

# Nearly Kirkman Triple Systems of Order 18

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## Abstract

We obtain 135 nonisomorphic nearly Kirkman triple systems of order 18 (the smallest order for which such a system exists), including all 119 systems of a well-defined subclass.

## 1 Introduction

Nearly Kirkman triple systems (NKTS) were introduced in [7] as an analogue of Kirkman triple systems (cf. Chapter 19 of [5]). An NKTS of order  $v = 6n$  is a partition of the edges of the cocktail-party graph  $K_{6n} - F$  into  $3n - 1$  (edge-disjoint) triangle-factors. Equivalently, an NKTS( $6n$ ) is a resolvable maximum partial triple system (MPT) of order  $6n$ , or a resolvable 3-GDD of type  $2^{3n}$  (cf. [4], [5]). The existence problem for NKTS was settled in a series of papers [7], [1], [3], [9]: an NKTS( $6n$ ) exists if and only if  $n \geq 3$ .

Thus the smallest order for which an NKTS exists is 18. An example of an NKTS(18) was given in [7] (cf. also [5]). As far as we are able to tell, this is, up to an isomorphism, the only example of an NKTS(18) appearing in the literature thus far.

In this article, we make the first step towards a complete enumeration of NKTS(18). We introduce a classification of NKTS based on invariants discussed in [6] (see also Section 2 below), and obtain a complete enumeration of NKTS(18) of one – well-defined – subclass. On the other hand, we also show that there exist further examples of NKTS(18) not belonging to this subclass.

We were motivated in part by our ultimate goal to achieve a complete enumeration of Kirkman triple systems of order 21 (of which so far 192 nonisomorphic systems are known, cf. [6]) in the hope that an approach similar to that used here might be fruitful for that end.

## 2 Classification of NKTS by type vectors

Thus an NKTS( $6n$ )  $(V, \mathcal{R})$ ,  $\mathcal{R} = \{R_1, \dots, R_{3n-1}, F\}$  consists of  $3n - 1$  (edge-) disjoint parallel classes  $R_1, \dots, R_{3n-1}$  of triples (sometimes referred to as *triangle-factors*), and one parallel class  $F$  of pairs (the *1-factor*).

Given two distinct parallel classes  $R, R'$  in an NKTS( $6n$ ), the *block-intersection graph*  $G(R, R')$  has as its vertices all triples of  $R \cup R'$ ; two vertices are adjacent if the corresponding two triples have a common element. Thus  $G(R, R')$  is a cubic bipartite graph with  $4n$  vertices and  $6n$  edges.

The type vector and type matrix are invariants which have been used successfully to distinguish Kirkman triple systems (see, e.g., [10], [8], [6]). They can be used, essentially without changes, to distinguish between nearly Kirkman triple systems.

Let  $C_1, \dots, C_m$  be the list of all nonisomorphic cubic graphs with  $4n$  vertices. With any NKTS( $6n$ ),  $(V, \mathcal{R})$  where  $\mathcal{R} = \{R_1, \dots, R_{3n-1}, F\}$ , associate the *type matrix*  $B = (b_{ij})$  of order  $3n - 1$  where  $b_{ii} = 0$ , and for  $i \neq j$ ,  $b_{ij} = k$  if  $G(R_i, R_j) \cong C_k$ , and the *type vector*  $T = (t_1, \dots, t_m)$  where  $t_k = |\{b_{ij} : i < j, b_{ij} = k\}|$  (cf.[6]).

In this paper, we concentrate on the case of NKTS(18). Any NKTS(18) contains 8 parallel classes of triples. Thus the type matrix is a (symmetric)  $8 \times 8$  matrix whose entries are from the set  $\{1, 2, 3, 4, 5, 6\}$ , as there are exactly 6 nonisomorphic cubic bipartite graphs with 12 vertices. One of the graphs is disconnected, the other five are connected (cf. Nos. 64, 66, 71, 74 and 83 in the listing of all 85 connected cubic graphs in [2]). The six nonisomorphic cubic graphs with 12 vertices are listed for convenience below.

Graph No.	Vertices					
	1	2	3	4	5	6
1	ABC	ABC	ABC	DEF	DEF	DEF
2	ABC	ABC	ABD	CEF	DEF	DEF
3	ABC	ABC	ADE	BDF	CEF	DEF
4	ABC	ABD	ACE	BDF	CEF	DEF
5	ABC	ABD	ACE	BEF	CDF	DEF
6	ABC	ABD	AEF	BEF	CDE	CDF

Table 1. The six 12-vertex bipartite cubic graphs

Each graph contains 6 white vertices 1,2,3,4,5,6, and 6 black vertices A,B,C,D,E,F. E.g., in the graph No.1, the white vertex 1 is adjacent to the black vertices A,B, and C, while in the graph No.4, the white vertex 3 is adjacent to the black vertices A,D, and E, etc.

For example, in this notation, the type matrix of the NKTS(18) given in [7] or [5] is

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 6 & 6 & 1 & 6 \\ 1 & 0 & 6 & 6 & 1 & 1 & 6 & 1 \\ 1 & 6 & 0 & 4 & 4 & 4 & 4 & 4 \\ 1 & 6 & 4 & 0 & 4 & 4 & 4 & 4 \\ 6 & 1 & 4 & 4 & 0 & 4 & 4 & 4 \\ 6 & 1 & 4 & 4 & 4 & 0 & 4 & 4 \\ 1 & 6 & 4 & 4 & 4 & 4 & 0 & 4 \\ 6 & 1 & 4 & 4 & 4 & 4 & 4 & 0 \end{pmatrix}$$

and its type vector is  $(7, 0, 0, 15, 0, 6)$ .

Here type 1 corresponds to the unique disconnected cubic bipartite graph on 12 vertices. In the next section we describe how we generated all NKTS(18) containing at least one pair (of parallel classes  $R, R'$  whose block-intersection graph  $G(R, R')$  is) of type 1.

### 3 All NKTS(18) containing type 1 graph

Suppose  $(V, \mathcal{R}), \mathcal{R} = \{R_1, R_2, \dots, R_8, F\}$ , is an NKTS(18) such that the intersection graph  $G(R_1, R_2)$  is of type 1. Without loss of generality, we may assume  $V = \{0, 1, 2, \dots, 17\}$ , and

$$R_1 = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, a, b, c, d, e, f, g, h\},$$

$$R_2 = \{0, 3, 6, 1, 4, 7, 2, 5, 8, 9, c, f, a, d, g, b, e, h\}.$$

Then we have:

**Lemma 1** *Each edge of  $F$  joins a vertex of  $V_1 = \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$  to a vertex of  $V_2 = \{9, a, b, c, d, e, f, g, h\}$ .*

**Proof.**  $(V, \mathcal{R})$  contains 36 triples in addition to those of  $R_1 \cup R_2$ . There are 81 pairs  $\{x, y\}$  with  $x \in V_1, y \in V_2$ . Each triple covering such a pair covers exactly two such pairs and contains two elements from  $V_1$  and one from  $V_2$ , or vice versa; it also covers one pair  $\{u, v\}$  with both  $u, v \in V_i, i \in \{1, 2\}$ . There are exactly 36 pairs of the latter kind to be covered by the 36 triples, and the statement of the lemma now follows easily.  $\Delta$

Thus the 1-factor  $F$  is of the form

$$F = \{\{x_i, y_i\} : x_i \in V_1, y_i \in V_2\}.$$

In order to generate all NKTS(18) containing at least one pair of parallel classes of type 1, we first established that there are exactly 65 nonisomorphic possibilities for the 1-factor  $F$  (provided  $R_1, R_2$  are fixed as above). For example, some of these are

$$F_0 = \{0, 9, 1, a, 2, b, 3, c, 4, d, 5, e, 6, f, 7, g, 8, h\}$$

$$F_1 = \{0, 9, 1, a, 2, b, 3, c, 4, d, 5, e, 6, f, 7, h, 8, g\}$$

.....

$$F_{26} = \{0, 9, 1, a, 2, b, 3, d, 4, e, 5, c, 6, h, 7, f, 8, g\}$$

.....

$$F_{64} = \{0, 9, 1, d, 2, h, 3, e, 4, f, 5, a, 6, g, 7, b, 8, c\}.$$

[A full listing of the 65 possible 1-factors can be found in the Appendix.]

Next, given the triples of  $R_1 \cup R_2$  and a particular 1-factor  $F_i$ , all possible completions to an MPT(18) were generated by an exhaustive backtrack. This always resulted in a huge number of MPT(18): on average, over 2 million MPTs were obtained for a given  $F_i$ , from 1,749,092 for  $F_{53}$ , to 3,408,192 for  $F_{64}$ . Each obtained MPT(18) was then analyzed for resolvability, and all NKTS(18) obtained were recorded. For each of  $F_0$  and  $F_{64}$ , 17 nonisomorphic NKTS(18) were generated while for 34 of the 65 possible 1-factors (e.g., for  $F_1$  shown above), none of the obtained MPT(18) was resolvable, and thus yielded no NKTS.

The total number of nonisomorphic NKTS(18) that we obtained is 135. However, not all of the obtained solutions contain a pair of parallel classes of type 1. This is due to the fact that even though we always started with such a pair, the completions to MPT(18) that were generated were only then tested for resolvability in all possible ways. In some solutions, the original pair of parallel classes gets "destroyed". Altogether, among the 135 solutions obtained, there are 16 that have this property. One of these solutions has the following 8 parallel classes:

$$\begin{aligned} &\{0\ 7\ g, 1\ 3\ b, 2\ a\ h, 4\ 6\ c, 5\ 9\ e, 8\ d\ f\} \\ &\{0\ 5\ f, 1\ 6\ g, 2\ 4\ d, 3\ 9\ h, 7\ a\ e, 8\ b\ c\} \\ &\{0\ a\ c, 1\ 5\ d, 2\ 3\ f, 4\ 9\ g, 6\ 7\ 8, b\ e\ h\} \\ &\{0\ d\ h, 1\ 8\ 9, 2\ 6\ e, 3\ 7\ c, 4\ a\ f, 5\ b\ g\} \\ &\{0\ 8\ e, 1\ c\ h, 2\ 7\ 9, 6\ b\ f, 3\ 4\ 5, a\ d\ g\} \\ &\{0\ 4\ b, 1\ e\ f, 2\ c\ g, 3\ 8\ a, 5\ 7\ h, 6\ 9\ d\} \\ &\{0\ 1\ 2, 3\ e\ g, 4\ 8\ h, 5\ 6\ a, 7\ b\ d, 9\ c\ f\} \\ &\{0\ 3\ 6, 1\ 4\ 7, 2\ 5\ 8, 9\ a\ b, c\ d\ e, f\ g\ h\}. \end{aligned}$$

Its type vector is  $(0, 0, 10, 3, 15, 0)$ .

On the other hand, it is clear that all solutions containing at least one pair of parallel classes of type 1 are among the 135 solutions generated. Thus we get:

**Theorem 1** *There are exactly 119 nonisomorphic NKTS(18) containing at least one pair of parallel classes of type 1.*

Of the 119 solutions, the first component of the type vector equals 1 in 111 cases, equals 2 in 5 cases, 4 in 2 cases, and 7 in a single case; the latter corresponds to the solution given originally in [7].

A complete list of the 119 solutions, with their type vectors, automorphism group orders, and their number of 5- and 6-cocliques is given in the Appendix.

The type vectors proved to be very efficient invariants: they alone distinguish the 119 solutions with type 1 pair of parallel classes, except between pairs of solutions in four cases, and between one quadruple of solutions. However, employing, in addition, the type matrix further distinguishes all solutions except of one single triple of solutions!

In addition to type vectors and type matrices, we also calculated, for each obtained solution, the number of sets of 5 (or 6, respectively) disjoint triples, each from a different parallel class (i.e., "transversal" sets of 5 or 6 triples). These distinguish the remaining solutions.

As can be seen from the list of type vectors corresponding to the individual solutions, in almost half of all cases (59) the type vectors have no zero component, and thus the corresponding NKTS contain at least one pair of parallel classes of each of the six possible types. In analogy with [6], these can be termed type-heterogeneous. On the other hand, the minimum number of nonzero components in the type vector of any obtained solution is 3 (there are three such solutions, including the one given above) which does not come even close to a type-uniform solution (cf. [6]).

## 4 Conclusion

Among the 16 solutions obtained that do not contain a pair of parallel classes of type 1, there are 12 that do not contain a pair of type 2 either, including one (the solution given above) which does not contain a pair of type 6, either. It is reasonable to expect further, yet unobtained, solutions to exist. It appears, however, that the approach and methods of this paper need to be extended and refined before a complete enumeration of NKTS(18) can be successfully performed.

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## 6 APPENDIX

The 65 non-isomorphic 1-factors for type 1 pair of parallel classes

09 1a 2b 3c 4d 5e 6f 7g 8h	09 1a 2b 3c 4d 5e 6f 7h 8g	09 1a 2b 3c 4d 5e 6g 7h 8f
09 1a 2b 3c 4d 5f 6e 7g 8h	09 1a 2b 3c 4d 5f 6e 7h 8g	09 1a 2b 3c 4d 5f 6g 7e 8h
09 1a 2b 3c 4d 5f 6g 7h 8e	09 1a 2b 3c 4d 5f 6h 7e 8g	09 1a 2b 3c 4d 5f 6h 7g 8e
09 1a 2b 3c 4d 5h 6g 7f 8e	09 1a 2b 3c 4e 5d 6g 7f 8h	09 1a 2b 3c 4e 5f 6d 7h 8g
09 1a 2b 3c 4e 5f 6g 7d 8h	09 1a 2b 3c 4e 5f 6g 7h 8d	09 1a 2b 3c 4e 5f 6h 7d 8g
09 1a 2b 3c 4e 5f 6h 7g 8d	09 1a 2b 3c 4e 5g 6h 7d 8f	09 1a 2b 3c 4e 5g 6h 7f 8d
09 1a 2b 3c 4e 5h 6f 7d 8g	09 1a 2b 3c 4e 5h 6g 7d 8f	09 1a 2b 3c 4f 5g 6d 7e 8h
09 1a 2b 3c 4f 5g 6e 7h 8d	09 1a 2b 3c 4f 5g 6h 7e 8d	09 1a 2b 3c 4h 5g 6f 7e 8d
09 1a 2b 3d 4c 5f 6e 7h 8g	09 1a 2b 3d 4c 5f 6h 7e 8g	09 1a 2b 3d 4c 5c 6h 7f 8g
09 1a 2b 3d 4e 5f 6g 7h 8c	09 1a 2b 3d 4e 5f 6h 7c 8g	09 1a 2c 3b 4d 5f 6e 7h 8g
09 1a 2c 3b 4d 5f 6g 7e 8h	09 1a 2c 3b 4d 5f 6g 7h 8e	09 1a 2c 3b 4d 5f 6h 7e 8g
09 1a 2c 3b 4d 5g 6f 7e 8h	09 1a 2c 3b 4d 5g 6f 7h 8e	09 1a 2c 3b 4d 5g 6h 7e 8f
09 1a 2c 3b 4d 5g 6h 7f 8e	09 1a 2c 3b 4d 5h 6g 7f 8e	09 1a 2c 3b 4e 5f 6d 7g 8h
09 1a 2c 3b 4e 5f 6g 7d 8h	09 1a 2c 3b 4e 5f 6g 7h 8d	09 1a 2c 3b 4e 5f 6h 7d 8g
09 1a 2c 3b 4e 5f 6h 7g 8d	09 1a 2c 3b 4e 5g 6d 7f 8h	09 1a 2c 3b 4e 5g 6d 7h 8f
09 1a 2c 3b 4e 5g 6h 7d 8f	09 1a 2c 3b 4f 5e 6d 7h 8g	09 1a 2c 3b 4f 5e 6g 7d 8h
09 1a 2c 3b 4f 5g 6f 7d 8g	09 1a 2c 3b 4f 5g 6d 7e 8h	09 1a 2c 3b 4f 5g 6d 7h 8e
09 1a 2c 3b 4f 5g 6h 7d 8g	09 1a 2c 3b 4g 5f 6h 7e 8d	09 1a 2c 3b 4h 5f 6e 7g 8d
09 1a 2c 3d 4b 5f 6h 7e 8g	09 1a 2c 3d 4b 5g 6h 7f 8e	09 1a 2c 3d 4b 5h 6g 7f 8e
09 1a 2c 3d 4c 5g 6h 7f 8b	09 1a 2c 3d 4e 5h 6g 7f 8b	09 1a 2c 3d 4f 5b 6h 7e 8g
09 1a 2c 3d 4f 5e 6h 7b 8g	09 1a 2c 3d 4f 5h 6e 7g 8b	09 1a 2c 3d 4f 5h 6g 7e 8b
09 1a 2e 3d 4f 5b 6h 7c 8g	09 1d 2h 3e 4f 5a 6g 7b 8c	

The 119 non-isomorphic NKTS(18) with at least one type 1 pair of parallel classes

- 1: 0ef 1bg 27a 3dh 48c 569 04b 13f 29d 57h 6ac 8eg 05g 1ch 24f 38a 6bd 79e 012 345 678 9cf adg beh  
07c 15d 26h 39g 4ae 8bf 08d 16e 2cg 37b 49h 5af 0ah 189 23e 46g 5bc 7df 036 147 258 9ab cde fgh
- 2: 04b 13f 29d 57h 6ac 8eg 05g 1ch 24f 38a 6bd 79e 012 345 678 9cf adg beh 07c 15d 26h 39g 4ae 8bf  
08d 16e 2cg 37b 49h 5af 0cf 189 27a 3dh 46g 5bc 0ah 1bg 23e 48c 569 7df 036 147 258 9ab cde fgh
- 3: 0ef 1bg 27a 3dh 48c 569 04b 13f 29d 57h 6ac 8eg 05g 1ch 24f 38a 6bd 79e 07c 15d 26h 39g 4ac 8bf  
012 345 678 9ab cde fgh 08d 16e 2cg 37b 49h 5af 0ah 189 23e 46g 5bc 7df 036 147 258 9cf adg beh
- 4: 04b 13f 29d 57h 6ac 8eg 05g 1ch 24f 38a 6bd 79e 07c 15d 26h 39g 4ac 8bf 012 345 678 9ab cde fgh  
08d 16e 2cg 37b 49h 5af 0cf 189 27a 3dh 46g 5bc 0ah 1bg 23e 48c 569 7df 036 147 258 9cf adg beh
- 5: 0ac 1df 24g 38b 56h 79e 012 345 678 9cf adg beh 036 147 258 9ab cde fgh 07h 189 26d 3ae 4bf 5cg  
05f 1ch 239 46a 7bd 8eg 0bg 16e 2ah 37f 48c 59d 04e 15b 27c 3dh 69g 8af 08d 13g 2ef 49h 57a 6bc
- 6: 0ac 1df 24g 38b 56h 79e 012 345 678 9ab cde fgh 036 147 258 9cf adg beh 07h 189 26d 3ae 4bf 5cg  
05f 1ch 239 46a 7bd 8eg 0bg 16e 2ah 37f 48c 59d 04e 15b 27c 3dh 69g 8af 08d 13g 2ef 49h 57a 6bc
- 7: 08d 13g 2ch 4bf 57a 69e 0ac 1ef 24g 38b 56h 79d 0bg 15c 26d 37e 49h 8af 05f 1dh 239 46a 7bc 8eg  
07h 16b 2ae 3df 48c 59g 012 345 678 9ab cde fgh 036 147 258 9ab cde fgh 04e 189 27f 3ah 5bd 6cg
- 8: 08d 13g 2ch 4bf 57a 69e 0ac 1ef 24g 38b 56h 79d 0bg 15c 26d 37e 49h 8af 036 147 258 9cf adg beh  
05f 1dh 239 46a 7bc 8eg 07h 16b 2ae 3df 48c 59g 012 345 678 9ab cde fgh 04e 189 27f 3ah 5bd 6cg
- 9: 07h 1df 2ae 39g 48c 56b 0bg 13e 26d 49h 57c 8af 0ac 16h 24g 38b 59d 7ef 036 147 258 9cf adg beh  
012 345 678 9ab cde fgh 04e 189 23f 5ab 6cg 7bd 08d 15g 2ch 37a 4bf 69c 05f 1bc 279 3dh 46a 8eg
- 10: 07h 1df 2ae 39g 48c 56b 0bg 13e 26d 49h 57c 8af 0ac 16h 24g 38b 59d 7ef 012 345 678 9cf adg beh  
036 147 258 9ab cde fgh 04c 189 23f 5ah 6cg 7bd 08d 15g 2ch 37a 4bf 69c 05f 1bc 279 3dh 46a 8eg
- 11: 05g 13h 24f 6bd 79e 8ac 08d 16e 27c 3bg 49h 5af 0ef 1cg 2dh 37a 48b 569 036 147 258 9cf adg beh  
0ah 189 23e 46g 5bc 7df 07b 15d 29g 38f 4ae 6ch 012 345 678 9ab cde fgh 04c 1bf 26a 39d 57h 8eg

12: 05g 13h 24f 6bd 79e 8ac 0ef 1cg 2dh 37a 48b 569 036 147 258 9cf adg beh 08d 16e 27c 345 9ab fgh  
 0ah 189 23e 46g 5bc 7df 07b 15d 29g 38f 4ae 6ch 012 678 cde 3bg 49h 5af 04c 1bf 26a 39d 57h 8eg  
 13: 05g 13h 24f 6bd 79e 8ac 08d 16e 27c 3bg 49h 5af 0ef 1cg 2dh 345 678 9ab 036 147 258 9cf adg beh  
 0ah 189 23e 46g 5bc 7df 07b 15d 29g 38f 4ae 6ch 012 cde fgh 37a 48b 569 04c 1bf 26a 39d 57h 8eg  
 14: 05g 13h 24f 6bd 79e 8ac 04c 1bf 2dh 37a 569 8eg 036 147 258 9cf adg beh 08d 16e 27c 345 9ab fgh  
 0ah 189 23e 46g 5bc 7df 07b 15d 29g 38f 4ae 6ch 012 678 cde 3bg 49h 5af 0ef 1cg 26a 39d 48b 57h  
 15: 05g 13h 24f 6bd 79e 8ac 08d 16e 27c 3bg 49h 5af 0ef 189 2dh 37a 46g 5bc 036 147 258 9cf adg beh  
 0ah 1cg 23e 48b 569 7df 07b 15d 29g 38f 4ae 6ch 012 345 678 9ab cde fgh 04c 1bf 26a 39d 57h 8eg  
 16: 07b 13e 29g 48f 5ac 6dh 012 345 678 9ab cde fgh 036 147 258 9cf adg beh 08c 1df 27e 39h 46a 5bg  
 05d 16b 2af 38g 49e 7ch 0ef 1cg 24h 37a 569 8bd 04g 15h 26c 3bf 79d 8ae 0ah 189 23d 4bc 57f 6eg  
 17: 07b 13e 29g 48f 5ac 6dh 012 345 678 9cf adg beh 036 147 258 9ab cde fgh 08c 1df 27e 39h 46a 5bg  
 05d 16b 2af 38g 49e 7ch 0ef 1cg 24h 37a 569 8bd 04g 15h 26c 3bf 79d 8ae 0ah 189 23d 4bc 57f 6eg  
 18: 012 345 678 9ab cde fgh 05h 1ef 26c 38a 49g 7bd 07e 189 23g 4ah 5bc 6df 08d 13b 24f 5eg 69h 7ac  
 036 147 258 9cf adg beh 04c 16g 2ae 3dh 579 8bf 0af 15d 27h 39e 46b 8cg 0bg 1ch 29d 37f 48e 56a  
 19: 05h 1ef 26c 38a 49g 7bd 07e 189 23g 4ah 5bc 6df 08d 13b 24f 5eg 69h 7ac 012 345 678 9cf adg beh  
 04c 16g 2ae 3dh 579 8bf 036 147 258 9ab cde fgh 0af 15d 27h 39e 46b 8cg 0bg 1ch 29d 37f 48e 56a  
 20: 012 345 678 9ab cde fgh 0af 15d 26c 37h 49e 8bg 08e 13b 2dh 46f 59g 7ac 036 147 258 9cf adg beh  
 0cg 189 23f 4ah 57e 6bd 05h 16g 2ae 39d 48c 7bf 04b 1ch 279 3eg 56a 8df 07d 1ef 24g 38a 5bc 69h  
 21: 0af 15d 26c 37h 49e 8bg 08e 13b 2dh 46f 59g 7ac 012 345 678 9cf adg beh 0cg 189 23f 4ah 57e 6bd  
 036 147 258 9ab cde fgh 05h 16g 2ae 39d 48c 7bf 04b 1ch 279 3eg 56a 8df 07d 1ef 24g 38a 5bc 69h  
 22: 012 345 678 9ab cde fgh 08d 19e 26g 37a 4bf 5ch 04h 16f 23e 59g 7bd 8ac 0af 13h 29d 48g 57e 6bc  
 036 147 258 9cf adg beh 0eg 18b 24c 3df 56a 79h 05b 1cg 27f 389 4ae 6dh 07c 15d 2ah 3bg 469 8ef  
 23: 08d 19e 26g 37a 4bf 5ch 04h 16f 23e 59g 7bd 8ac 0af 13h 29d 48g 57e 6bc 012 345 678 9cf adg beh  
 036 147 258 9ab cde fgh 0eg 18b 24c 3df 56a 79h 05b 1cg 27f 389 4ae 6dh 07c 15d 2ah 3bg 469 8ef  
 24: 012 345 678 9ab cde fgh 04h 1bf 27c 38g 59d 6ae 036 147 258 9cf adg beh 0af 16d 2eg 39h 48c 57b  
 0cg 189 24a 3bd 56h 7ef 07d 15g 23f 49e 6bc 8ah 08b 13e 2dh 46f 5ac 79g 05e 1ch 269 37a 4bg 8df  
 25: 04h 1bf 27c 38g 59d 6ae 012 345 678 9cf adg beh 0af 16d 2eg 39h 48c 57b 036 147 258 9ab cde fgh  
 0cg 189 24a 3bd 56h 7ef 07d 15g 23f 49e 6bc 8ah 08b 13e 2dh 46f 5ac 79g 05e 1ch 269 37a 4bg 8df  
 26: 0af 18g 23h 49e 57c 6bd 07b 13e 24f 59g 6ac 8dh 012 345 678 9cf adg beh 08c 15b 2ae 3df 46g 79h  
 0eg 16f 27d 389 4bc 5ah 036 147 258 9ab cde fgh 05d 1ch 269 3bg 48a 7ef 04h 19d 2cg 37a 56c 8bf  
 27: 012 345 678 9ab cde fgh 0af 18g 23h 49e 57c 6bd 07b 13e 24f 59g 6ac 8dh 036 147 258 9cf adg beh  
 08c 15b 2ae 3df 46g 79h 0eg 16f 27d 389 4bc 5ah 05d 1ch 269 3bg 48a 7ef 04h 19d 2cg 37a 56c 8bf  
 28: 012 345 678 9ab cde fgh 0ch 1ef 26g 389 4bd 57a 07e 1bg 23d 48f 59h 6ac 036 147 258 9cf adg beh  
 0df 18c 2ah 37b 49g 56e 05g 13h 24c 6bf 79d 8ae 08b 15d 29e 3af 46h 7cg 04a 169 27f 3eg 5bc 8dh  
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- 101: 0eg 1df 26b 389 4ch 57a 04a 16g 23h 59d 7bc 8ef 012 345 678 9cf adg beh 05c 13b 2af 48g 6dh 79e  
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- 102: 012 345 678 9ab cde fgh 08c 1df 26b 3ah 49e 57g 05f 19g 27a 38e 4bc 6dh 036 147 258 9cf adg beh

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 05b 189 2eg 3ch 46a 7df 012 345 678 9ab cde fgh 0ef 1bc 26d 37a 48h 59g 0ac 16e 279 38f 4bg 5dh  
 111: 07h 13g 2af 49e 56c 8bd 08g 1ah 24c 39d 57e 6bf 04d 15f 23b 69h 7cg 8ae 012 345 678 9cf adg beh  
 05b 189 2eg 3ch 46a 7df 036 147 258 9ab cde fgh 0ef 1bc 26d 37a 48h 59g 0ac 16e 279 38f 4bg 5dh  
 112: 07g 13c 29d 46b 5ef 8ah 0af 16h 27c 39g 48d 5bc 08e 1bf 2cg 37a 49h 56d 036 147 258 9cf adg beh  
 04c 1ac 26f 3dh 579 8bg 05h 189 23b 4eg 6ac 7df 012 345 678 9ab cde fgh 0bd 15g 24a 38f 69e 7ch  
 113: 07g 13c 29d 46b 5ef 8ah 0af 16h 27c 39g 48d 5bc 08e 1bf 2cg 37a 49h 56d 036 147 258 9ab cde fgh  
 04c 1ac 26f 3dh 579 8bg 05h 189 23b 4eg 6ac 7df 012 345 678 9cf adg beh 0bd 15g 24a 38f 69e 7ch  
 114: 07g 13c 29d 46b 5ef 8ah 08e 16h 24a 39g 5bc 7df 036 147 258 9cf adg beh 05h 1bf 2cg 37a 48d 69e  
 04c 1ac 26f 3dh 579 8bg 012 345 678 9ab cde fgh 0af 189 23b 4eg 56d 7ch 0bd 15g 27e 38f 49h 6ac  
 115: 07g 13c 29d 46b 5ef 8ah 08e 16h 24a 39g 5bc 7df 036 147 258 9ab cde fgh 05h 1bf 2cg 37a 48d 69e  
 04c 1ac 26f 3dh 579 8bg 012 345 678 9cf adg beh 0af 189 23b 4eg 56d 7ch 0bd 15g 27e 38f 49h 6ac  
 116: 0dh 13c 26f 4ac 579 8bg 0cg 1bf 2ac 37h 489 56d 08f 16a 27e 39g 4bd 5ch 04c 15g 23b 69e 7df 8ah  
 036 147 258 9cf adg beh 07a 19h 24g 38d 5ef 6bc 012 345 678 9ab cde fgh 05b 18c 29d 3af 46h 7cg  
 117: 07a 13c 29d 46h 5ef 8bg 0eg 1bf 2ac 37h 489 56d 08f 16a 27e 39g 4bd 5ch 04c 15g 23b 69e 7df 8ah  
 036 147 258 9cf adg beh 012 345 678 9ab cde fgh 0dh 18e 24g 3af 579 6bc 05b 19h 26f 38d 4ae 7cg  
 118: 0dh 13c 26f 4ac 579 8bg 0eg 1bf 2ac 37h 489 56d 08f 16a 27e 39g 4bd 5ch 04c 15g 23b 69e 7df 8ah  
 036 147 258 9ab cde fgh 07a 19h 24g 38d 5ef 6bc 012 345 678 9cf adg beh 05b 18c 29d 3af 46h 7cg  
 119: 07a 13c 29d 46h 5ef 8bg 0eg 1bf 2ac 37h 489 56d 08f 16a 27e 39g 4bd 5ch 04c 15g 23b 69e 7df 8ah  
 036 147 258 9ab cde fgh 012 345 678 9cf adg beh 0dh 18e 24g 3af 579 6bc 05b 19h 26f 38d 4ae 7cg

Type vectors, group orders and 5- and 6-spreads of our class of NKTS(18)'s

No.	type vector	G	$P_5$	$P_6$	No.	type vector	G	$P_5$	$P_6$	No.	type vector	G	$P_5$	$P_6$
1	2 4 4 14 2 2	2	88 0	41	1 1 9 4 9 4	1	92 0	81	1 2 11 3 10 1	1	78 0			
2	2 2 4 19 0 1	2	94 0	42	1 3 11 5 8 0	1	84 0	82	1 1 9 1 14 2	1	72 0			
3	2 8 0 14 0 4	2	88 0	43	1 3 9 5 9 1	1	73 1	83	1 2 7 1 16 1	1	87 0			
4	2 4 4 16 0 2	2	98 0	44	1 1 4 2 13 7	2	74 0	84	1 6 4 3 13 1	1	84 0			
5	1 10 2 12 2 1	4	110 3	45	1 1 6 4 11 5	2	72 0	85	1 2 7 5 12 1	1	77 0			
6	1 6 6 14 0 1	4	98 5	46	1 1 8 4 9 5	2	82 2	86	1 0 7 3 15 2	1	73 0			
7	1 3 10 6 5 3	2	90 0	47	1 1 10 4 9 3	2	86 2	87	1 0 8 6 10 3	1	78 0			
8	1 1 12 8 3 3	2	82 0	48	1 0 5 5 11 6	1	93 1	88	1 0 12 3 10 2	1	78 0			
9	1 0 6 8 12 1	2	84 0	49	1 0 7 4 11 5	1	94 0	89	1 1 5 4 15 2	1	71 0			
10	1 0 10 12 4 1	2	90 0	50	1 3 10 4 7 3	2	78 2	90	1 4 14 0 8 1	4	62 0			
11	7 0 0 15 0 6	18	90 0	51	1 5 8 4 7 3	2	94 2	91	1 0 14 6 6 1	4	54 0			
12	4 0 6 10 6 2	6	60 0	52	1 4 2 4 16 1	2	58 0	92	1 0 6 12 8 1	4	82 2			
13	1 0 12 15 0 0	18	78 0	53	1 4 6 10 6 1	2	70 0	93	1 0 6 10 10 1	4	66 2			
14	2 0 14 8 4 0	2	72 0	54	1 14 6 6 0 1	4	106 3	94	1 1 13 3 8 2	1	92 0			
15	4 0 12 10 0 2	6	96 0	55	1 10 10 4 2 1	4	108 5	95	1 1 11 5 8 2	1	95 0			
16	1 12 0 9 0 6	6	66 0	56	1 4 5 17 0 1	1	86 0	96	1 1 9 5 10 2	1	81 0			
17	1 6 0 9 12 0	6	60 0	57	1 4 5 15 1 2	1	75 0	97	1 1 12 4 8 2	1	91 0			
18	1 1 4 9 10 3	2	74 0	58	1 6 6 9 3 3	3	66 0	98	1 2 9 2 14 0	1	85 0			
19	1 1 6 7 12 1	2	62 0	59	1 0 12 9 6 0	3	72 0	99	1 2 13 2 10 0	1	92 0			
20	1 1 4 9 10 3	2	92 2	60	1 0 6 9 12 0	3	78 0	100	1 1 9 2 11 4	1	73 0			
21	1 1 6 7 12 1	2	84 2	61	1 0 9 6 12 0	3	69 0	101	1 1 8 1 11 6	1	62 0			
22	1 2 12 4 7 2	1	79 0	62	1 0 6 9 9 3	6	84 6	102	1 2 5 3 13 4	1	75 0			
23	1 2 13 3 7 2	1	79 0	63	1 0 6 6 12 3	6	66 3	103	1 1 8 4 11 3	1	73 0			
24	1 2 7 4 14 0	1	73 1	64	1 1 4 1 16 5	1	93 0	104	1 0 6 12 8 1	4	90 3			
25	1 2 11 5 9 0	1	83 0	65	1 1 6 0 15 5	1	94 0	105	1 0 6 10 10 1	4	94 2			
26	1 0 7 4 14 2	1	68 0	66	1 1 11 6 8 1	1	104 1	106	1 4 6 6 10 1	4	74 4			
27	1 0 7 1 19 0	1	65 0	67	1 3 9 7 7 1	1	108 1	107	1 4 10 4 8 1	4	90 3			
28	1 3 7 5 7 5	1	87 0	68	1 0 7 3 13 4	1	78 0	108	1 0 10 6 10 1	4	82 0			
29	1 1 14 3 6 3	1	85 0	69	1 1 8 3 12 3	1	86 0	109	1 0 14 4 8 1	4	82 0			
30	1 4 7 7 6 3	1	85 1	70	1 1 11 4 9 2	1	79 0	110	1 0 6 4 16 1	4	90 2			
31	1 5 7 4 7 4	1	82 1	71	1 2 6 6 10 3	1	89 0	111	1 0 6 4 12 5	4	86 3			
32	1 1 6 5 15 0	1	75 0	72	1 4 4 4 10 5	1	78 0	112	1 0 14 4 8 1	4	66 0			
33	1 2 7 4 13 1	1	74 0	73	1 4 6 3 10 4	1	76 0	113	1 4 14 2 6 1	4	74 3			
34	1 0 14 4 6 3	1	78 0	74	1 1 13 2 9 2	1	94 1	114	1 0 10 4 12 1	4	90 2			
35	1 0 15 1 7 4	1	99 1	75	1 0 12 4 8 3	1	83 0	115	1 0 10 6 10 1	4	70 3			
36	1 0 11 8 7 1	1	84 0	76	1 0 11 4 10 2	1	87 0	116	1 0 6 12 0 9	36	126 15			
37	1 3 11 7 5 1	1	98 0	77	1 0 10 5 11 1	1	79 0	117	1 0 6 12 0 9	38	126 6			
38	1 3 12 7 4 1	2	76 1	78	1 2 4 6 13 2	1	80 0	118	1 0 6 12 0 9	38	90 6			
39	1 5 12 5 4 1	2	92 3	79	1 1 6 5 13 2	1	74 0	119	1 0 6 12 0 9	36	90 6			
40	1 1 9 3 11 3	1	86 0	80	1 1 14 6 5 1	1	82 0							