Minimum Distance Bounds for Linear Codes over GF(7)[1]

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Abstract

Let $[n,k,d;q]$-codes be linear codes of length n, dimension k and minimum Hamming distance d over $GF(q)$. Let $d_7(n,k)$ be the maximum possible minimum Hamming distance of a linear $[n,k,d;7]$-code for given values of n and k. In this paper, fifty-eight new linear codes over $GF(7)$ are constructed, the nonexistence of sixteen linear codes is proved and a table of $d_7(n,k)$ $k \leq 7$, $n \leq 100$ is presented.

1 Introduction

Let $GF(q)$ denote the Galois field of q elements, and let $V(n,q)$ denote the vector space of all ordered n-tuples over $GF(q)$. A linear code C of length n and dimension k over $GF(q)$ is a k-dimensional subspace of $V(n,q)$. Such a code is called an $[n,k,d;q]$-code if it has minimum Hamming distance d.

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A central problem in coding theory is that of optimizing one of the parameters n, k and d for given values of the other two. Two versions are:

Problem 1: Find $d_q(n, k)$, the largest value of d for which there exists an $[n, k, d; q]$-code.

Problem 2: Find $n_q(k, d)$, the smallest value of n for which there exists an $[n, k, d; q]$-code.

A code which achieves one of these two values is called optimal.

For the case of linear codes over $GF(7)$, Problem 2 has been solved for $k \leq 3$ (see [11]). In this paper we consider the next four dimensions. Fifty-eight new linear codes over $GF(7)$ are constructed, the nonexistence of sixteen linear codes is proved and a table of $d_7(n, k)$ $k \leq 7$, $n \leq 100$ is presented.

2 Preliminary results

A well-known lower bound for $n_q(k, d)$ is the Griesmer bound

$$n_q(k,d) \geq g_q(k,d) = \sum_{j=0}^{k-1} \lceil \frac{d}{q^j} \rceil$$

where $\lceil x \rceil$ denotes the smallest integer $\geq x$.

Theorem 1 [14] *(MacWilliams' identities) Let C be an $[n, k, d; 7]$-code and A_i and B_i denote the number of codewords of weight i in C and in its dual $C^\perp$, respectively. Then*

$$\sum_{i=0}^{n} K_t(i) A_i = 7^k B_t, \qquad \text{for } 0 \leq t \leq n$$

where

$$K_t(i) = \sum_{j=0}^{t} (-1)^j \binom{n-i}{t-j} \binom{i}{j} 6^{t-j}$$

are the Krawtchouk polynomials.

Lemma 1 [13] *For an $[n, k, d; 7]$-code, $B_i = 0$ for each value of $i, 1 \leq i \leq k$, such that there does not exist an $[n-i, k-i+1, d; 7]$-code.*

Definition Let C be an $[n, k, d; q]$-code with generator matrix G and let c be a row of G. The residual code of C with respect to c is the code generated by the restriction of G to the columns where c has a zero entry. The residual code of C with respect to c is denoted $Res(C, c)$ or $Res(C, w)$ if the weight of c is w ($wt(c) = w$).

Lemma 2 [5] *Let C be an $[n, k, d; 7]$-code with $c \in C$, $wt(c) = w$ and $w < d + \lceil \frac{w}{7} \rceil$. Then $Res(C, w)$ has parameters $[n - w, k - 1, d^\circ]$, where $d^\circ \geq d - w + \lceil \frac{w}{7} \rceil$.*

Lemma 3 [13] *Let C be an $[n, k, d; 7]$-code with $k \geq 2$. Then:*

a) $A_i = 0$ or 6 for $i > (7n - 6d)/2$

b) If $A_i = 6$, then $A_j = 0$ for $j + i > 7n - 6d$ and $i \neq j$.

Lemma 4 [13] *Let C be a $[g_7(k, d), k, d; 7]$-code. Then $B_1 = 0$ for all d and $B_i = 0$ if $1 < i < k + 1$ and if $d \leq 7^{k-i+1}$.*

Corollary 4.1 If $k = 4$, then $B_1 = 0$, $B_2 = 0$ for $d \leq 343$, $B_3 = 0$ for $d \leq 49$, $B_4 = 0$ for $d \leq 7$.

Let C be an $[n, k, d; q]$-code.

A *punctured code* of C is a code obtained by deleting a coordinate from every codeword of C.

A *shortened code* of C is a code obtained by taking only those codewords of C having a zero in a given coordinate position and then deleting that coordinate.

From these constructions, we have

a) $d_q(n + 1, k) \leq d_q(n, k) + 1$

b) $d_q(n + 1, k + 1) \leq d_q(n, k)$.

A code C is said to be quasi-cyclic (QC) if a cyclic shift of any codeword by p positions is also a codeword in C. A cyclic code is a QC code with $p = 1$. The length n of a QC code is a multiple of p, i.e., $n = mp$. With a suitable permutation of coordinates, many QC codes can be characterized in terms of $(m \times m)$ circulant matrices. In this case, a QC code can be transformed into an equivalent code with generator matrix

$$G = [R_0;\ R_1;\ R_2;\ \dots;\ R_{p-1}], \tag{1}$$

where $R_i, i = 0, 1, \dots, p-1$ is a circulant matrix of the form

$$R = \begin{bmatrix} r_0 & r_1 & r_2 & \cdots & r_{m-1} \\ r_{m-1} & r_0 & r_1 & \cdots & r_{m-2} \\ r_{m-2} & r_{m-1} & r_0 & \cdots & r_{m-3} \\ \vdots & \vdots & \vdots & & \vdots \\ r_1 & r_2 & r_3 & \cdots & r_0 \end{bmatrix}. \tag{2}$$

The algebra of $m \times m$ circulant matrices over $GF(q)$ is isomorphic to the algebra of polynomials in the ring $GF(q)[x]/(x^m - 1)$ if R is mapped onto the polynomial, $r(x) = r_0 + r_1x + r_2x^2 + \cdots + r_{m-1}x^{m-1}$, formed from the entries in the first row of R [14]. The $r_i(x)$ associated with a QC code are called the ***defining polynomials*** [7]. A number of QC codes over GF(5) were presented in [9].

If the defining polynomials $r_i(x)$ contain a common factor which is also a factor of $x^m - 1$, then the QC code is called *degenerate* [7]. Define the *order* of this QC code as [15]

$$h(x) = \frac{x^m - 1}{\gcd\{x^m - 1, r_0(x), r_1(x), \cdots, r_{p-1}(x)\}}. \tag{3}$$

The dimension of the QC code, k, is equal to the degree of $h(x)$. If $h(x)$ has degree m, the dimension of the code is m, and (1) is a generator matrix. If $\deg(h(x)) = k < m$, a generator matrix for the code can be constructed by deleting $m - k$ rows of (1).

In this paper, new QC codes are constructed using a nonexhaustive heuristic combinatorial search, similar to that in [8],[10],[4].

3 Bounds for $d_7(n, k)$.

A. Lower bounds.

Theorem 2 *There exist quasi-cyclic codes with parameters:*

$$[8,4,5;7], [13,4,9;7], [20,4,15;7], [25,4,19;7], [32,4,25;7],$$

$$[66,4,54;7],[75,4,62;7],[80,4,66;7],[90,4,75;7],[100,4,84;7].$$

Proof. The coefficients of the defining polynomials of these codes are as follows:

A [8,4,5;7]-code:

00011432;

A [13,4,9;7]-code:

0001110246245;

A [20,4,15;7]-code:

0112, 0001, 1162, 1214, 0155;

A [25,4,19;7]-code:

42116, 45426, 41144, 44346, 40433;

A [32,4,25;7]-code:

1346, 0113, 0166, 0103, 0011, 1523, 1113, 0163;

A [66,4,54;7]-code:

654643, 065021, 064434, 062652, 652161, 064236, 645634, 060602, 063246, 006512, 065325;

A [80,4,66;7]-code:

01066466, 14113531, 01324514, 01666103, 01652632, 01603344, 00156651, 01531634, 14355534, 01345226;

A [90,4,75;7]-code:

065513, 065632, 066654, 006104, 006055, 065632, 063252, 654353, 065464, 651413, 064204, 655662, 656446, 643356, 064442

A [100,4,84;7]-code:

6651311264, 0622601551, 0665501122, 6514212635, 6542412353, 0654501232, 0663301144, 0060100106, 6646211315, 0626301514;

Theorem 3 *There exist quasi-twisted codes with parameters:*

$$[50,4,42;7],[60,4,49;7],[75,4,62;7].$$

Proof. **A [50,4,42;7]-code:** ($\alpha = 6$)

11242, 12514, 01166, 12525, 11231, 11535, 12342, 11645, 11462, 11513;

A [60,4,49;7]-code: ($\alpha = 6$)

01415, 13515, 01353, 01111, 01331, 01441, 12364, 11143, 00165, 11561; 11612, 00143.

A [75,4,62;7]-code: ($\alpha = 6$)

11165, 01243, 12635, 01166, 01122, 01463, 01144, 01635, 01265, 11121, 01613, 12353, 01525, 01602, 00121;

Remark: For quasi-twisted codes see [12].

Theorem 4 *There exist quasi-cyclic codes with parameters:*

$$[18,5,12;7],[20,5,13;7],[25,5,17;7],[40,5,30;7],[45,5,34;7],$$

$$[50,5,38;7],[55,5,42;7],[64,5,50;7],[66,5,51;7],[72,5,56;7],$$

$$[80,5,63;7],[88,5,70;7],[90,5,71;7],[96,5,76;7],[100,5,79;7].$$

Proof. The coefficients of the defining polynomials of these codes are as follows:

A [18,5,12;7]-code:

111613, 112552, 116314;

A [20,5,13;7]-code:

00014, 11112, 00145, 00132;

A [25,5,17;7]-code:

00106, 01663, 00101, 12432, 01635;

A [32,5,23;7]-code:

01414523, 01234222, 01502065, 01026604;

A [40,5,30;7]-code:

01335642, 01554113, 00014036, 01226412, 01353524;

A [45,5,34;7]-code:

14256, 00124, 11426, 11566, 00113, 12513, 11123, 01241, 01324;

A [50,5,38;7]-code:

11114, 00153, 01155, 12136, 11415, 01631, 13215, 01133, 01132, 11653;

A [55,5,42;7]-code:

00162, 00014, 01322, 11266, 01133, 13143, 11652, 11564, 01514, 00163, 01031;

A [64,5,50;7]-code:

01522202, 01106336, 01051515, 01215566, 01643136, 15246121, 01261465, 12414142;

A [66,5,51;7]-code:

000106, 015543, 015532, 013552, 116523, 013464, 112123, 011352, 001133, 121462, 011403;

A [72,5,56;7]-code:

113454, 001602, 014344, 011502, 016126, 014234, 011462, 014311, 123453, 112156, 115236, 014542;

A [80,5,63;7]-code:

00010142, 01365221, 01423114, 00011623, 15241453, 01604266, 01653201, 01222525, 01363666, 00133331;

A [88,5,70;7]-code:

01633064, 01665605, 15635521, 01124211, 01213234, 15322666, 16441432, 15661213, 01013426, 01066325, 00136444;

A [90,5,71;7]-code:

01216, 00113, 01023, 01063, 01253, 11524, 00015, 01611, 12632, 01425, 01515, 01466, 01113, 11165, 12543, 14346, 12623, 12123;

A [96,5,76;7]-code:

15262441, 00100646, 01251323, 15123632, 01135134, 00122415, 15661213, 01445653, 15521623, 01353524, 11114224, 00136444

A [100,5,79;7]-code:

00153, 01253, 01124, 11625, 01241, 14343, 00145, 12123, 01531, 11343, 00106, 13132, 14216, 01226, 01254, 00115, 01555, 01525, 11345, 01055;

Theorem 5 *There exist quasi-cyclic codes with parameters:*

$$[18,6,11;7],[21,6,13;7],[24,6,15;7],[30,6,20;7],[36,6,25;7],$$

$$[54,6,39;7],[60,6,44;7],[66,6,49;7],[72,6,54;7],[78,6,59;7],$$
$$[84,6,63;7],[90,6,68;7],[96,6,73;7].$$

Proof. The coefficients of the defining polynomials of these codes are as follows:

A [18,6,11;7]-code:

000114, 111512, 132563;

A [21,6,13;7]-code:

0112163, 0001105, 0112424;

A [24,6,15;7]-code:

012464, 016125, 120622, 000001;

A [30,6,20;7]-code:

001403, 116213, 016125, 120622, 000001;

A [36,6,25;7]-code:

011446, 010154, 116213, 016125, 120622, 000001;

A [54,6,39;7]-code:

001021, 113352, 001635, 011446, 010154, 116213, 016125, 120622,000001;

A [60,6,44;7]-code:

010223, 010363, 113352, 001635, 011446, 010154, 116213, 016125, 120622, 000001;

A [66,6,49;7]-code:

015135, 010234, 010363, 113352, 001635, 011446, 010154, 116213, 016125, 120622, 000001;

A [72,6,54;7]-code:

001631, 111554, 010234, 010363, 113352, 001635, 011446, 010154, 116213, 016125, 120622, 000001;

A [78,6,59;7]-code:

001355, 142346, 111554, 010234, 010363, 113352, 001635, 011446, 010154, 116213, 016125, 120622, 000001;

A [84,6,63;7]-code:

015166, 001355, 142346, 111554, 010234, 010363, 113352, 001635, 011446, 010154, 116213, 016125, 120622, 000001;

A [90,6,68;7]-code:

001636, 016616, 001355, 142346, 111554, 010234, 010363, 113352, 001635, 011446, 010154, 116213, 016125, 120622, 000001;

A [96,6,73;7]-code:

001622, 001636, 016616, 001355, 142346, 111554, 010234, 010363, 113352, 001635, 011446, 010154, 116213, 016125, 120622, 000001;

Theorem 6 *There exist quasi-cyclic codes with parameters:*

$$[14,7,7;7],[21,7,12;7],[24,7,14;7],[28,7,17;7],[32,7,20;7],$$

$$[35,7,22;7],[40,7,26;7],[56,7,39;7],[63,7,44;7],[70,7,50;7],$$

$$[77,7,56;7],[84,7,61;7],[91,7,67;7],[98,7,73;7].$$

Proof. The coefficients of the defining polynomials of these codes are as follows:

A [14,7,7;7]-code:

0113652, 0000001;

A [21,7,12;7]-code:

0123642, 0114362, 0000001;

A [24,7,14;7]-code:

00163141, 00126346, 00000011;

A [28,7,17;7]-code:

0001664, 0123642, 0114362, 0000001;

A [32,7,20;7]-code:

00125554, 00114642, 00126346, 00000011;

A [35,7,22;7]-code:

0011224, 0115225, 0123642, 0114362, 0000001;

A [56,7,39;7]-code:

0011262, 0011363, 0102436, 0011224, 0115225, 0123642, 0114362, 0000001;

A [63,7,44;7]-code:

1111150, 0011262, 0011363, 0102436, 0011224, 0115225, 0123642, 0114362, 0000001;

A [70,7,50;7]-code:

0161614, 0011133, 0011262, 0011363, 0102436, 0011224, 0115225, 0123642, 0114362, 0000001;

A [77,7,56;7]-code:

1325653, 0161614, 0011133, 0011262, 0011363, 0102436, 0011224, 0115225, 0123642, 0114362, 0000001;

A [84,7,61;7]-code:

0012342, 1325653, 0161614, 0011133, 0011262, 0011363, 0102436, 0011224, 0115225, 0123642, 0114362, 0000001;

A [91,7,67;7]-code:

1116656, 1141645, 1325653, 0161614, 0011133, 0011262, 0011363, 0102436, 0011224, 0115225, 0123642, 0114362, 0000001;

A [98,7,73;7]-code:

1120662, 1116656, 1141645, 1325653, 0161614, 0011133, 0011262, 0011363, 0102436, 0011224, 0115225, 0123642, 0114362, 0000001;

B. Upper bounds.

Theorem 7 $d_7(98,4) = 82.$

Proof. Suppose there exists a $[g_7(4,83)$=98,4,83;7]-code C. By Lemma 1 and Table I or by Corollary 4.1 , $B_1 = B_2 = 0$. By Lemma 2 and Table I $A_i = 0$ for $i \in \{85,86,87,88,89,92,93,94,95\}$ The first three MacWilliams identities are:

$$\begin{aligned}
e_0: &\quad A_{83} + A_{84} + A_{90} + A_{91} + A_{96} + A_{97} + A_{98} = 2400 \\
e_1: &\quad 7.A_{83} - 42.A_{90} - 49.A_{91} - 84.A_{96} - 91.A_{97} - 98.A_{98} = -588 \\
e_2: &\quad -287.A_{83} - 294.A_{84} + 693.A_{90} + 1029.A_{91} + 3444.A_{96} + 4074.A_{97} \\
&\quad +4753.A_{98} = -171108.
\end{aligned}$$

Calculating the next linear combination

$$(42.e_0 - e_1/7 + e_2/7)/7,$$

we obtain

$$21.A_{90} + 28.A_{91} + 78.A_{96} + 91.A_{97} + 105.A_{98} = 10920.$$

It follows now that $A_{91} \leq 390$. By Lemma 3 $A_i \in \{0,6\}$ for $i = 96, 97, 98$. With the aid of a computer we have determined that no solution exists for the MacWilliams identities in non-negative integer multiples of 6.

By Theorem 2 there exist [100,4,84;7]-codes and so $d_7(98,4) = 82$.

Theorem 8 $d_7(83,4) \leq 69, \quad d_7(91,4) \leq 76.$

Proof. The proof of Theorem 8 is similar to the proof of Theorem 7.

Recently Dodunekov and Landjev [6] proved the nonexistence of $[16,4,12;7]$-codes.

The next codes do not exist via their residual codes:

$$[52,4,43;7], [45,5,36;7], [94,5,78;7], [31,7,22;7].$$

The next codes do not exist by the Linear Programming Bound (for this bound see [14], [1], [3]):

$$[60,5,49;7], [67,5,55;7], [74,5,61;7], [81,5,67;7], [88,5,73;7],$$

$$[81,6,66;7], [87,6,71;7], [94,6,77;7], [54,7,42;7], [61,7,48;7],$$

$$[67,7,53;7], [74,7,59;7], [81,7,65;7].$$

There exist [48,7,34;7] codes and there do not exist [9,4,6;7], [9,5,5;7], [31,5,24;7], [37,5,29;7], [9,6,4;7] and [9,7,3;7] codes. [2].

The results presented in this paper are compiled in Table I.

Table I. Bounds on Minimum Distance

Lower and Upper Bounds on $d_7(n,k)$ for Linear Codes over $GF(7)$

n,k	1	2	3	4	5	6	7
1	1						
2	2	1					
3	3	2	1				
4	4	3	2	1			
5	5	4	3	2	1		
6	6	5	4	3	2	1	
7	7	6	5	4	3	2	1
8	8	7	6	5	4	3	2
9	9	7	6	5	4	3	2
10	10	8	7	6	5	4	3
11	11	9	8	7	6	5	4
12	12	10	9	8	7	6	5
13	13	11	10	9	8	7	6
14	14	12	11	9-10	8-9	7-8	7
15	15	13	12	10-11	9-10	8-9	7-8
16	16	14	12	11	10-11	9-10	8-9
17	17	14	13	12	11	10-11	9-10
18	18	15	14	13	12	11	9-11
19	19	16	15	14	12-13	11-12	10-11
20	20	17	16	15	13-14	12-13	11-12
21	21	18	17	15-16	13-15	13-14	12-13
22	22	19	18	16-17	14-16	13-15	12-14
23	23	20	18	17-18	15-17	14-16	13-15
24	24	21	19	18	16-18	15-17	14-16
25	25	21	20	19	17-18	15-18	14-17
n,k	1	2	3	4	5	6	7

n,k	1	2	3	4	5	6	7
26	26	22	21	19-20	17-19	16-18	15-18
27	27	23	22	20-21	18-20	17-19	16-18
28	28	24	23	21-22	19-21	18-20	17-19
29	29	25	24	22-23	20-22	19-21	17-20
30	30	26	24	23-24	21-23	20-22	18-21
31	31	27	25	24	22-23	20-23	19-21
32	32	28	26	25	23-24	21-23	20-22
33	33	28	27	25-26	23-25	22-24	20-23
34	34	29	28	26-27	24-26	23-25	21-24
35	35	30	29	27-28	25-27	24-26	22-25
36	36	31	30	28-29	26-28	25-27	22-26
37	37	32	30	29-30	27-28	25-28	23-27
38	38	33	31	30	28-29	25-28	24-28
39	39	34	32	31	29-30	26-29	25-28
40	40	35	33	32	30-31	27-30	26-29
41	41	35	34	33	30-32	28-31	27-30
42	42	36	35	34	31-33	29-32	28-31
43	43	37	36	35	32-34	30-33	29-32
44	44	38	37	36	33-35	31-34	30-33
45	45	39	38	37	34-35	32-35	31-34
46	46	40	39	38	34-36	33-35	32-35
47	47	41	40	39	35-37	34-36	33-35
48	48	42	41	40	36-38	34-37	34-36
49	49	42	42	41	37-39	35-38	34-37
50	50	43	42	42	38-40	35-39	35-38
n,k	1	2	3	4	5	6	7

n,k	1	2	3	4	5	6	7
51	51	44	43	42	38-41	36-40	35-39
52	52	45	44	42	39-42	37-41	35-40
53	53	46	45	42-43	40-42	38-42	36-41
54	54	47	46	43-44	41-43	39-42	37-41
55	55	48	47	44-45	42-44	39-43	38-42
56	56	49	48	45-46	42-45	40-44	39-43
57	57	49	49	45-47	43-46	41-45	39-44
58	58	50	49	46-48	44-47	42-46	39-45
59	59	51	49	47-49	45-48	43-47	40-46
60	60	52	50	48-49	46-48	44-48	41-47
61	61	53	51	49-50	47-49	44-48	42-47
62	62	54	52	50-51	48-50	45-49	43-48
63	63	55	53	51-52	49-51	46-50	44-49
64	64	56	54	52-53	50-52	47-51	44-50
65	65	56	55	53-54	50-53	48-52	45-51
66	66	57	56	53-55	51-54	49-53	46-52
67	67	58	56	54-56	51-54	49-54	47-52
68	68	59	57	55-56	52-55	50-54	48-53
69	69	60	58	56-57	53-56	51-55	49-54
70	70	61	59	57-58	54-57	52-56	50-55
71	71	62	60	58-59	55-58	53-57	50-56
72	72	63	61	59-60	56-59	54-58	51-57
73	73	63	62	59-61	56-60	54-59	52-58
74	74	64	63	60-62	57-60	55-60	53-58
75	75	65	63	61-63	58-61	56-60	54-59
n,k	1	2	3	4	5	6	7

n,k	1	2	3	4	5	6	7
76	76	66	64	62-63	59-62	57-61	55-60
77	77	67	65	63-64	60-63	58-62	56-61
78	78	68	66	64-65	61-64	59-63	56-62
79	79	69	67	65-66	62-65	59-64	56-63
80	80	70	68	66-67	63-66	59-65	57-64
81	81	70	69	66-68	63-66	60-65	58-64
82	82	71	70	67-69	64-67	61-66	59-65
83	83	72	70	68-69	65-68	62-67	60-66
84	84	73	71	69-70	66-69	63-68	61-67
85	85	74	72	70-71	67-70	63-69	61-68
86	86	75	73	71-72	68-71	64-70	62-69
87	87	76	74	72-73	69-72	65-70	63-70
88	88	77	75	73-74	70-72	66-71	64-70
89	89	77	76	74-75	70-73	67-72	65-71
90	90	78	77	75-76	71-74	68-73	66-72
91	91	79	77	75-76	71-75	68-74	67-73
92	92	80	78	76-77	72-76	69-75	67-74
93	93	81	79	77-78	73-77	70-76	68-75
94	94	82	80	78-79	74-77	71-76	69-76
95	95	83	81	79-80	75-78	72-77	70-76
96	96	84	82	80-81	76-79	73-78	71-77
97	97	84	83	81-82	76-80	73-79	72-78
98	98	85	84	82	77-81	74-80	73-79
99	99	86	84	83	78-82	75-81	73-80
100	100	87	85	84	79-83	76-82	73-81
n,k	1	2	3	4	5	6	7

Remark: Recently a table for $d_7(n, k)$ has been constructed by Brouwer [2], but the bounds in this table are weak compared with the bounds given in this paper.

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