On Minimum Degree of Strongly k-Extendable Graphs

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Abstract. Let G be a simple connected graph on 2n vertices with a perfect matching. For a positive integer k, $1 \le k \le n - 1$, G is *k-extendable* if for every matching M of size k in G, there is a perfect matching in G containing all the edges of M. For an integer k, $0 \le k \le n - 2$, G is *strongly k-extendable* if G - {u, v} is k-extendable for every pair of vertices u and v of G. The problem that arises is that of characterizing k-extendable graphs and strongly k-extendable graphs. The first of these problems has been considered by several authors whilst the latter has only been recently studied by the author. In a recent paper, we established a number of properties of strongly k-extendable graphs including some sufficient conditions for strongly k-extendable graphs. In this paper, we focus on a necessary condition, in terms of minimum degree, for strongly k-extendable graphs. Further, we determine the set of realizable values for minimum degree of strongly k-extendable graphs. A complete characterization of strongly k-extendable graphs on 2n vertices for k = n - 2 and n - 3 is also established.

1. Introduction

All graphs considered in this paper are finite, connected, loopless and have no multiple edges. For the most part our notation and terminology follows that of Bondy and Murty [4]. Thus G is a graph with vertex set V(G), edge set E(G), $\nu(G)$ vertices, $\varepsilon(G)$ edges, minimum degree $\delta(G)$ and independence number $\alpha(G)$. For $V' \subseteq V(G)$, G[V′] denotes the subgraph induced by V′. Similarly G[E′] denotes the subgraph induced by the edge set E′ of G. $N_G(u)$ denotes the neighbour set of u in G and $\overline{N}_G(u)$ the non-neighbours of u. Note that $\overline{N}_G(u) = V(G) \setminus (N_G(u) \cup \{u\})$. The *join* $G \vee H$ of disjoint graphs G and H is the graph obtained from $G \cup H$ by joining each vertex of G to each vertex of H.

A *matching* M in G is a subset of E(G) in which no two edges have a vertex in common. M is a *maximum matching* if $|M| \ge |M'|$ for any other matching M′ in G. A vertex v is *saturated* by M if some edge of M is incident to v; otherwise, v is said to be *unsaturated.* A matching M is *perfect* if it saturates every vertex of the graph. For simplicity we let V(M) denote the vertex set of the subgraph G[M] induced by M.

Let G be a simple connected graph on 2n vertices with a perfect matching. For a given positive integer k, $1 \leq k \leq n - 1$, G is *k-extendable* if for every matching M of size k in G, there exists a perfect matching in G containing all the edges of M. For convenience, a graph with a perfect matching is said to be 0-extendable. For an integer k, $0 \leq k \leq n - 2$, we say that G is *strongly k-extendable* or simply *k*-extendable* if for every pair of vertices u and v of G, G - {u, v} is k-extendable. A graph G is *bicritical* if G - {u, v} has a perfect matching for every pair of vertices u and v. Clearly, 0*-extendable graphs are bicritical and a concept of k*-extendable graphs is a generalization of bicritical graphs.

Observe that the complete graph K_{2n} of order 2n is k*-extendable for all k, $0 \leq k \leq n - 2$ whilst the complete bipartite graph $K_{n,n}$ with bipartition (X, Y) is k-extendable, $0 \leq k \leq n - 2$, but not k*-extendable since a deletion of any two distinct vertices of X results in a graph $K_{n-2,n}$ which clearly has no perfect matching.

A number of authors have studied k-extendable graphs. Excellent surveys are the papers of Plummer [9, 10]. Lovász [5], Lovász and Plummer [6, 7] and Plummer [8] have studied k*-extendable graphs for k = 0 (bicritical graphs) whilst k*-extendable graphs for $k \geq 1$ have been recently studied by the author [3]. This paper is a continuation of work initiated in [3] which established a number of properties of k*-extendable graphs, including some sufficient conditions. Here we establish a necessary condition, in terms of minimum degree, for k*-extendable graphs. Further, a characterization of k*-extendable graphs on 2n vertices is given for k = n - 2 and n - 3.

Section 2 contains some preliminary results that we make use of in establishing our results. In Section 3, we establish a necessary condition, in terms of minimum degree, for k*-extendable graphs. Further, we determine the set of realizable values for minimum degree of k*-extendable graphs. A complete characterization of k*-extendable graphs on 2n vertices for k = n - 2 and n - 3 is given in Section 4.

2. Preliminaries

In this section we state a number of results which we make use of in our work. We begin with a fundamental result of k-extendable graphs proved by Plummer [8]:

Theorem 2.1: Let G be a k-extendable graph on 2n vertices, $1 \leq k \leq n - 1$. Then
- (i) G is (k - 1)-extendable;
- (ii) G is (k + 1)-connected. ❑

Our next three results are properties of k*-extendable graphs proved by Ananchuen [3].

Lemma 2.2 : If G is a k*-extendable graph on 2n vertices; $0 \le k \le n - 2$, then G is (k + 1)-extendable. ❑

Lemma 2.3: Let G be a graph on 2n vertices and $0 \le k \le n - 2$. If $\delta(G) \ge n + k + 1$, then G is k*-extendable. ❑

Theorem 2.4: Let G be a graph on 2n vertices with $\delta(G) = n + k$; $0 \le k \le n - 2$. If n - k - 1 is even and $\alpha(G) \le n - k - 1$, then G is k*-extendable. ❑

We conclude this section by stating a characterization of (n - 2)-extendable graphs on 2n vertices proved by Ananchuen and Caccetta [1].

Theorem 2.5: Let G be a graph on $2n \ge 10$ vertices with a perfect matching. Then G is (n - 2)-extendable if and only if G:

(i) is $K_{n,n}$ or K_{2n}, or
(ii) is a bipartite graph with minimum degree n - 1, or
(iii) has minimum degree 2n - 3 and $\alpha(G) \le 2$, or
(iv) has minimum degree 2n - 2. ❑

3. Minimum Degree of k*-Extendable Graphs

In this section we establish a necessary condition, in terms of the minimum degree, for k*-extendable graphs. We start with a following lemma:

Lemma 3.1: Let G be a k*-extendable graph on 2n vertices; $1 \le k \le n - 2$. Then G is (k + 3)-connected.

Proof: Since G – {u, v} is k-extendable for every pair of vertices u and v of G, G – {u, v} is (k + 1)–connected by Theorem 2.1. Thus G is (k + 3)-connected. ❑

Remark 3.1: Note that for any positive integer r, a graph $K_2 \vee 2K_{2r}$ is 0*-extendable which is 2-connected. Thus the bound on k in Lemma 3.1 is sharp. However, if G is 0*-extendable, then $\delta(G) \ge 3$ by the definition of 0*-extendable graphs. This fact together with Lemma 3.1 assures that if G is a k*-extendable graph on 2n vertices, $0 \le k \le n - 2$, then $\delta(G) \ge k + 3$.

Our next result concerns the size of a maximum matching in an induced subgraph of a neighbour set of a vertex in a k*-extendable graph.

Lemma 3.2: Let G be a k*-extendable graph on 2n vertices; $0 \le k \le n - 2$, and u a vertex of degree $k + t$; $3 \le t \le k + 2$, of G. Then $G[N_G(u)]$ has a matching of size at most $t - 3$.

Proof: Suppose to the contrary that there exists a vertex u of G of degree $k + t$; $3 \le t \le k + 2$ such that $G[N_G(u)]$ has a maximum matching of size at least $t - 2$.

Let M be a maximum matching in $G[N_G(u)]$ of size $s \ge t - 2$. Now $s < k$, since G is k*-extendable and $d_G(u) \le 2k + 2$. Further, $|N_G(u) \setminus V(M)| \ge 3$. Suppose $|\overline{N}_G(u)| = 1$. Then $d_G(u) = |N_G(u)| = 2n - 2$. Since $d_G(u) \le 2k + 2$ and the assumption on k, $k = n - 2$. Let $u' \in \overline{N}_G(u)$. Because G is k*-extendable, $G - \{u, u'\} = G[N_G(u)]$ contains a perfect matching. Thus $n - 1 = |M| = s < k = n - 2$, a contradiction. Hence, $|\overline{N}_G(u)| \ge 2$. Let x and y be vertices of $\overline{N}_G(u)$ and $G^* = G - \{x, y\}$. Clearly G^* is k-extendable. Further, $N_G(u) = N_{G^*}(u)$ and $G[N_G(u)] = G^*[N_{G^*}(u)]$.

Let F be a perfect matching in G^* containing M. Then there exists a vertex v of $N_G(u) \setminus V(M)$ such that $uv \in F$. Put

$$F_1 = \{ab \in F \mid a \in N_G(u) \setminus (V(M) \cup \{v\}), b \in \overline{N}_G(u)\}.$$

Since $|N_G(u) \setminus V(M)| \ge 3$, $|F_1| = k + t - 2s - 1 \ge 2$. Let $wz \in F_1$ where $w \in N_G(u) \setminus (V(M) \cup \{v\})$ and $z \in \overline{N}_G(u)$. Consider $F_2 = M \cup (F_1 \setminus \{wz\})$. Since $s \ge t - 2$,

$$|F_2| = s + (k + t - 2s - 1) - 1 = k + t - s - 2 \le k.$$

But then F_2 does not extend to a perfect matching in $G - \{v, w\}$ as u becomes an isolated vertex in $G - (\{v, w\} \cup V(F_2))$. This contradicts k*-extendability of G and completes the proof of our lemma. ❑

We now prove the main result in this section.

Theorem 3.3: If G is a k*-extendable graph on 2n vertices; $0 \le k \le n - 2$, then $k + 3 \le \delta(G) \le n - 2$ or $\delta(G) \ge 2k + 3$.

Proof: The assertion is true for $k = 0$ by Remark 3.1. Suppose to the contrary that G is a k*-extendable graph on 2n vertices, $1 \le k \le n - 2$, with $n - 1 \le \delta(G) \le 2k + 2$. Let u be a vertex of G with $d_G(u) = \delta(G) = r$ and M a maximum matching in $G[N_G(u)]$. By Lemma 3.2, $|M| \le r - k - 3 \le k - 1$ and $|N_G(u) \setminus V(M)| \ge r - 2(r - k - 3) = 2k - r + 6 \ge 4$.

By applying similar argument as in the proof of Lemma 3.2, $|\overline{N}_G(u)| \ge 2$. Let $x, y \in \overline{N}_G(u)$ and $G_1^* = G - \{x, y\}$. Since $|N_G(u) \setminus V(M)| \ge 4$, there is a vertex $v \in N_G(u) \setminus V(M)$. Because G is k*-extendable and $M \cup \{uv\}$ is a matching in G of size at most k, there is a perfect matching F in G_1^* containing $M \cup \{uv\}$. Let

$$F_1 = \{ab \in F \mid a \in N_G(u) \setminus (V(M) \cup \{v\}), b \in \overline{N}_G(u) \setminus \{x, y\}\}$$

and

$$F_2 = \{ab \in F \mid a, b \in \overline{N}_G(u) \setminus \{x, y\}\}.$$

Clearly, $|F_1| = r - 2|M| - 1$

and

$$|F_2| = \frac{1}{2}[(2n - r - 3) - (r - 2|M| - 1)]$$
$$= n - r + |M| - 1.$$

Suppose $G[\overline{N}_G(u)]$ contains M' as a matching of size $n - r + |M| \le n - k - 3 \le k$. Since

$$\begin{aligned}|\overline{N}_G(u) \setminus V(M')| &= (2n - r - 1) - 2(n - r + |M|)\\ &= r - 2|M| - 1\\ &\ge r - 2(r - k - 3) - 1\\ &= 2k - r + 5\\ &\ge 3,\end{aligned}$$

there exist vertices $x', y' \in \overline{N}_G(u) \setminus V(M')$. But then M' does not extend to a perfect matching in $G_2^* = G - \{x', y'\}$, since $G_2^*[N_G(u) \setminus V(M)] = G[N_G(u) \setminus V(M)]$ is an independent set of order $r - 2|M|$ and

$$\begin{aligned}|\overline{N}_{G_2^*}(u) \setminus (V(M') \cup \{x', y'\})| &= (2n - r - 1) - [2(n - r + |M|) + 2]\\ &= r - 2|M| - 3.\end{aligned}$$

Hence, the size of a maximum matching in $G[\overline{N}_G(u)]$ is $n - r + |M| - 1$.

Now let w, z be vertices of $N_G(u) \setminus (V(M) \cup \{v\})$ and $G_3^* = G - \{w, z\}$. Clearly, $M'' = M \cup \{uv\}$ is a matching of size at most k in G_3^*. Suppose that M'' extends to a perfect matching F' in G_3^*. Since $N_G(u) \setminus (V(M) \cup \{v, w, z\})$ is an independent set of order $r - 2|M| - 3$ and $\overline{N}_G(u)$ has a maximum matching of size $n - r + |M| - 1$, M'' has to map the $r - 2|M| - 3$ vertices of $N_G(u) \setminus (V(M) \cup \{v, w, z\})$ onto $(2n - r - 1) - 2(n - r + |M| - 1) = r - 2|M| + 1$ vertices of $\overline{N}_{G_3^*}(u) \setminus V(M''')$ where M''' is a maximum matching in $G[\overline{N}_G(u)]$, which is impossible. This contradicts the k^*-extendability of G and completes the proof of our theorem. ❑

Corollary 3.4: Let G be a k^*-extendable graph on $2n$ vertices; $0 \le k \le n - 2$. Then G is complete or $n \ge k + 3$.

Proof: Suppose $n \le k + 2$. By Theorem 3.3, $\delta(G) \ge 2k + 3$. This implies that $\nu(G) = 2k + 4$. Consequently, G is complete. ❑

Corollary 3.5: Let G be a k^*-extendable graph on $2n$ vertices with $\delta(G) \le 2k + 2$. Then $2n \ge 4k + 8$.

Proof: By Theorem 3.3, it follows that $2k + 2 \leq n - 2$. Thus $2n \geq 4k + 8$ as required. ❑

Next we consider the realizability problem associated with Theorem 3.3. We start with the following lemma.

Lemma 3.6: For any non-negative integers n, k, r with $2k + 3 \leq r \leq 2n - 1$, there exists a k*-extendable graph on 2n vertices with minimum degree r.

Proof: Let $G_1 = K_1$, $G_2 = K_r$ and $G_3 = K_{2n-r-1}$. Then $G = G_1 \vee G_2 \vee G_3$ is a graph on 2n vertices with minimum degree r. Figure 3.1 depicts the graph G. Note that in our diagrams a "double line" denotes the join.

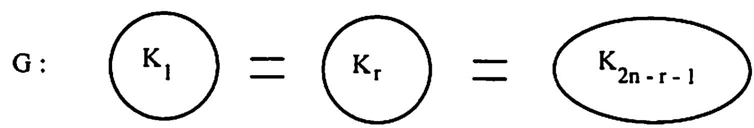

Figure 3.1

Let u and v be any pair of vertices of G and M a matching of size k in $G - \{u, v\}$. Put

$$A = \{u, v\} \cup V(M).$$

If $V(G_1) \subseteq A$, then $G - A = K_{2n-2k-2}$ has a perfect matching. Next we suppose that $V(G_1) \cap A = \emptyset$. Let $s = |A \cap V(G_2)|$. Then $G - A = K_1 \vee K_{r-s} \vee K_{2n+s-r-2k-3}$. Clearly, $0 \leq s \leq 2k + 2 < r$ and $2k + 2 - s \leq 2n - r - 1$. Thus $r - s \geq 1$ and $2n + s - r - 2k - 3 \geq 0$. Consequently, $G - A$ contains a perfect matching. Hence, G is k*-extendable as required. ❑

Lemma 3.7: For any positive integers n, k, r with $k + 3 \leq r \leq 2k + 2$ and $2n \geq 4k + 8$, there exists a k*-extendable graph on 2n vertices with minimum degree r.

Proof: Let $2n = 4k + 2s + 8$ for some integer $s \geq 0$. For integer t with $3 \leq t \leq k + 2$, let $G_1 = K_1$, $G_2 = \overline{K}_{t-3}$, $G_3 = \overline{K}_{k+3}$ and $G_4 = K_{3k-t+2s+7}$. Then $G = G_1 \vee (G_2 \vee G_3) \vee G_4$ is a graph of order 2n with minimum degree $k + t$. Figure 3.2 illustrates our notation.

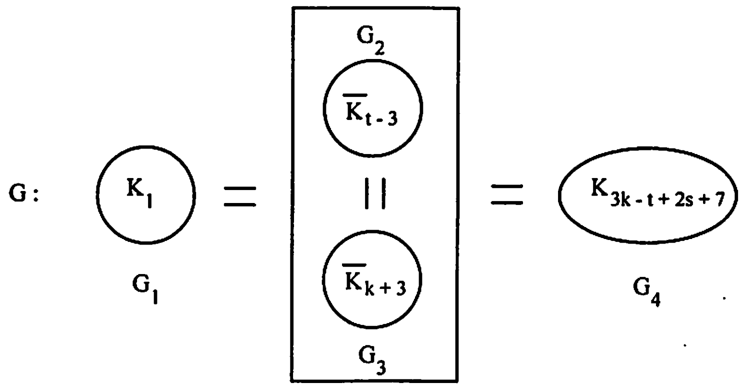

Figure 3.2

Observe that $G_2 \vee G_3$ contains a maximum matching of size $t - 3 \le k - 1$. We will show that G is k^*-extendable. Let $u, v \in V(G)$ and M a matching of size k in $G - \{u, v\}$. To complete the proof of our lemma we need to show that $G' = G - (V(M) \cup \{u, v\})$ contains a perfect matching. Let

$$A = V(M) \cup \{u, v\}$$
$$a_1 = |V(G_1) \cap A|$$
$$a_2 = |V(G_2) \cap A|$$
$$a_3 = |V(G_3) \cap A|$$

and
$$a_4 = |V(G_4) \cap A|.$$

Notice that $a_1 + a_2 + a_3 + a_4 = |A| = 2k + 2$ and $0 \le a_1 \le 1$. We distinguish two cases according to a_1.

Case 1: $a_1 = 1$.
Then $G' = (\overline{K}_{t-3-a_2} \vee \overline{K}_{k+3-a_3}) \vee K_{k+2s+6-t+a_2+a_3}$. Let M_1 be a maximum matching in $G'[(V(G_2) \cup V(G_3)) \setminus A]$. Then

$$|M_1| = \min\{t - 3 - a_2, k + 3 - a_3\}.$$

Consider $B = (V(G_2) \cup V(G_3)) \setminus (A \cup V(M_1))$. Clearly,

$$|B| = (k + t) - (a_2 + a_3 + 2|M_1|)$$
$$= \begin{cases} k - t + a_2 - a_3 + 6, \text{ for } |M_1| = t - 3 - a_2 \\ t - k - a_2 + a_3 - 6, \text{ for } |M_1| = k + 3 - a_3. \end{cases}$$

Since $t \le k + 2$,

$$|V(G_4) \setminus A| - |B|$$
$$= k + 2s + 6 - t + a_2 + a_3 - |B|$$

$$= \begin{cases} 2s + 2a_3 \ge 0, & \text{for } |M_1| = t - 3 - a_2 \\ 2k - 2t + 2s + 2a_2 + 12 \ge 8, & \text{for } |M_1| = k + 3 - a_3 \end{cases}$$

and then there is a matching M_2 which maps each vertex of B to a vertex of $V(G_4) \setminus A$. Clearly,

$$G'[V(G_4) \setminus (A \cup V(M_2))] = \begin{cases} K_{2s+2a_3}, & \text{for } |M_1| = t-3-a_2 \\ K_{2k-2t+2s+2a_2+12}, & \text{for } |M_1| = k+3-a_3 \end{cases}$$

contains a perfect matching M_3. Hence, $M_1 \cup M_2 \cup M_3$ forms a perfect matching in G'.

Case 2: $a_1 = 0$.
If $V(G_3) \setminus A = \varnothing$, then $|M| \geq (k+3) - 2 = k+1$, a contradiction. Thus $V(G_3) \setminus A \neq \varnothing$. Let $xy \in E(G)$ where $x \in V(G_1)$ and $y \in V(G_3) \setminus A$. Clearly, $G' - \{x, y\} = (\overline{K}_{t-3-a_2} \vee \overline{K}_{k+2-a_3}) \vee K_{k+2s+5-t+a_2+a_3}$. A similar argument as that used in Case 1 establishes that $G' - \{x, y\}$ contains F as a perfect matching in $G' - \{x, y\}$. Hence, $F \cup \{xy\}$ forms a perfect matching in G'. This completes the proof of our lemma. ❑

Let G be a k*-extendable graph on 2n vertices, $0 \leq k \leq n-2$, with minimum degree r. By Theorem 3.3 and Corollary 3.5, notice that

$$r \in \begin{cases} [k+3, 2n-1], & \text{for } n \geq 2k+4 \\ [2k+3, 2n-1], & \text{for } n \leq 2k+3. \end{cases} \tag{3.1}$$

Corollary 3.5 and Lemmas 3.6 and 3.7 yield the following theorem:

Theorem 3.8: For any integers n, k and r with $0 \leq k \leq n-2$, there exists a k*-extendable graph on 2n vertices with minimum degree r if r satisfies (3.1). ❑

4. A Characterization of (n - 2)*-Extendable and (n - 3)*-Extendable Graphs

We now turn our attention to a characterization of k*-extendable graphs on 2n vertices for k = n - 2 and n - 3. We begin with (n - 2)*-extendable graphs.

Theorem 4.1: G is an (n - 2)*-extendable graph on $2n \geq 4$ vertices if and only if G is K_{2n}.

Proof: It follows directly from Corollary 3.4 and the fact that K_{2n} is k*-extendable for $0 \leq k \leq n-2$. ❑

Our next result concerns the independence number of k*-extendable graphs which is useful for establishing a characterization of (n - 3)*-extendable graphs.

Lemma 4.2: Let G be a k^*-extendable graph on 2n vertices; $0 \le k \le n - 2$. Then $\alpha(G) \le n - k - 1$.

Proof: The case $k = n - 2$ is obvious since the only $(n-2)^*$-extendable graph is K_{2n}. So we only need to consider the case $0 \le k \le n - 3$. Suppose to the contrary that $\alpha(G) \ge n - k$. Let S be an independent set of vertices of G of order n - k. Further let $u \in S$ and $v \in N_G(u)$. Since G is k^*-extendable, there is a perfect matching F containing the edge uv. Let $F_1 = \{xy \in F \mid x \in S\}$. Then $|F \setminus F_1| = k$. Let zz′, ww′ be edges of F_1 with $z', w' \in S$. Then $G' = G - (V(F \setminus F_1) \cup \{z, w\})$ contains S as an independent set of order n - k. Since $\nu(G') = 2n - 2k - 2$, G′ has no perfect matching. This implies that $F \setminus F_1$ cannot extend to a perfect matching in $G - \{z, w\}$, a contradiction to the extendability of G. Hence, $\alpha(G) \le n - k - 1$, completing the proof of our lemma. ❑

Lemma 4.2 is best possible since there exists a k^*-extendable graph G with $\alpha(G) = n - k - 1$. Such a graph is $K_{2k+2} \vee (\overline{K}_{n-k-1} \vee \overline{K}_{n-k-1})$.

We now characterize $(n - 3)^*$-extendable graphs on 2n vertices.

Theorem 4.3: Let G be a graph on $2n \ge 6$ vertices. Then G is $(n - 3)^*$-extendable if and only if G:

(i) is K_{2n}, or

(ii) has minimum degree 2n - 2, or

(iii) has minimum degree 2n - 3 and $\alpha(G) \le 2$.

Proof: The necessity follows directly from Theorem 3.3 and Lemma 4.2. Now we prove the sufficiency. Clearly, K_{2n} is $(n - 3)^*$-extendable. If $\delta(G) = 2n - 2$, then, by Lemma 2.3, G is $(n - 3)^*$-extendable. The last case follows directly from Theorem 2.4. This completes the proof of our theorem. ❑

Remark 4.1: There exist $(n - 3)^*$-extendable graphs for each type specified in Theorem 4.3. Clearly, $2K_1 \vee K_{2n-2}$ satisfies type (ii) and $2K_2 \vee K_{2n-4}$ is of type (iii).

A consequence of Theorems 2.5 and 4.3 is the following theorem:

Theorem 4.4: Let G be a graph on $2n \ge 10$ vertices. Then G is $(n - 3)^*$-extendable if and only if G is (n - 2)-extendable non-bipartite . ❑

Let $\mathcal{G}(2n, k)$ and $\mathcal{G}^*(2n, k^*)$ denote the classes of k-extendable non-bipartite graphs and k^*-extendable graphs on 2n vertices, respectively. Theorem 4.4 assures that for $2n \ge 10$

$$\mathcal{G}(2n, n-2) = \mathcal{G}^*(2n, (n-3)^*).$$

The bound on the number of vertices of a graph in Theorem 4.4 is best possible since there exist (n - 2)-extendable non-bipartite graphs on 2n = 6 and 2n = 8 vertices which are not (n - 3)*-extendable. Such graphs are displayed in Figure 4.1

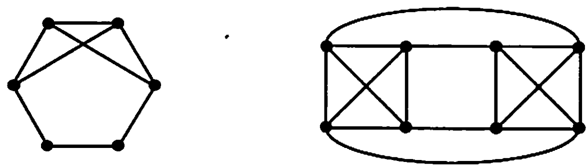

Figure 4.1

For 2n ≥ 6, Lemma 2.2 implies that $\mathcal{G}^*(2n, (n-3)^*) \subseteq \mathcal{G}(2n, n-2)$. The graphs in Figure 4.1 ensure that $\mathcal{G}^*(2n, (n-3)^*)$ is the proper subclass of $\mathcal{G}(2n, n-2)$. By take advantage of a characterization of (n - 2)-extendable graphs on 2n ≥ 6 vertices, proved by Ananchuen and Caccetta [1, 2], we can now state the following corollary.

Corollary 4.5: For 2n ≥ 6, $| \mathcal{G}(2n, n-2) \setminus \mathcal{G}^*(2n, (n-3)^* | = 11$. Such graphs are displayed in Figure 4.2.

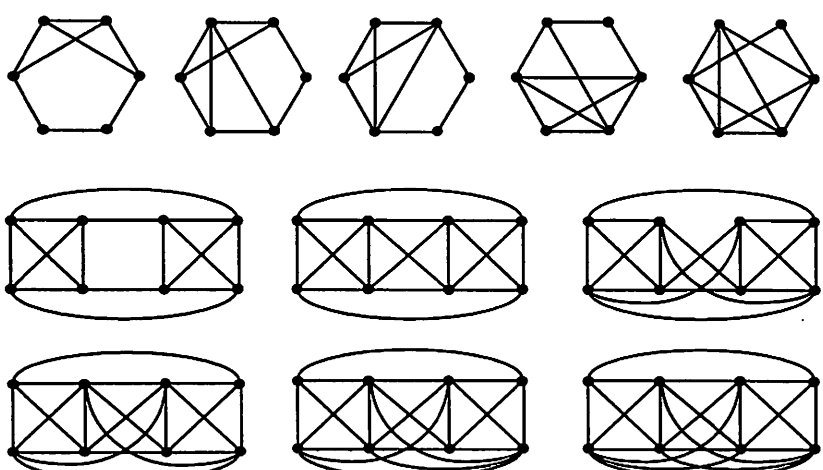

Figure 4.2 ❑

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