TRIVALENT SYMMETRIC GRAPHS ON UP TO 768 VERTICES

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Abstract

A complete list is given of all finite trivalent arc-transitive connected graphs on up to 768 vertices, completing and extending the Foster census. Several previously undiscovered graphs appear, including one on 448 vertices which is the smallest arc-transitive trivalent graph having no automorphism of order 2 which reverses an arc. The graphs on the list are classified according to type (as described by Djokovic and Miller in terms of group amalgams), and were produced with the help of a parallel program which finds all normal subgroups of low index in a finitely-presented group. Further properties of each graph are also given: its girth, diameter, Hamiltonicity, and whether or not it is bipartite.

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1. Introduction

Let Γ be an undirected simple graph, let G be a group of automorphisms of Γ. If G acts transitively on the vertices of Γ, and the stabilizer in G of a vertex v acts transitively on the vertices adjacent to v, then G is said to act *symmetrically* on Γ. In this case G acts transitively on the arcs (ordered edges) of Γ, and Γ is called a *symmetric* graph. Generalising for $s \geq 1$, an s-arc in Γ is a sequence $(v_0, v_1, \ldots, v_s)$ of vertices of Γ such that $\{v_{i-1}, v_i\}$ is an edge of Γ for $1 \leq i \leq s$, and $v_{i-1} \neq v_{i+1}$ for $1 \leq i < s$ (or in other words, such that any two consecutive v_i are adjacent and any three consecutive v_i are distinct), and then Γ is said to be *s-arc-transitive* if its automorphism group acts transitively on the set of all s-arcs of Γ. In particular, a 1-arc-transitive graph is precisely a symmetric graph.

By a well known theorem of Tutte (see [**1**; §18]), a finite connected symmetric graph of degree 3 can be at most 5-arc-transitive; indeed its automorphism group acts regularly on s-arcs for some $s \leq 5$, in which case the graph Γ itself is called s-arc-regular. Tutte's theorem was extended by Djokovic and Miller [**8**], who classified finite connected trivalent symmetric graphs into seven types (according to the level of s-arc-regularity and the existence or otherwise of an involutory automorphism reversing an arc), and these types were later described in a unified way by Conder and Lorimer [**5**] in terms of generators and relations for their automorphism groups .

Conversely, given any group G containing a subgroup H and an element a such that $a^2 \in H$, we may construct a graph $\Gamma = \Gamma(G, H, a)$ on which G acts symmetrically, as follows: take as vertices of Γ the right cosets of H in G, and join two cosets Hx and Hy by an edge in Γ whenever $xy^{-1} \in HaH$. Defined in this way, Γ is an undirected graph on which the group G acts as a group of automorphisms under the action $g : Hx \rightarrow Hxg$ for each $g \in G$ and each coset Hx in G. The stabilizer in G of the vertex H is the subgroup H itself, and as this acts transitively on the set of neighbours of H (which are all of the form Hah for $h \in H$), it follows that Γ is symmetric.

The above construction (part of the folk-lore of algebraic graph theory) is given in more detail in [**10**]. The graph $\Gamma = \Gamma(G, H, a)$ is connected if and only if G is generated by HaH (or equivalently, by $H \cup \{a\}$), and is regular of degree d where $d = |H : H \cap a^{-1}Ha|$ is the number of right cosets of H contained in the double coset HaH. Similarly, other properties of Γ (such as its girth and diameter) depend on the choice of G, H and a, and in particular on relations satisfied in G by a and elements of H.

In this paper we provide a complete list of all finite connected trivalent symmetric graphs on up to 768 vertices. This list was determined with the help of a program which finds all normal subgroups of up to a given index in a finitely-presented group (see [**4**]). Our list completes and extends the "Foster census", a partial list of such graphs on up to 512 vertices, compiled by R.M. Foster over many years and published with a few additions (but no claim of completeness) in [**3**]. It also extends the full list of all such graphs of order up to 240 produced by Conder and Morton using similar methods in [**6**], and fills the gaps for graphs of order up to 768 in a database maintained by Gordon Royle [**11**] — or more precisely, the gaps which existed before we produced the complete list.

Remarkably, and as pointed out in [**6**], Foster's hand-prepared list had only one omission for graphs of order up to 240. On the other hand, for order between 240 and 512 there were (understandably) several omissions, some of which have been corrected by Gordon Royle (personal communication), and the remainder of which we have found. Those graphs of order up to 512 which do not appear in the Foster census are labelled 408B, 432E, 448C, 480C, 480D, 486D, 512D, 512E, 512F and 512G. In particular, the "new" graph 448C is the only one on the complete list which is arc-transitive but has no involutory automorphism reversing an arc, and hence is the smallest such graph. The previously smallest known (and first known) example of this type was one on 6652800 vertices (see [**5**]).

The complete list is described in Section 3, following some more background material and a description of our methods in Section 2. The girth and diameter and the existence (or otherwise) of a bipartition of each graph is given, and these properties were determined with the help of the MAGMA system [**2**]. Also the Hamiltonicity of the graphs was checked using a computer program kindly supplied to us by Nick Wormald, but is not documented. Further details are available from the authors upon request.

2. Background and methods

As outlined in [**5**] (and again in [**6**]), if the group G acts symmetrically on the finite connected trivalent graph Γ, then G is a homomorphic image of one of the following seven finitely-presented groups, G_1, G_2^1, G_2^2, G_3, G_4^1, G_4^2 or G_5, depending on the type of Γ:

(i) G_1 is the modular group, generated by two elements h, a subject to the relations $h^3 = a^2 = 1$,

(ii) G_2^1 is the extended modular group, generated by three elements h, a, p subject to the relations $h^3 = a^2 = p^2 = 1$, $apa = p$, and $php = h^{-1}$,

(iii) G_2^2 is generated by three elements h, a, p subject to the relations $h^3 = p^2 = 1$, $a^2 = p$, and $php = h^{-1}$,

(iv) G_3 is generated by four elements h, a, p, q subject to the relations $h^3 = a^2 = p^2 = q^2 = 1$, $qp = pq$, $h^{-1}ph = p$, $qhq = h^{-1}$, and $apa = q$,

(v) G_4^1 is generated by five elements h, a, p, q, r subject to the relations $h^3 = a^2 = p^2 = q^2 = r^2 = 1$, $pq = qp$, $pr = rp$, $rq = pqr$, $h^{-1}ph = q$, $h^{-1}qh = pq$, $rhr = h^{-1}$, $apa = p$, and $aqa = r$,

(vi) G_4^2 is generated by five elements h, a, p, q, r subject to the relations $h^3 = p^2 = q^2 = r^2 = 1$, $a^2 = p$, $pq = qp$, $pr = rp$, $rq = pqr$, $h^{-1}ph = q$, $h^{-1}qh = pq$, $rhr = h^{-1}$, and $a^{-1}qa = r$,

(vii) G_5 is generated by six elements h, a, p, q, r, s subject to the relations $h^3 = a^2 = p^2 = q^2 = r^2 = s^2 = 1$, $pq = qp$, $pr = rp$, $ps = sp$, $qr = rq$, $qs = sq$, $sr = pqrs$, $h^{-1}ph = p$, $h^{-1}qh = r$, $h^{-1}rh = pqr$, $shs = h^{-1}$, $apa = q$, and $ara = s$.

In fact the relations show G_2^2 and G_4^2 can be generated by just h and a, while G_3, G_4^1 and G_5 can be generated by h, a and p, since for example in G_4^2 we have $p = a^2$, $q = h^{-1}ph$, and $r = a^{-1}qa$. As pointed out in [**6**], however, the given presentations are very useful and have been used in the construction of numerous examples using the general method described in the Introduction (see [**5**] and other references listed in [**6**]).

The key to such constructions is the fact that the converse also holds: if G is any non-degenerate finite homomorphic image of one of the seven groups above (non-degenerate meaning that the homomorphism preserves the orders of the generators h, a etc.), and H is the subgroup generated by (the images of) all the given generators except a, then G acts symmetrically on the connected graph $\Gamma = \Gamma(G, H, a)$, with the subgroup H stabilizing the vertex H, and with any vertex Hw being adjacent to each of Haw, $Hahw$ and $Hah^{-1}w$ in Γ. Note here that for notational convenience we are using the same symbols for the images of the generators as for the generators themselves.

Hence the search for finite connected trivalent symmetric graphs of up to a particular order can be reduced to the problem of finding all normal subgroups of up to a specific index in each of the above seven finitely-presented groups. In fact if Γ is a such a graph of order at most n, then its full automorphism group $G = \mathrm{Aut}\,\Gamma$ is isomorphic to the factor group of G_1, G_2^1, G_2^2, G_3, G_4^1, G_4^2 or G_5 by a normal subgroup K of index up to $3n$, $6n$, $6n$, $12n$, $24n$, $24n$ or $48n$ respectively, depending on the type of Γ.

The development and implementation of an algorithm for determining all normal subgroups of up to a given index in a finitely-presented group have been described in detail in [**4**]. This algorithm is an adaptation of the one due to Charles Sims and others for finding conjugacy classes of *all* subgroups of up to a given index (see [**7**] or [**12**]), but with additional subgroup generators being treated as relators — representatives of conjugacy classes of elements generating a normal subgroup — rather than simply elements which may generate a subgroup that is not normal. Such an adaptation was signalled in [**6**].

In fact we have used a "mixed" adaptation of the low index subgroups algorithm, which finds each normal subgroup K of up to a given index in a finitely-presented group G, but using coset enumeration over the subgroup HK for a fixed (constant) subgroup H in G. In this case whenever the algorithm forces two cosets HKx and HKy to coincide, the deduction $xy^{-1} \in HK$ gives rise to multiple possibilities for the additional relator, namely all elements of the form wxy^{-1} where w is an element of the fixed subgroup H, and each one needs to be checked as a possible new branch of the search tree.

Also in the context of this symmetric graphs problem, the fixed subgroup H is precisely the subgroup corresponding to a vertex-stabilizer (generated by all the given generators other than a). In particular, the index of a subgroup HK of the required form is equal to the order of the corresponding graph, and so in each case we need only search for subgroups (of this form) of index up to the same index n. Moreover, the fixed subgroup H has order at most 48 (attained in the case of the group G_5), and this limits the number of branches at each node of the search tree. While 48 might seem large, the tight structure of the group G_5 makes the computation proceed very fast for G_5, indeed faster than the same computation for G_2^1, which appears to be the slowest case (for the same index).

As the computations were carried out on a distributed processing system using at times well over 100 separate processors (the "Kaláka" system developed by Peter Dobcsányi and described in his PhD thesis [9]) , it is very difficult to give an accurate description of the computation times involved, however the following times taken by the longest branch of each computation provide reasonable estimates of these:

Type	Time required for longest branch at first level
G_1	13 hrs 09 mins
G_2^1	43 hrs 33 mins
G_2^2	3 hrs 48 mins
G_3	10 hrs 22 mins
G_4^1	4 hrs 38 mins
G_4^2	2 hrs 06 mins
G_5	3 hrs 48 mins

Finally, note that some trivalent arc-transitive graphs can be constructed from factor groups of more than one of the generic groups (G_1, G_2^1, etc.) associated with the seven types of graph, reflecting the fact that their full automorphism groups contain proper arc-transitive subgroups, and hence we require methods for dealing with such multiplicity. One way is to directly compute the (full) automorphism groups and test for graph isomorphism, with the help of the MAGMA system [2]. In many cases, however, it is possible to incorporate further information about the seven generic groups into an extension of the low index subgroups computation.

For example, by Proposition 4.1 of [5] it is known that if $\Gamma = \Gamma(G, H, a)$ is a connected finite trivalent symmetric graph constructible from a non-degenerate factor group G of G_2^1, and H and a are as described above for the latter group, then Γ has a larger, 3-arc-transitive group of automorphisms if and only if there is a group automorphism $\phi : G \to G$ which centralises H

and takes a to ap. The existence of such an automorphism may be checked using the coset table for H in G (or equivalently the natural permutation representation of G on right cosets of G by right multiplication): simply replace the column (or permutation) corresponding to the generator a by one which gives the action of ap on cosets, and check whether or not the resulting coset table (or permutation representation) is equivalent to the original. If it is, then such an automorphism exists, and the graph Γ is 3-arc-transitive, while on the other hand if they are not equivalent, then no such automorphism exists; but in the latter case we find $\Gamma = \Gamma(G, H, a)$ is isomorphic to the (dual) graph $\Gamma = \Gamma(G, H, ap)$, and hence will arise twice in the list of graphs produced by the low index subgroups computation for G_2^1 (and one of these two occurrences can then be eliminated).

Analogous methods apply for the groups G_2^2, G_4^1 and G_4^2, and similarly for the group G_1: check for a group automorphism $\phi : G \to G$ centralising a and taking h to h^{-1}, to see if Γ has a 2-arc-regular group of automorphisms. Other possibilities (such as a graph arising from the group G_1 having a 4-arc-transitive group of automorphisms) can still be dealt with using the graph automorphism facilities available in MAGMA [2].

3. Results

Results of our computations are summarised below, with the graphs classified according to the seven types described earlier. In each case we give a label for the graph, indicating its order, in a form consistent with the Foster census [3] and Gordon Royle's database [11]. This label is followed by representatives of conjugacy classes which generate the corresponding normal subgroup K of the appropriate finitely-presented group (G_1, G_2^1, etc.) — or equivalently, additional relator(s) which produce the full automorphism group of the graph when inserted into the appropriate presentation.

Each label consists of the order of the graph, plus an alphabetic letter (A, B, C etc.) if there is more than one isomorphism class of connected trivalent symmetric graphs of that order, and an asterisk if the graph does not appear in the Foster census. For example, the label F448B is used for the graph on 448 vertices called 448B in the Foster census, while F448C* is a new one of order 448.

The graphs appear under each type in ascending order with respect to girth (the length of the shortest cycle in the graph), and for given girth in ascending order of graph order (the number of vertices). Exactly one graph from each isomorphism class is listed, and listed just once, according to its type.

For brevity we use the symbols u and v (as in [6]) to denote the products ha and $h^{-1}a$ respectively. Accordingly the girth of each graph is equal to the smallest total number of occurrences of u and v in any of the additional relators, usually the shortest (and always the first) additional relator in each case. Hence for example, graphs obtained from one of the seven groups with relator v^6 adjoined (along with other relators) are members of the family of graphs of girth 6 described by Miller in [10].

Note that there is exactly one graph of type (iii) on the list, namely F448C* (of order 448), and hence this is the smallest example of a finite trivalent graph which is 2-arc-transitive but has no involutory automorphism reversing an arc. On the other hand, no graphs of type (vi) appear: all graphs found from factor groups of G_4^2 turned out to be 5-arc-transitive; nevertheless, graphs of type (vi) do exist (see [5]). Also note that all of the graphs listed have girth at most 14, for in each case there is always an additional relator of length at most 14 in terms of u and v.

Case (i): G_1 : 1-arc-regular graphs

F026	$v^6,\ u^2(vu)^2v^2$
F038	$v^6,\ u^2v^2u^2vuv^2$
F042	$v^6,\ u^2(vu)^3v^2$
F056A	$v^6,\ u^2v^2u^2(vu)^2v^2$
F062	$v^6,\ u^2(vu)^4v^2$
F074	$v^6,\ u^2v^2uvu^2(vu)^2v^2$
F078	$v^6,\ u^2vuv^2u^2(vu)^2v^2$
F086	$v^6,\ u^2(vu)^5v^2$
F098A	$v^6,\ u^2v^2uvu^2(vu)^3v^2$
F104	$v^6,\ u^2vuv^2u^2(vu)^3v^2$
F114	$v^6,\ u^2(vu)^6v^2$
F122	$v^6,\ u^2v^2(uv)^2u^2(vu)^3v^2$
F126	$v^6,\ u^2vuv^2uvu^2(vu)^3v^2$
F134	$v^6,\ u^2(vu)^2v^2u^2(vu)^3v^2$
F146	$v^6,\ u^2(vu)^7v^2$
F152	$v^6,\ u^2v^2(uv)^2u^2(vu)^4v^2$
F158	$v^6,\ u^2vuv^2uvu^2(vu)^4v^2$
F168A	$v^6,\ u^2(vu)^2v^2u^2(vu)^4v^2$
F182B	$v^6,\ u^2(vu)^8v^2$
F182A	$v^6,\ u^2v^2(uv)^3u^2(vu)^4v^2$
F186	$v^6,\ u^2vuv^2(uv)^2u^2(vu)^4v^2$
F194	$v^6,\ u^2(vu)^2v^2uvu^2(vu)^4v^2$
F206	$v^6,\ u^2(vu)^3v^2u^2(vu)^4v^2$
F218	$v^6,\ u^2v^2(uv)^3u^2(vu)^5v^2$

F222	$v^6,\ u^2(vu)^9v^2$
F224A	$v^6,\ u^2vuv^2(uv)^2u^2(vu)^5v^2$
F234A	$v^6,\ u^2(vu)^2v^2uvu^2(vu)^5v^2$
F248	$v^6,\ u^2(vu)^3v^2u^2(vu)^5v^2$
F254	$v^6,\ u^2v^2(uv)^4u^2(vu)^5v^2$
F258	$v^6,\ u^2vuv^2(uv)^3u^2(vu)^5v^2$
F266B	$v^6,\ u^2(vu)^{10}v^2$
F266A	$v^6,\ u^2(vu)^2v^2(uv)^2u^2(vu)^5v^2$
F278	$v^6,\ u^2(vu)^3v^2uvu^2(vu)^5v^2$
F294B	$v^6,\ u^2(vu)^4v^2u^2(vu)^5v^2$
F296	$v^6,\ u^2v^2(uv)^4u^2(vu)^6v^2$
F302	$v^6,\ u^2vuv^2(uv)^3u^2(vu)^6v^2$
F312A	$v^6,\ u^2(vu)^2v^2(uv)^2u^2(vu)^6v^2$
F314	$v^6,\ u^2(vu)^{11}v^2$
F326	$v^6,\ u^2(vu)^3v^2uvu^2(vu)^6v^2$
F338A	$v^6,\ u^2v^2(uv)^5u^2(vu)^6v^2$
F342	$v^6,\ u^2vuv^2(uv)^4u^2(vu)^6v^2$
F344	$v^6,\ u^2(vu)^4v^2u^2(vu)^6v^2$
F350	$v^6,\ u^2(vu)^2v^2(uv)^3u^2(vu)^6v^2$
F362	$v^6,\ u^2(vu)^3v^2(uv)^2u^2(vu)^6v^2$
F366	$v^6,\ u^2(vu)^{12}v^2$
F378A	$v^6,\ u^2(vu)^4v^2uvu^2(vu)^6v^2$
F386	$v^6,\ u^2v^2(uv)^5u^2(vu)^7v^2$
F392A	$v^6,\ u^2vuv^2(uv)^4u^2(vu)^7v^2$
F398	$v^6,\ u^2(vu)^5v^2u^2(vu)^6v^2$
F402	$v^6,\ u^2(vu)^2v^2(uv)^3u^2(vu)^7v^2$
F416	$v^6,\ u^2(vu)^3v^2(uv)^2u^2(vu)^7v^2$
F422	$v^6,\ u^2(vu)^{13}v^2$
F434B	$v^6,\ u^2(vu)^4v^2uvu^2(vu)^7v^2$
F434A	$v^6,\ u^2v^2(uv)^6u^2(vu)^7v^2$
F438	$v^6,\ u^2vuv^2(uv)^5u^2(vu)^7v^2$
F446	$v^6,\ u^2(vu)^2v^2(uv)^4u^2(vu)^7v^2$
F456A	$v^6,\ u^2(vu)^5v^2u^2(vu)^7v^2$
F458	$v^6,\ u^2(vu)^3v^2(uv)^3u^2(vu)^7v^2$
F474	$v^6,\ u^2(vu)^4v^2(uv)^2u^2(vu)^7v^2$
F482	$v^6,\ u^2(vu)^{14}v^2$
F488	$v^6,\ u^2v^2(uv)^6u^2(vu)^8v^2$
F494B	$v^6,\ u^2(vu)^5v^2uvu^2(vu)^7v^2$
F494A	$v^6,\ u^2vuv^2(uv)^5u^2(vu)^8v^2$
F504A	$v^6,\ u^2(vu)^2v^2(uv)^4u^2(vu)^8v^2$
F518A*	$v^6,\ u^2(vu)^6v^2u^2(vu)^7v^2$
F518B*	$v^6,\ u^2(vu)^3v^2(uv)^3u^2(vu)^8v^2$
F536*	$v^6,\ u^2(vu)^4v^2(uv)^2u^2(vu)^8v^2$

F542*	$v^6,\ u^2v^2(uv)^7u^2(vu)^8v^2$
F546A*	$v^6,\ u^2(vu)^{15}v^2$
F546B*	$v^6,\ u^2vuv^2(uv)^6u^2(vu)^8v^2$
F554*	$v^6,\ u^2(vu)^2v^2(uv)^5u^2(vu)^8v^2$
F558*	$v^6,\ u^2(vu)^5v^2uvu^2(vu)^8v^2$
F566*	$v^6,\ u^2(vu)^3v^2(uv)^4u^2(vu)^8v^2$
F582*	$v^6,\ u^2(vu)^4v^2(uv)^3u^2(vu)^8v^2$
F584*	$v^6,\ u^2(vu)^6v^2u^2(vu)^8v^2$
F602B*	$v^6,\ u^2(vu)^5v^2(uv)^2u^2(vu)^8v^2$
F602A*	$v^6,\ u^2v^2(uv)^7u^2(vu)^9v^2$
F608*	$v^6,\ u^2vuv^2(uv)^6u^2(vu)^9v^2$
F614*	$v^6,\ u^2(vu)^{16}v^2$
F618*	$v^6,\ u^2(vu)^2v^2(uv)^5u^2(vu)^9v^2$
F626*	$v^6,\ u^2(vu)^6v^2uvu^2(vu)^8v^2$
F632*	$v^6,\ u^2(vu)^3v^2(uv)^4u^2(vu)^9v^2$
F650A*	$v^6,\ u^2(vu)^4v^2(uv)^3u^2(vu)^9v^2$
F654*	$v^6,\ u^2(vu)^7v^2u^2(vu)^8v^2$
F662*	$v^6,\ u^2v^2(uv)^8u^2(vu)^9v^2$
F666*	$v^6,\ u^2vuv^2(uv)^7u^2(vu)^9v^2$
F672B*	$v^6,\ u^2(vu)^5v^2(uv)^2u^2(vu)^9v^2$
F674*	$v^6,\ u^2(vu)^2v^2(uv)^6u^2(vu)^9v^2$
F686A*	$v^6,\ u^2(vu)^{17}v^2$
F686C*	$v^6,\ u^2(vu)^3v^2(uv)^5u^2(vu)^9v^2$
F698*	$v^6,\ u^2(vu)^6v^2uvu^2(vu)^9v^2$
F702B*	$v^6,\ u^2(vu)^4v^2(uv)^4u^2(vu)^9v^2$
F722A*	$v^6,\ u^2(vu)^5v^2(uv)^3u^2(vu)^9v^2$
F728B*	$v^6,\ u^2(vu)^7v^2u^2(vu)^9v^2$
F728A*	$v^6,\ u^2v^2(uv)^8u^2(vu)^{10}v^2$
F734*	$v^6,\ u^2vuv^2(uv)^7u^2(vu)^{10}v^2$
F744B*	$v^6,\ u^2(vu)^2v^2(uv)^6u^2(vu)^{10}v^2$
F746*	$v^6,\ u^2(vu)^6v^2(uv)^2u^2(vu)^9v^2$
F758*	$v^6,\ u^2(vu)^3v^2(uv)^5u^2(vu)^{10}v^2$
F762*	$v^6,\ u^2(vu)^{18}v^2$
F448A	$v^7,\ (uv)^8,\ u^2v^2u^2vu^2(vu)^2v^2uv^2$
F144A	$v^8,\ (u^2v)^2(uv^2)^2$
F400A	$(uv)^4,\ v^{12},\ u^5vu^2v^2uv^5$
F432E*	$v^8,\ u^2v^2u^3(vu)^2v^3$
F576B*	$v^8,\ (uv)^6,\ (u^2v^2)^4,\ uvuv^3u^3v^2uv^2uv^3$
F720D*	$v^8,\ u^2v^3u^3vuv^2uv^3$
F504B	$v^9,\ u^2vuv^2u^2vuv^2,\ uv^2(uv)^2u^3v^3uv^3$
F112C	$(uv^4)^2,\ u^2v^2u^2(vu)^2v^2$
F208	$(uv^4)^2,\ u^2vuv^2u^2(vu)^3v^2$
F304	$(uv^4)^2,\ u^2v^2(uv)^2u^2(vu)^4v^2$

F336C	$(uv^4)^2,\ u^2(vu)^2v^2u^2(vu)^4v^2$
F448B	$(uv^4)^2,\ u^2vuv^2(uv)^2u^2(vu)^5v^2$
F496	$(uv^4)^2,\ u^2(vu)^3v^2u^2(vu)^5v^2$
F592*	$(uv^4)^2,\ u^2v^2(uv)^4u^2(vu)^6v^2$
F624B*	$(uv^4)^2,\ u^2(vu)^2v^2(uv)^2u^2(vu)^6v^2$
F688*	$(uv^4)^2,\ u^2(vu)^4v^2u^2(vu)^6v^2$
F720F*	$v^{10},\ u^3vu^2v^2uv^2uv^4,\ (uv^2u^2v^3)^2$
F162B	$u^4v^2u^2v^4$
F168E	$u^3v^2uvu^2v^3,\ (uv^3)^3,\ v^{12}$
F256D	$u^2v^2u^3vuv^3,\ v^{12}$
F312B	$(uv^3)^3,\ v^{12},\ u^2v^2u^2(vu)^3v^2$
F336F	$(u^2v)^2(uv^2)^2,\ (u^2v^5)^2$
F378B	$u^6v^6,\ (uv^3)^3,\ (u^2v^2)^2u^2v(uv)^2uv^2$
F432C	$(u^2v)^2(uv^2)^2,\ (uv)^6$
F456B	$(uv^3)^3,\ v^{12},\ u^2v(uv)^2u^2v^2uvu^2v^3$
F504D	$(uv^3)^3,\ v^{12},\ (uv)^2u^2(vu)^5v^2$
F648D*	$(uv)^2u^2(vu)^2v^2,\ (uv^3)^3,\ u^{10}v^2u^2v^{10}$
F648E*	$(uv^3)^3,\ uvu^5vuv^5,\ u^2vuv^2u^3v^2u^2v^3$
F672A*	$(uv^3)^3,\ uvu^5vuv^5,\ u^4v^2u^2v^2u^2v^4$
F672G*	$(uv^3)^3,\ v^{12},\ u^2v^2u^2(vu)^6v^2$
F702A*	$u^6v^6,\ (uv^3)^3,\ u^2(vu)^2v^2uvu^2(vu)^5v^2$
F744A*	$(uv^3)^3,\ v^{12},\ u^2v^2uvu^2v^2u^2(vu)^4v^2$
F768D*	$v^{12},\ (uv^2uv^3)^2,\ uvu^2v^2uvu^2(vu)^2v^2$
F512E*	$u^3vuvu^2vuv^4$
F624A*	$u^2v^2u^2(vu)^3v^2,\ (u^2v^5)^2,\ (uvuv^4)^2$

Case (ii): G_2^1 : 2-arc-regular graphs (with an edge-flip of order 2)

F004	$pvuv$
F008	v^4
F020A	v^5
F016	u^3v^3
F024	$v^6,\ uvu^2vuv^2$
F032	$v^6,\ (uv)^4$
F050	$v^6,\ (uv)^5$
F054	$v^6,\ (uv)^2u^2(vu)^2v^2$
F072	$v^6,\ (uv)^6$
F096A	$v^6,\ (uv)^3u^2(vu)^3v^2$
F098B	$v^6,\ (uv)^7$
F128A	$v^6,\ (uv)^8$
F150	$v^6,\ (uv)^4u^2(vu)^4v^2$
F162A	$v^6,\ (uv)^9$
F200	$v^6,\ (uv)^{10}$

F216A	$v^6,\ (uv)^5u^2(vu)^5v^2$
F242	$v^6,\ (uv)^{11}$
F288A	$v^6,\ (uv)^{12}$
F294A	$v^6,\ (uv)^6u^2(vu)^6v^2$
F338B	$v^6,\ (uv)^{13}$
F384A	$v^6,\ (uv)^7u^2(vu)^7v^2$
F392B	$v^6,\ (uv)^{14}$
F450	$v^6,\ (uv)^{15}$
F486A	$v^6,\ (uv)^8u^2(vu)^8v^2$
F512A	$v^6,\ (uv)^{16}$
F578*	$v^6,\ (uv)^{17}$
F600B*	$v^6,\ (uv)^9u^2(vu)^9v^2$
F648A*	$v^6,\ (uv)^{18}$
F722B*	$v^6,\ (uv)^{19}$
F726*	$v^6,\ (uv)^{10}u^2(vu)^{10}v^2$
F056B	$v^7,\ (uv)^4$
F084	$v^7,\ pv(uv)^4$
F168B	$v^7,\ uv(u^2v)^2(uv^2)^2$
F182C	$v^7,\ pv(uv)^6$
F364A	$v^7,\ (uv)^6$
F364B	$v^7,\ (uv)^7$
F364C	$v^7,\ (uvuv^2)^3$
F048	u^4v^4
F064	$v^8,\ uvu^3vuv^3$
F112A	$v^8,\ uv^2u^3v^2uv^3$
F120A	$v^8,\ pv^2(u^2v^2)^2$
F168C	$v^8,\ (u^2v^2)^3,\ pv^3(uv^2)^2uv^3$
F240A	$v^8,\ (uv)^5,\ u^2v^3u^3vuv^2uv^3$
F256A	$v^8,\ (uv^2)^4$
F336B	$v^8,\ (u^2v^2)^3$
F336A	$v^8,\ (u^2v^2uv^2)^2$
F360A	$v^8,\ (uv)^5,\ pv^3(uv^2)^3uv^3$
F512F*	$v^8,\ u^2vuvu^3v^2uvuv^3$
F672C*	$v^8,\ pv^2uv^2u^2vu^2v^2uv^2$
F720B*	$v^8,\ (u^2v)^2u^2v^2(uv^2)^2$
F720E*	$v^8,\ (uv)^5$
F060	$pv^3u^3v^3,\ v^{10}$
F108	$v^9,\ u^2vu^2v^2uv^2$
F168D	$v^9,\ (uvuv^3)^2$
F240B	$pv^3u^3v^3,\ pv(uv)^7$
F408A	$v^9,\ pvuv^2uvu^2vuv^2$
F480A	$pv^3u^3v^3$
F504C	$v^9,\ u^2v^2u^3v^2u^2v^3$

F570B*	v^9, $(uv)^5$, $pv^3(u^3v^3)^2$
F120B	u^5v^5, $(uv)^5$
F128B	$(u^2v^3)^2$, $(uv^2)^4$
F144B	$(uv^4)^2$, $u^2v(uv)^3u^2v^3$
F192B	$(uv^4)^2$, $(uv)^3u^2(vu)^3v^2$
F216B	$(u^2v^3)^2$, v^{12}
F220B	v^{10}, $(u^3v^3)^2$, $pvuv^2uvu^2vuv^2$
F220A	v^{10}, $pv(uv)^5$, $(uv^2)^4$
F240C	u^5v^5
F250	v^{10}, $(uv)^2u^2(vu)^2v^2$
F256B	$(u^2v^3)^2$, $(uv)^6$
F256C	$(uv^4)^2$, $(uv)^8$
F400B	$(uv^4)^2$, $u^2v(uv)^7u^2v^3$
F432A	$(u^2v^3)^2$, $(uv)^2u^2(vu)^2v^2uv^2uv^2$
F432B	$(uv^4)^2$, $(uv)^5u^2(vu)^5v^2$
F440B	v^{10}, $(u^3v^3)^2$
F440A	v^{10}, $(uv^2)^4$, $u^2v^2u^3v^2u^2v^3$
F480D*	v^{10}, $(uv)^2u^3(vu)^2v^3$, $u^3vu^2v^2uv^2uv^4$
F500	$(u^2v^3)^2$, $pv(uv)^7$
F512B	$(u^2v^3)^2$, v^{16}
F576A*	$(uv^4)^2$, $(uv)^{12}$
F660*	v^{10}, $(uv^2)^4$, $pv^4uv^3uv^4$
F720C*	v^{10}, $uv^3u^4v^3uv^4$
F768A*	$(uv^4)^2$, $(uv)^7u^2(vu)^7v^2$
F168F	$pv^2uvu^2v^6$
F192C	$u^2vu^3v^2uv^3$, v^{12}
F216C	u^6v^6, $(uv)^6$, $(uv^3)^3$
F224B	$u^2vu^2v^3uv^3$
F336D	$(u^3v^3)^2$, $(u^2v^2)^3$
F336E	$(uv^5)^2$, $uv(u^2v)^2(uv^2)^2$
F364F	$(u^2v^2)^3$, v^{12}, $pvuv^2uvu^2vuv^2$
F364E	$(uv^5)^2$, $pv^2(u^2v^2)^3$
F364D	$(uv^5)^2$, $pv(uv)^6$, $pvuv^2u^2v^2u^2vuv^3$
F384C	u^6v^6, $(uv^2uv^3)^2$, $(uv)^3u^2(vu)^3v^2$
F384B	$(uv^3)^3$, v^{12}, $u^2v^2uvu^2v^2u^2vuv^2$
F432D	u^6v^6, $u^2vuvu^2v^2uvuv^2$
F480B	$(u^2v^4)^2$, $uv^2(uv)^4uv^2uv^3$
F480C*	$(u^2v^4)^2$, $pv(uv)^7$
F486B	u^6v^6, $(uv^3)^3$, $(uv)^9$
F512D*	$(u^3v^3)^2$, $(uv^2uv^3)^2$
F512G*	v^{12}, $u^3vu^2vuv^2uv^3$
F512C	u^6v^6, $(uv^2uv^3)^2$, $(uv)^8$
F576C*	v^{12}, $u^2vuvu^2v^2uvuv^2$, uvu^5vuv^5

F600A*	$(uv^3)^3,\ v^{12},\ u^2v^2(uv)^3u^2(vu)^3v^2$
F648B*	$u^6v^6,\ (uv^3)^3,\ (uv)^5u^2(vu)^5v^2$
F648C*	$(uv^3)^3,\ v^{12},\ (uv)^9$
F648F*	$v^{12},\ u^3vu^3v^3uv^3,\ uvu^4vu^2v^2uv^4$
F672F*	$(u^2v^2)^3,\ u^7v^7$
F672D*	$(u^2v^2)^3,\ (u^3v^4)^2$
F680B*	$pv^4u^4v^4,\ v^{15}$
F686B*	$u^6v^6,\ (uv)^7$
F728C*	$(uvuv^3)^2,\ v^{12}$
F728F*	$(uv^5)^2,\ u^2vu^2vuvu^2vu^2v^3$
F728E*	$(uv^5)^2,\ (uv)^7,\ (uvu^2vuv^2)^2$
F728D*	$(u^2v^2)^3,\ v^{12},\ (uv)^7,\ uv^2u^3v^2uv^3uv^3$
F750*	$u^6v^6,\ uvuv^3uv^2uv^2uv^3,\ (uv)^4u^2(vu)^4v^2$
F768F*	$(uv^3)^3,\ (u^2v^5)^2,\ u^2v^2uvu^2v^2u^2vuv^2$
F768E*	$v^{12},\ uvu^5vuv^5,\ u^2v^2uvu^2v^2u^2vuv^2$
F768B*	$v^{12},\ (uv^2uv^3)^2,\ u^2v^2uvu^2v^2u^2vuv^2$
F768C*	$v^{12},\ (uv^2uv^3)^2,\ (uv)^3u^2(vu)^3v^2$
F504E	$u^7v^7,\ uv(u^2v)^2(uv^2)^2,\ (uv^4)^3$
F672E*	$(u^2v^2uv^2)^2,\ (uv^6)^2$
F768G*	$u^2vuvu^2v^2uvuv^2,\ uvu^5vuv^5,\ u^2v^2u^4v^2u^2v^4$

Case (iii): G_2^2 : 2-arc-regular graphs (with no edge-flip of order 2)

F448C*	$(vu)^7,\ pvuv^3u^5(vu)^2$

Case (iv): G_3 : 3-arc-regular graphs

F006	u^2v^2
F010	v^5
F018	v^6
F020B	$pqv^2u^2v^2$
F028	$puv^2u^2v^2$
F040	$(uv^3)^2$
F056C	$v^8,\ pqv^3uv^2uv^3$
F096B	$(u^2v^2)^2,\ v^{12}$
F112B	v^8
F192A	$(u^2v^2)^2$
F408B*	$qv(u^2v^2)^2$
F570A*	v^9
F080	u^5v^5
F110	$qv^2(uv^3)^2$
F220C	$pqv^2(u^2v^2)^2,\ v^{12}$
F640*	$v^{10},\ (u^2v^2)^3$

F680A*	$pqv^2(u^2v^2)^2,\ (uvuv^5)^2,\ v^{17}$
F506A	$v^{11},\ puv^2(u^2v^2)^2$
F162C	u^6v^6
F182D	qv^4uvuv^5
F224C	$u^2vuvu^2v^5$
F288B	$(uv^3)^3,\ v^{12}$
F360B	$(u^2v^2)^3,\ uvu^4vuv^4$
F364G	$(u^3v^3)^2,\ pqvu^3vuv^3uv^4$
F384D	$u^2vu^3v^2uv^3$
F440C	$v^{12},\ (u^3v^4)^2$
F486C	$(u^2v^2)^3,\ (uv^3)^3,\ uvu^5vuv^5$
F486D*	$(uv^3)^3,\ uvu^5vuv^5,\ u^3v^2uvu^2vu^2v^4$
F576D*	$(uv^3)^3,\ (u^2v^5)^2$
F720A*	uvu^4vuv^4
F728G*	$(u^3v^3)^2$

Case (v): G_4^1 : 4-arc-regular graphs (with an edge-flip of order 2)

F014	pu^3v^3
F102	v^9
F204	$pqu^3(vu)^2v^5$
F506B	$pqu^2vuv^2uv^7$
F620*	$v^{15},\ rvu^5(vu)^2v^5,\ qrvuvu^2vuvu^2v^6$

Case (vi): G_4^2 : 2-arc-regular graphs (with no edge-flip of order 2)

None of order ≤ 768

Case (vii): G_5 : 5-arc-regular graphs

F030	pqu^4v^4
F090	v^{10}
F234B	$v^{12},\ qrsv^2u^2vu^4v^4$
F468	v^{12}
F650B*	$qsvu^2vu^4v^4.$

4. Summary and final comments

Our results may also be conveniently summarised by the following table, which provides the complete list in a form consistent with the Foster census [3] and Gordon Royle's database [11], and filling all the gaps for order up to 768.

Each row of the table describes one graph on the list, giving it a label as described in the previous Section. Recall that an asterisk indicates the graph does not appear in the Foster census. This is followed by the order of the graph, the order of its full automorphism group, the largest s for which the graph is s-arc-transitive, the girth and diameter of the graph, an indication of whether or not the graph is bipartite, and an indication whether or not it has a Hamilton cycle.

As noted earlier, these properties were determined with the help of the MAGMA system [2], with the exception of Hamiltonicity which was checked using a computer program kindly supplied to us by Nick Wormald. Of the 298 graphs on the complete list, only two are non-Hamiltonian: the Petersen graph (on 10 vertices) and Coxeter's graph (on 28 vertices), both of which are well known to have Hamilton paths but no Hamilton cycles.

TABLE 1: **All trivalent symmetric graphs on up to 768 vertices**

Graph	Order	Automs	s-trans	Girth	Diameter	Bipartite?	Hamilton?
F004	4	24	2	3	1	No	Yes
F006	6	72	3	4	2	Yes	Yes
F008	8	48	2	4	3	Yes	Yes
F010	10	120	3	5	2	No	No
F014	14	336	4	6	3	Yes	Yes
F016	16	96	2	6	4	Yes	Yes
F018	18	216	3	6	4	Yes	Yes
F020A	20	120	2	5	5	No	Yes
F020B	20	240	3	6	5	Yes	Yes
F024	24	144	2	6	4	Yes	Yes
F026	26	78	1	6	5	Yes	Yes
F028	28	336	3	7	4	No	No
F030	30	1440	5	8	4	Yes	Yes
F032	32	192	2	6	5	Yes	Yes
F038	38	114	1	6	5	Yes	Yes
F040	40	480	3	8	6	Yes	Yes
F042	42	126	1	6	6	Yes	Yes
F048	48	288	2	8	6	Yes	Yes
F050	50	300	2	6	7	Yes	Yes
F054	54	324	2	6	6	Yes	Yes
F056A	56	168	1	6	7	Yes	Yes
F056B	56	336	2	7	6	No	Yes
F056C	56	672	3	8	7	Yes	Yes

Graph	Order	Automs	s-trans	Girth	Diameter	Bipartite?	Hamilton?
F060	60	360	2	9	5	No	Yes
F062	62	186	1	6	7	Yes	Yes
F064	64	384	2	8	6	Yes	Yes
F072	72	432	2	6	8	Yes	Yes
F074	74	222	1	6	7	Yes	Yes
F078	78	234	1	6	8	Yes	Yes
F080	80	960	3	10	8	Yes	Yes
F084	84	504	2	7	7	No	Yes
F086	86	258	1	6	9	Yes	Yes
F090	90	4320	5	10	8	Yes	Yes
F096A	96	576	2	6	8	Yes	Yes
F096B	96	1152	3	8	7	Yes	Yes
F098A	98	294	1	6	9	Yes	Yes
F098B	98	588	2	6	9	Yes	Yes
F102	102	2448	4	9	7	No	Yes
F104	104	312	1	6	9	Yes	Yes
F108	108	648	2	9	7	No	Yes
F110	110	1320	3	10	7	Yes	Yes
F112A	112	672	2	8	7	Yes	Yes
F112B	112	1344	3	8	10	Yes	Yes
F112C	112	336	1	10	7	Yes	Yes
F114	114	342	1	6	10	Yes	Yes
F120A	120	720	2	8	8	Yes	Yes
F120B	120	720	2	10	9	Yes	Yes
F122	122	366	1	6	9	Yes	Yes
F126	126	378	1	6	10	Yes	Yes
F128A	128	768	2	6	11	Yes	Yes
F128B	128	768	2	10	8	Yes	Yes
F134	134	402	1	6	11	Yes	Yes
F144A	144	432	1	8	7	Yes	Yes
F144B	144	864	2	10	8	Yes	Yes
F146	146	438	1	6	11	Yes	Yes
F150	150	900	2	6	10	Yes	Yes
F152	152	456	1	6	11	Yes	Yes
F158	158	474	1	6	11	Yes	Yes
F162A	162	972	2	6	12	Yes	Yes
F162B	162	486	1	12	7	Yes	Yes
F162C	162	1944	3	12	8	Yes	Yes
F168A	168	504	1	6	12	Yes	Yes
F168B	168	1008	2	7	9	No	Yes

Graph	Order	Automs	s-trans	Girth	Diameter	Bipartite?	Hamilton?
F168C	168	1008	2	8	8	No	Yes
F168D	168	1008	2	9	7	No	Yes
F168E	168	504	1	12	7	Yes	Yes
F168F	168	1008	2	12	8	Yes	Yes
F182A	182	546	1	6	11	Yes	Yes
F182B	182	546	1	6	13	Yes	Yes
F182C	182	1092	2	7	8	No	Yes
F182D	182	2184	3	12	9	Yes	Yes
F186	186	558	1	6	12	Yes	Yes
F192A	192	2304	3	8	12	Yes	Yes
F192B	192	1152	2	10	10	Yes	Yes
F192C	192	1152	2	12	8	Yes	Yes
F194	194	582	1	6	13	Yes	Yes
F200	200	1200	2	6	13	Yes	Yes
F204	204	4896	4	12	9	Yes	Yes
F206	206	618	1	6	13	Yes	Yes
F208	208	624	1	10	9	Yes	Yes
F216A	216	1296	2	6	12	Yes	Yes
F216B	216	1296	2	10	9	Yes	Yes
F216C	216	1296	2	12	8	Yes	Yes
F218	218	654	1	6	13	Yes	Yes
F220A	220	1320	2	10	9	No	Yes
F220B	220	1320	2	10	9	Yes	Yes
F220C	220	2640	3	10	10	Yes	Yes
F222	222	666	1	6	14	Yes	Yes
F224A	224	672	1	6	13	Yes	Yes
F224B	224	1344	2	12	9	Yes	Yes
F224C	224	2688	3	12	10	Yes	Yes
F234A	234	702	1	6	14	Yes	Yes
F234B	234	11232	5	12	8	No	Yes
F240A	240	1440	2	8	10	Yes	Yes
F240B	240	1440	2	9	10	No	Yes
F240C	240	1440	2	10	11	Yes	Yes
F242	242	1452	2	6	15	Yes	Yes
F248	248	744	1	6	15	Yes	Yes
F250	250	1500	2	10	10	Yes	Yes
F254	254	762	1	6	13	Yes	Yes
F256A	256	1536	2	8	10	Yes	Yes
F256B	256	1536	2	10	10	Yes	Yes
F256C	256	1536	2	10	11	Yes	Yes
F256D	256	768	1	12	9	Yes	Yes

Graph	Order	Automs	s-trans	Girth	Diameter	Bipartite?	Hamilton?
F258	258	774	1	6	14	Yes	Yes
F266A	266	798	1	6	15	Yes	Yes
F266B	266	798	1	6	15	Yes	Yes
F278	278	834	1	6	15	Yes	Yes
F288A	288	1728	2	6	16	Yes	Yes
F288B	288	3456	3	12	9	Yes	Yes
F294A	294	1764	2	6	14	Yes	Yes
F294B	294	882	1	6	16	Yes	Yes
F296	296	888	1	6	15	Yes	Yes
F302	302	906	1	6	15	Yes	Yes
F304	304	912	1	10	11	Yes	Yes
F312A	312	936	1	6	16	Yes	Yes
F312B	312	936	1	12	9	Yes	Yes
F314	314	942	1	6	17	Yes	Yes
F326	326	978	1	6	17	Yes	Yes
F336A	336	2016	2	8	10	Yes	Yes
F336B	336	2016	2	8	13	Yes	Yes
F336C	336	1008	1	10	12	Yes	Yes
F336D	336	2016	2	12	9	Yes	Yes
F336E	336	2016	2	12	12	Yes	Yes
F336F	336	1008	1	12	12	Yes	Yes
F338A	338	1014	1	6	15	Yes	Yes
F338B	338	2028	2	6	17	Yes	Yes
F342	342	1026	1	6	16	Yes	Yes
F344	344	1032	1	6	17	Yes	Yes
F350	350	1050	1	6	17	Yes	Yes
F360A	360	2160	2	8	11	Yes	Yes
F360B	360	4320	3	12	10	Yes	Yes
F362	362	1086	1	6	17	Yes	Yes
F364A	364	2184	2	7	11	No	Yes
F364B	364	2184	2	7	12	No	Yes
F364C	364	2184	2	7	13	No	Yes
F364D	364	2184	2	12	9	No	Yes
F364E	364	2184	2	12	9	Yes	Yes
F364F	364	2184	2	12	10	Yes	Yes
F364G	364	4368	3	12	12	Yes	Yes
F366	366	1098	1	6	18	Yes	Yes
F378A	378	1134	1	6	18	Yes	Yes
F378B	378	1134	1	12	10	Yes	Yes

Graph	Order	Automs	s-trans	Girth	Diameter	Bipartite?	Hamilton?
F384A	384	2304	2	6	16	Yes	Yes
F384B	384	2304	2	12	10	Yes	Yes
F384C	384	2304	2	12	10	Yes	Yes
F384D	384	4608	3	12	12	Yes	Yes
F386	386	1158	1	6	17	Yes	Yes
F392A	392	1176	1	6	17	Yes	Yes
F392B	392	2352	2	6	19	Yes	Yes
F398	398	1194	1	6	19	Yes	Yes
F400A	400	1200	1	8	10	Yes	Yes
F400B	400	2400	2	10	13	Yes	Yes
F402	402	1206	1	6	18	Yes	Yes
F408A	408	2448	2	9	10	No	Yes
F408B*	408	4896	3	9	10	No	Yes
F416	416	1248	1	6	19	Yes	Yes
F422	422	1266	1	6	19	Yes	Yes
F432A	432	2592	2	10	12	Yes	Yes
F432B	432	2592	2	10	14	Yes	Yes
F432C	432	1296	1	12	10	Yes	Yes
F432D	432	2592	2	12	12	Yes	Yes
F432E*	432	1296	1	8	12	Yes	Yes
F434A	434	1302	1	6	17	Yes	Yes
F434B	434	1302	1	6	19	Yes	Yes
F438	438	1314	1	6	18	Yes	Yes
F440A	440	2640	2	10	11	Yes	Yes
F440B	440	2640	2	10	12	Yes	Yes
F440C	440	5280	3	12	10	Yes	Yes
F446	446	1338	1	6	19	Yes	Yes
F448A	448	1344	1	7	11	No	Yes
F448B	448	1344	1	10	13	Yes	Yes
F448C*	448	2688	2	14	10	Yes	Yes
F450	450	2700	2	6	20	Yes	Yes
F456A	456	1368	1	6	20	Yes	Yes
F456B	456	1368	1	12	10	Yes	Yes
F458	458	1374	1	6	19	Yes	Yes
F468	468	22464	5	12	13	Yes	Yes
F474	474	1422	1	6	20	Yes	Yes
F480A	480	2880	2	9	15	No	Yes
F480B	480	2880	2	12	11	Yes	Yes
F480C*	480	2880	2	12	10	No	Yes
F480D*	480	2880	2	10	10	Yes	Yes
F482	482	1446	1	6	21	Yes	Yes

Graph	Order	Automs	s-trans	Girth	Diameter	Bipartite?	Hamilton?
F486A	486	2916	2	6	18	Yes	Yes
F486B	486	2916	2	12	12	Yes	Yes
F486C	486	5832	3	12	12	Yes	Yes
F486D*	486	5832	3	12	12	Yes	Yes
F488	488	1464	1	6	19	Yes	Yes
F494A	494	1482	1	6	19	Yes	Yes
F494B	494	1482	1	6	21	Yes	Yes
F496	496	1488	1	10	15	Yes	Yes
F500	500	3000	2	10	12	No	Yes
F504A	504	1512	1	6	20	Yes	Yes
F504B	504	1512	1	9	10	No	Yes
F504C	504	3024	2	9	12	No	Yes
F504D	504	1512	1	12	12	Yes	Yes
F504E	504	3024	2	14	10	No	Yes
F506A	506	6072	3	11	11	No	Yes
F506B	506	12144	4	14	10	Yes	Yes
F512A	512	3072	2	6	21	Yes	Yes
F512B	512	3072	2	10	12	Yes	Yes
F512C	512	3072	2	12	11	Yes	Yes
F512D*	512	3072	2	12	11	Yes	Yes
F512E*	512	1536	1	14	12	Yes	Yes
F512F*	512	3072	2	8	12	Yes	Yes
F512G*	512	3072	2	12	10	Yes	Yes
F518A*	518	1554	1	6	21	Yes	Yes
F518B*	518	1554	1	6	21	Yes	Yes
F536*	536	1608	1	6	21	Yes	Yes
F542*	542	1626	1	6	19	Yes	Yes
F546A*	546	1638	1	6	22	Yes	Yes
F546B*	546	1638	1	6	20	Yes	Yes
F554*	554	1662	1	6	21	Yes	Yes
F558*	558	1674	1	6	22	Yes	Yes
F566*	566	1698	1	6	21	Yes	Yes
F570A*	570	6840	3	9	11	No	Yes
F570B*	570	3420	2	9	11	No	Yes
F576A*	576	3456	2	10	16	Yes	Yes
F576B*	576	1728	1	8	12	Yes	Yes
F576C*	576	3456	2	12	12	Yes	Yes
F576D*	576	6912	3	12	14	Yes	Yes
F578*	578	3468	2	6	23	Yes	Yes
F582*	582	1746	1	6	22	Yes	Yes

Graph	Order	Automs	s-trans	Girth	Diameter	Bipartite?	Hamilton?
F584*	584	1752	1	6	23	Yes	Yes
F592*	592	1776	1	10	15	Yes	Yes
F600A*	600	3600	2	12	12	Yes	Yes
F600B*	600	3600	2	6	20	Yes	Yes
F602A*	602	1806	1	6	21	Yes	Yes
F602B*	602	1806	1	6	23	Yes	Yes
F608*	608	1824	1	6	21	Yes	Yes
F614*	614	1842	1	6	23	Yes	Yes
F618*	618	1854	1	6	22	Yes	Yes
F620*	620	14880	4	15	10	No	Yes
F624A*	624	1872	1	14	12	Yes	Yes
F624B*	624	1872	1	10	16	Yes	Yes
F626*	626	1878	1	6	23	Yes	Yes
F632*	632	1896	1	6	23	Yes	Yes
F640*	640	7680	3	10	12	Yes	Yes
F648A*	648	3888	2	6	24	Yes	Yes
F648B*	648	3888	2	12	14	Yes	Yes
F648C*	648	3888	2	12	12	Yes	Yes
F648D*	648	1944	1	12	13	Yes	Yes
F648E*	648	1944	1	12	12	Yes	Yes
F648F*	648	3888	2	12	10	Yes	Yes
F650A*	650	1950	1	6	23	Yes	Yes
F650B*	650	31200	5	12	11	Yes	Yes
F654*	654	1962	1	6	24	Yes	Yes
F660*	660	3960	2	10	11	No	Yes
F662*	662	1986	1	6	21	Yes	Yes
F666*	666	1998	1	6	22	Yes	Yes
F672A*	672	2016	1	12	12	Yes	Yes
F672B*	672	2016	1	6	24	Yes	Yes
F672C*	672	4032	2	8	12	No	Yes
F672D*	672	4032	2	12	12	Yes	Yes
F672E*	672	4032	2	14	12	Yes	Yes
F672F*	672	4032	2	12	13	Yes	Yes
F672G*	672	2016	1	12	12	Yes	Yes
F674*	674	2022	1	6	23	Yes	Yes
F680A*	680	8160	3	10	11	No	Yes
F680B*	680	4080	2	12	10	No	Yes
F686A*	686	2058	1	6	25	Yes	Yes
F686B*	686	4116	2	12	12	Yes	Yes
F686C*	686	2058	1	6	23	Yes	Yes

Graph	Order	Automs	s-trans	Girth	Diameter	Bipartite?	Hamilton?
F688*	688	2064	1	10	17	Yes	Yes
F698*	698	2094	1	6	25	Yes	Yes
F702A*	702	2106	1	12	14	Yes	Yes
F702B*	702	2106	1	6	24	Yes	Yes
F720A*	720	8640	3	12	12	Yes	Yes
F720B*	720	4320	2	8	12	Yes	Yes
F720C*	720	4320	2	10	10	Yes	Yes
F720D*	720	2160	1	8	12	Yes	Yes
F720E*	720	4320	2	8	16	Yes	Yes
F720F*	720	2160	1	10	11	Yes	Yes
F722A*	722	2166	1	6	25	Yes	Yes
F722B*	722	4332	2	6	25	Yes	Yes
F726*	726	4356	2	6	22	Yes	Yes
F728A*	728	2184	1	6	23	Yes	Yes
F728B*	728	2184	1	6	25	Yes	Yes
F728C*	728	4368	2	12	12	Yes	Yes
F728D*	728	4368	2	12	12	Yes	Yes
F728E*	728	4368	2	12	13	Yes	Yes
F728F*	728	4368	2	12	14	Yes	Yes
F728G*	728	8736	3	12	14	Yes	Yes
F734*	734	2202	1	6	23	Yes	Yes
F744A*	744	2232	1	12	13	Yes	Yes
F744B*	744	2232	1	6	24	Yes	Yes
F746*	746	2238	1	6	25	Yes	Yes
F750*	750	4500	2	12	12	Yes	Yes
F758*	758	2274	1	6	25	Yes	Yes
F762*	762	2286	1	6	26	Yes	Yes
F768A*	768	4608	2	10	18	Yes	Yes
F768B*	768	4608	2	12	11	Yes	Yes
F768C*	768	4608	2	12	11	Yes	Yes
F768D*	768	2304	1	12	12	Yes	Yes
F768E*	768	4608	2	12	12	Yes	Yes
F768F*	768	4608	2	12	13	Yes	Yes
F768G*	768	4608	2	14	12	Yes	Yes

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