$, contradicting Claim 2. Hence we may assume that u and x are adjacent. But then u and x have no common neighbor in G, so $\{u, x\}$ is a dominating set in $\overline{G}$. If $\overline{u}$ and w are not adjacent, then $\{\overline{u}, y\}$ is a dominating set in $\overline{G}$. On the other hand, if $\overline{u}$ and w are adjacent, then $\{\overline{u}, w\}$ is a dominating set in $\overline{G}$. In both cases we contradict Claim 2. We deduce, therefore, that $\langle T \rangle$ is $(K_4 - e)$-free. □

Claim 4 *$\langle T \rangle$ is K_3-free.*

Proof: Assume, to the contrary, that T contains three pairwise adjacent vertices x, y and z. Let $w \in T - \{x, y, z\}$. Suppose that xw is an edge. By Claim 3, w cannot be adjacent to y or z. But then x and w have no common neighbor in G, contradicting Claim 1. Hence each of x, y and z must be adjacent to a vertex of S.

We show next that every vertex of S is adjacent in G to at most one of x, y and z. If this is not the case, then we may assume ux and uy are edges of G, for a vertex $u \in S$. By Claim 1, x and w have a common neighbor which can only be u. Thus u is adjacent to only x, y and w, and hence, $\{u, w\}$ is a dominating set of $\overline{G}$. We now consider the partner $\overline{u}$ of u. Either $\overline{u}$ and z are not adjacent, in which case $\{\overline{u}, x\}$ is a dominating set of $\overline{G}$, or $\overline{u}$ and z are adjacent, in which case $\{\overline{u}, z\}$ is a dominating set of $\overline{G}$. In any event, we contradict Claim 2. Hence every vertex of S is adjacent to at most one of x, y and z.

By Claim 1, w has a common neighbor in S with each of x, y and z. Furthermore, we have shown that these common neighbors are distinct. Let x_1, y_1 and z_1 be the common neighbors of w with x, y and z, respectively. Thus $N(w) = \{x_1, y_1, z_1\}$. Since each set $D_i \in D$ dominates $\overline{G}$, x_1 cannot be a partner with y_1 or z_1, and y_1 and z_1 cannot be partners.

Since G is a cubic graph, the subgraph induced by x_1, y_1, z_1 contains at most one edge. We may assume that x_1 is not adjacent to y_1 or z_1. Thus x_1 and w have no common neighbor, so $\{x_1, w\}$ dominates $\overline{G}$. If $\overline{x_1}$ is not adjacent to x_1, then $\overline{x_1}$ and x have no common neighbor, whence $\{x, \overline{x_1}\}$ dominates $\overline{G}$. If $\overline{x_1}$ is not adjacent to y_1 (z_1), then $\overline{x_1}$ and y (respectively, z) have no common neighbor, whence $\{y, \overline{x_1}\}$ (respectively, $\{z, \overline{x_1}\}$) dominates $\overline{G}$. Hence if $\overline{x_1}$ is not adjacent to one of x_1, y_1, z_1, then we contradict Claim 2. Hence we may assume that $\overline{x_1}$ is adjacent to each of x_1, y_1, z_1. But then y_1 and w have no common neighbor, so $\{y_1, w\}$ dominates $\overline{G}$, while the partner $\overline{y_1}$ of y_1 and y have no common neighbor, so $\{y, \overline{y_1}\}$ dominates $\overline{G}$, contradicting Claim 2. We deduce, therefore, that $\langle T \rangle$ is K_3-free. □

Claim 5 *$\langle T \rangle$ is $K_{1,2}$-free.*

Proof: Assume, to the contrary, that T contains three vertices x, y, z that induce a $K_{1,2}$. We may assume that x and y are nonadjacent. By Claim 1, x and z have a common neighbor u, say. By Claim 4, we know that u belongs to S. Thus z is adjacent to only x, y, u. Furthermore, y and z have a common neighbor which must necessarily be u. Thus, u is adjacent only to x, y, z. Let $w \in T - \{x, y, z\}$. By Claim 1, w and z have a common neighbor which must be either x or y. We may assume that wx is an edge. But then w and x have no common neighbor which contradicts Claim 1. Hence $\langle T \rangle$ is $K_{1,2}$-free. □

Claim 6 *T is independent.*

Proof: Assume, to the contrary, that T contains adjacent vertices x and y. By Claims 4 and 5, we know that $\langle T \rangle$ has maximum degree 1. By Claim 1, x and y have a common neighbor a, say, which we know belongs to S. Again let $w, z \in T - \{x, y\}$. Since G is cubic, we may assume that aw is not an edge. By Claim 1, x and w have a common neighbor b, say, which must belong to S. Thus $N(x) = \{a, b, y\}$.

We show that by cannot be an edge. Suppose to the contrary that $by \in E(G)$. Then $N(b) = \{x, y, w\}$ and $N(y) = \{a, b, x\}$. By Claim 1, x and z have a common neighbor which can only be a. Hence $N(a) = \{x, y, z\}$. Since a and b have a common neighbor, they cannot be partners. But now both $\{b, w\}$ and $\{\overline{b}, x\}$ dominate $\overline{G}$, contradicting Claim 2. Hence $by \notin E(G)$.

By Claim 1, w and y have a common neighbor c, say, which we know belongs to S and is distinct from a and b. Thus $N(y) = \{a, x, c\}$. We now consider the vertex z.

We show that $az \notin E(G)$. If this is not the case, then az is an edge and $N(a) = \{x, y, z\}$. Thus, $\{a, z\}$ dominates $\overline{G}$. If $\overline{a}$ is not adjacent to b, then $\{\overline{a}, x\}$ dominates $\overline{G}$, while if $\overline{a}$ is not adjacent to c, then $\{\overline{a}, y\}$ dominates $\overline{G}$. Hence if $\overline{a}$ is not adjacent to b or c, then we contradict Claim 2. Thus, $\overline{a}$ is adjacent to both b and c. Hence b is adjacent only to $x, w, \overline{a}$ and c

is adjacent only to $y, w, \overline{a}$. But now both $\{c, y\}$ and $\{\overline{c}, x\}$ dominate $\overline{G}$, contradicting Claim 2. Thus $az \notin E(G)$.

By Claim 1, x and z have a common neighbor which can only be b, and y and z have a common neighbor which can only be c. Thus, $N(b) = \{x, w, z\}$ and $N(c) = \{y, w, z\}$. In particular, we note that $\{b, x\}$ dominates $\overline{G}$. If a and $\overline{b}$ are not adjacent, then $\{\overline{b}, y\}$ dominates $\overline{G}$, contradicting Claim 2. Hence $a\overline{b}$ must be an edge. Thus a is adjacent to only $x, y, \overline{b}$. But now both $\{c, y\}$ and $\{\overline{c}, x\}$ dominate $\overline{G}$, contradicting Claim 2. Hence the vertices of T must be pairwise nonadjacent. □

By Claim 6, we know that T is an independent set. Let $\{w, x, y, z\} \subseteq T$.

Claim 7 *$N(t)$ is independent for every $t \in T$.*

Proof: For $x \in T$, let $N(x) = \{a, b, c\}$ and assume to the contrary that $ab \in E(G)$. Now x must have a common neighbor with each $t \in T$. Since G is cubic, c is adjacent to at most two additional vertices from T. Hence at least one of y, w, and z must be adjacent to a or b and at least one must be adjacent to c. Without loss of generality, we may assume that y is adjacent to a and z is adjacent to c. Furthermore, b is adjacent to at most one of w and z. Thus, either $\{a, w\}$ or $\{a, z\}$ dominates $\overline{G}$. If $\overline{a}$ and x have no common neighbor, then $\{\overline{a}, x\}$ also dominates $\overline{G}$, a contradiction. Hence $\overline{a}$ is adjacent to b or c. But $\overline{a}b \notin E(G)$ since a and $\overline{a}$ have no common neighbors. Hence $\overline{a}c \in E(G)$ and since w and x must have a common neighbor we have $wb \in E(G)$. But now $\{\overline{b}, x\}$ and $\{b, z\}$ dominate $\overline{G}$, contradicting Claim 2. Hence $N(t)$ is independent for each $t \in T$. □

Claim 8 *At least one vertex of S is adjacent to three vertices of T.*

Proof: Assume, to the contrary, that every vertex of S is adjacent to at most two vertices of T. By Claim 1, every two vertices of T have a common neighbor. Hence there must exist six distinct vertices a, b, c, d, e, f of S such that a is the common neighbor of x and y, b the common neighbor of x and w, c the common neighbor of x and z, d the common neighbor of y and w, e the common neighbor of y and z, and f the common neighbor of w and z.

From Claim 7, $N(t)$ is independent for each $t \in T$. We show next that $N(T)$ is independent. If this is not the case, then we may assume that af is an edge. Thus, a is adjacent to only x, y, f and f is adjacent to only w, z, a. If $\overline{a}$ and x have no common neighbor, then both $\{a, y\}$ and $\{\overline{a}, x\}$ dominate $\overline{G}$, contradicting Claim 2. Hence $\overline{a}$ must be adjacent to b or c (implying that $\overline{a} \neq f$). Similarly, $\overline{f}$ must be adjacent to b or c (otherwise both $\{f, z\}$ and $\{\overline{f}, x\}$ dominate $\overline{G}$, a contradiction). Suppose $\overline{a}$ is adjacent to b. Then $\overline{f}$ must be adjacent to c. But then both $\{b, w\}$ and $\{\overline{b}, x\}$ dominate $\overline{G}$, again contradicting Claim 2. On the other hand, if $\overline{a}$ is adjacent to c, then $\overline{f}$ must be adjacent to b, and once again both $\{b, w\}$ and $\{\overline{b}, x\}$ dominate $\overline{G}$, a contradiction. Hence $N(T)$ is independent.

Since T and $N(T)$ are both independent, $\{t, s\}$ dominates $\overline{G}$ for each $t \in T$ and $s \in N(T)$. Thus if $\overline{a}$ and x have no common neighbor, then $\{\overline{a}, x\}$ and $\{a, z\}$ dominate $\overline{G}$. Note that this implies that $\overline{a} \notin N(T)$. Similarly, x and each of $\overline{b}$, $\overline{c}$, $\overline{d}$, $\overline{e}$, and $\overline{f}$ must share a common neighbor. But at most three of these vertices can share a common neighbor with x, a contradiction. We deduce, therefore, that at least one vertex of S is adjacent to three vertices of T. □

By Claim 8, at least one vertex of S is adjacent to three vertices of T. Let $a \in S$ have all its neighbors in T, say $N(a) = \{x, y, w\}$. Using Claim 6 we see that $\{a, t\}$ dominates $\overline{G}$ for all $t \in T$. Thus $\overline{a}$ and t must have a common neighbor for all $t \in T$, for otherwise we contradict Claim 2.

By Claim 1, z has a common neighbor with each of x, y and w. Let b, c, d be a common neighbor of z with x, y and w, respectively.

Claim 9 *The vertices b, c, d are distinct.*

Proof: Consider first the vertex b. We show that b cannot be adjacent to y or w. If this is not the case, then we may assume that by is an edge. So $N(b) = \{x, y, z\}$. If $\overline{a}$ and x do not have a common neighbor, then $\{\overline{a}, x\}$ and $\{a, y\}$ both dominate $\overline{G}$, contradicting Claim 2. Hence $\overline{a}$ and x must have a common neighbor, u say. Similarly, $\overline{b}$ and x must have a common neighbor which must be u. Thus u is adjacent to only $\overline{a}, \overline{b}, x$. Furthermore, $\overline{a}$ and y must have a common neighbor, v say, which must also be adjacent to $\overline{b}$. Hence $N(v) = \{\overline{a}, \overline{b}, y\}$. But now both $\{u, x\}$ and $\{\overline{u}, y\}$ dominate $\overline{G}$, contradicting Claim 2. Hence b is not adjacent to y or w. Similarly, we may show that c cannot be adjacent to x or w and that d cannot be adjacent to x or y. Hence b, c, d are distinct vertices. □

By Claim 9, we know that b, c, d are all distinct vertices. Thus, $N(z) = \{b, c, d\}$. By Claim 7, the set $\{b, c, d\}$ is independent. Note that $\overline{a} \notin \{b, c, d\}$.

We observed earlier that $\overline{a}$ and t must have a common neighbor for all $t \in T$. In particular, $\overline{a}$ and z must have a common neighbor. We may assume that $\overline{a}b$ is an edge. Thus, $N(b) = \{x, z, \overline{a}\}$. We now consider the vertex $\overline{b}$. Since $\overline{b}$ has no common neighbor with b, $\overline{b} \notin \{a, c, d\}$. If $\overline{b}$ and x do not have a common neighbor, say e, then both $\{b, z\}$ and $\{\overline{b}, x\}$ dominate $\overline{G}$, contradicting Claim 2. Similarly, since both $\{c, z\}$ and $\{d, z\}$ dominate $\overline{G}$, each of $\overline{c}$ and $\overline{d}$ must share a common neighbor with x implying that $\{\overline{c}e, \overline{d}e\} \subset E(G)$, contradicting the fact that G is cubic.

We deduce, therefore, that our initial assumption that $\ell \leq n/2 - 2$ must be false. Hence $\ell = n/2 - 1$ or $\ell = n/2$. If $\ell = n/2$, then $d(\overline{G}) = n/2$, while if $\ell = n/2 - 1$, then $d(\overline{G}) = n/2 - 1$. □

The lower bound is sharp as can be seen in Figure 1.

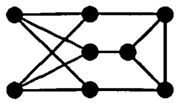

Figure 1: A graph G_1 satisfying $d(\overline{G_1}) = n/2 - 1$.

3 Characterization

In this section, we characterize those cubic graphs of order n with codomatic number $n/2 - 1$. We show firstly that the order n of such a cubic graph cannot be too large.

Lemma 3 *If $G = (V, E)$ is a cubic graph of order n satisfying $d(\overline{G}) = n/2 - 1$, then $n \leq 10$.*

Proof: Let G be a cubic graph with $d(\overline{G}) = n/2-1$. We follow the notation introduced in the proof of Theorem 2. By Theorem 2, we know that there exists a set $D = \{D_1, \ldots, D_\ell\}$ of $\ell = n/2-1$ disjoint 2-element dominating sets of $\overline{G}$. We can therefore partition V into the two sets S and T where $S = \cup_{i=1}^{\ell} D_i$ and $T = V - S$ (so $|T| = 2$). Let $T = \{x, y\}$.

Let $v \in S$. If v has a common neighbor with x but not y (respectively, with y but not x), then we call v a type-I vertex (respectively, type-II vertex). We call v a type-III vertex if v has a common neighbor with both x and y. We now consider the set $\{v, \overline{v}\} \in D$. If, without loss of generality, v has no common neighbor with x and $\overline{v}$ has no common neighbor with y, then $D' = (D - \{v, \overline{v}\}) \cup \{x, v\} \cup \{y, \overline{v}\}$ is a domatic partition of $\overline{G}$ of cardinality $n/2$. Hence each set $\{v, \overline{v}\} \in D$ must be exactly one of the following types:

Type 1. Both v and $\overline{v}$ are type-I vertices.

Type 2. Both v and $\overline{v}$ are type-II vertices.

Type 3. At least one of v and $\overline{v}$ is a type-III vertex.

Let t_1, t_2, and t_3 be the number of Type 1, Type 2, and Type 3 sets, respectively, in D, so $|D| = t_1 + t_2 + t_3$. We show that $t_1 + t_2 + t_3 \leq 5$. Since T does not dominate $\overline{G}$, x and y share a common neighbor in G. Therefore, since G is a cubic graph, $t_1 \leq 2$, $t_2 \leq 2$, and $t_3 \leq 5$. If $t_1 = 2$, then $t_2 \leq 2$ and $t_3 \leq 1$ implying that $|D| \leq 5$. Similarly, if $t_2 = 2$, the count is the same. Hence assume $t_1 \leq 1$ and $t_2 \leq 1$. If $t_1 = 1$, then $t_2 \leq 1$

and $t_3 \leq 3$ whence $|D| \leq 5$. Similarly, if $t_2 = 1$, then $|D| \leq 5$. Next assume $t_1 = t_2 = 0$. Then $t_3 \leq 5$ and so $|D| \leq 5$. Hence, $|D| = t_1 + t_2 + t_3 \leq 5$ always. Thus, $n = |S| + |T| = 2|D| + 2 \leq 12$. If $n = 12$, then one of the following is true: (1) $t_1 = t_2 = 2$, $t_3 = 1$; (2) $t_1 = t_2 = 1$, $t_3 = 3$; or (3) $t_1 = t_2 = 0$, $t_3 = 5$. A lengthy argument similar to the one in the proof to Theorem 2 yields a contradiction for each case. It follows that $n \leq 10$. □

We are finally in a position to characterize those cubic graphs G of order n for which $d(\overline{G}) = n/2 - 1$.

Theorem 4 *If G is a cubic graph of order n, then $d(\overline{G}) = n/2$, unless $G \in \{K_4, G_1\}$.*

Proof: The sufficiency is straightforward to check. To prove the necessity, let G be a cubic graph of order n satisfying $d(\overline{G}) = n/2 - 1$. Then G has even order and, by Lemma 3, $n \leq 10$. If $n = 4$, then $G \cong K_4$ whence $d(\overline{G}) = d(\overline{K_4}) = 1 = n/2 - 1$. Suppose, then, that $n \geq 6$. If $n = 6$, then $\overline{G}$ is a 2-regular graph, so $\overline{G} \cong C_6$ or $\overline{G} \cong K_3 \cup K_3$. However, in both cases $d(\overline{G}) = 3 = n/2$, a contradiction. Hence either $n = 8$ or $n = 10$. By inspection of the 27 (nonisomorphic) cubic graphs of order n for $8 \leq n \leq 10$ (which can be found in [11]), we find that each of these graphs except for the graph G_1 in Figure 1 has codomatic number equal to $n/2$. Hence if $n \geq 6$, then $G \cong G_1$. □

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