Some designs of small orders and their codes

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Abstract

The binary and ternary codes spanned by the rows of the point by block, pair by block, block by point incidence matrices of some 2-designs of small orders and their orthogonal complements are studied. Among some results, it is shown that if the code is properly chosen, then the weight distribution of the code serves as an appropriate design isomorphism invariant. The automorphism groups of the codes and the design are computed.

1. Introduction

Let v, k, and t be positive integers such that $v > k > t > 0$. Let X be a v-set. The elements of X are called *points* and the k-subsets of X are called *blocks*.

Let λ be a positive integer. A t-(v, k, λ) *design* D (or briefly a t-*design*) is a pair $(X, \mathcal{B})$, where $\mathcal{B}$ is a collection of blocks with the property that every t-subset of X occurs in exactly λ blocks of $\mathcal{B}$. v is called the *order* of the t-design. A t-design is *simple* if it contains no repeated blocks. It is well known that any t-design is also an i-design for $0 \leq i \leq t$. Two t-designs $(X, \mathcal{B})$ and $(X', \mathcal{B}')$ are *isomorphic* if and only if there exists a permutation $\sigma : X \longrightarrow X'$ such that $\sigma(\mathcal{B}) = \mathcal{B}'$, i.e. σ transforms the blocks of $\mathcal{B}$ onto the blocks of $\mathcal{B}'$. An *automorphism* of a t-design is a permutation on X which preserves its set of blocks.

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One can associate to every t-design the following set of incidence matrices: Let $D = (X, \mathcal{B})$ be a simple t-(v, k, λ) design with $|\mathcal{B}| = b$. Then for any i, $0 < i \leq t$, we denote by $\mathcal{D}_i$ a $\binom{v}{i} \times b, (0,1)$-matrix whose rows and columns are indexed by the i-subsets I of X and the blocks B of D, respectively, with its entries defined as follows:

$$\mathcal{D}_i(I, B) = \begin{cases} 1 & \text{if } I \subseteq B, \\ 0 & \text{otherwise.} \end{cases}$$

Now, a few words about codes: A *linear* $[n, k]$ *code* C of length n is a k-dimensional subspace of an n-dimensional vector space over a given finite field $F = GF(q)$.

The (Hamming) *weight* of a codeword $x \in C$, denoted by $w(x)$, is the number of non-zero components of x. Let $d = \min\{w(x) | x \in C, x \neq 0\}$. Let W_i denote the number of codewords in C of weight i. Then $\{W_i\}_{i=1}^{n}$ is called the *weight distribution* of C. The minimum weight is an important parameter of a code, and its weight distribution carries important information about it. Therefore, an $[n, k, d]$ code is an $[n, k]$ code with the minimum weight d.

Two codes are *isomorphic* if one of them can be obtained from the other one by a permutation of the n coordinate positions. An automorphism of a code is any permutation of the coordinates that preserves the code as a set of vectors. The set of all permutations forms under composition, the *automorphism group of* C.

The codes over $GF(2)$ and $GF(3)$ are called *binary* and *ternary* codes, respectively.

In this paper, we consider only (binary or ternary) codes arising from the row spaces of the incidence matrices $\mathcal{D}_i, \mathcal{D}_i^T, \mathcal{D}_i^{\perp}$, and $\mathcal{D}_i^{\perp T}$ of a given 2-design D. Associated codes are denoted by $C_i, C_i^T, C_i^{\perp}$, and $C_i^{\perp T}$, respectively.

Tonchev *et al.* [14], studied the binary codes C_1 and C_1^T arising from $\mathcal{D}_1$'s of the 80 non-isomorphic 2-$(15, 3, 1)$ designs. Among many interesting computational results, they noted that all the resulting codes C_1 are non-isomorphic, whereas there are only 5 non-isomorphic classes of codes generated by the associated $\mathcal{D}_1^T$'s. Some further studies ,regarding the codes from Steiner triple systems, have also been reported in [1,2,5,6,13].

More recently, two classes of codes coming from trivial designs were studied [7,8]. For a given v, the simple design $D = t\text{-}(v, k, \binom{v-t}{k-t})$ is called the trivial design. In [8], the family of designs $D = 2\text{-}(v, 3, v-2)$ were considered and the class of ternary codes $C_2^{\perp}$ associated with $\mathcal{D}_2$ were studied and some interesting information about these codes were reported. Also in [7], the class of binary codes C_1 again associated with the above mentioned trivial designs were studied and some optimal codes were found.

In this paper, following the computational style of [14], we consider all the 2-designs of small orders listed in [11], then we choose an "appropriate" incidence matrix from 4 different types of matrices mentioned above and then we construct the binary or ternary codes arising from them.

Our results show that the weight distribution of these codes can serve as a very reliable design isomorphism invariant. In particular, we study the two families of designs more carefully, namely, 2-(15, 3, 1) and 2-(9, 3, 4) designs. Meanwhile, we compute automorphism groups of designs and their related codes.

For further definitions, we refer the reader to [14].

2. A Computational lemma

Let C be an $[n, k, d]$ linear code. Let A_i, $d \leq i \leq n$, denote all the codewords of weight i (A_i could be empty). Then we can represent C as the following matrix in which every row is a codeword:

$$C = \begin{pmatrix} A_d \\ \hline A_{d+1} \\ \hline \vdots \\ \hline A_n \end{pmatrix}.$$

The automorphism of each A_i or some union of them is defined to be similar to the automorphism of C.

Proposition. Let D be a t-design, then

(i) The number of designs included in the code C_i (i=1 or 2) which are isomorphic to D is $|\text{Aut}C_i|/|\text{Aut}D|$;

(ii) $|\text{Aut}C| = |\text{Aut}C^{\perp}|$;

(iii) $D \cong D' \Leftrightarrow C \cong C'$. □

Lemma. Let C be a code obtained from a t-design D. Then

(i) If a union of A_i's, $\cup_I A_i$, contains a basis for the code C, then $\text{Aut}(\cup_I A_i) = \text{Aut}C$;

(ii) If the dimension of $\cup_I A_i$ as a matrix is less than the rank of C and $|\text{Aut}D| = |\text{Aut}(\cup_I A_i)|$, then $\text{Aut}D = \text{Aut}C$.

Proof. For every $\sigma \in \text{Aut}(\cup_I A_i)$, we apply σ to the columns of C and obtain a new code C'. C' has a basis for C since $\sigma(\cup_I A_i) = \cup_I A_i$. Therefore, $C' = C$. This means that $\sigma \in \text{Aut}C$. Clearly every $\sigma \in \text{Aut}C$ only commutes the rows of each A_i internally. This completes the proof of (i). For (ii), it is sufficient to note that

$$|\text{Aut}D| \leq |\text{Aut}C| \leq |\text{Aut}(\cup_I A_i)|.$$

Definition. The *weight matrix* of a code is a matrix $W = (w_{ij})$, where w_{ij} is the weight of the j-th column of the submatrix A_i.

Let $B = \cup_{i=d}^{s} A_i$ be the first union of A_i's that contains a basis for the code C. Clearly, if $\sigma \in \text{Aut}C$, then σ just commutes similar columns of the matrix W. To determine $\text{Aut}C$, we apply all such permutations to B, and using Lemma (i), we check whether they belong to $\text{Aut}C$. Sometimes too many columns in W are similar. Therefore, the method involves too much work, so one might use program *nauty* [12] instead. For instance, in Table 2, we have applied this method for exactly 312 cases and in 20 other cases we have employed *nauty* and in only one special case we had to use Lemma (ii).

3. Ternary codes from 2-$(15, 3, 1)$ designs

As mentioned before, Tonchev, *et al.* have studied the binary codes C_1 arising from non-isomorphic 2-$(15, 3, 1)$ designs. In this section, we study the ternary codes C_1 of these designs. The results are reported in Table 1. The labeling of these designs is the same as in [11]. To show that the weight distributions of these codes are different, it was sufficient to go up to W_{13}. Also, a few points of interest are worth to be mentioned here:

- The weight distributions of these codes are different, whence this is not so in the binary case[14]. Therefore it provides a suitable design isomorphism invariant for this classical case.
- We note that for the whole case, $|\text{Aut}D| = |\text{Aut } C|$. By Proposition (i), clearly every code contains only one design.
- Contrary to the binary case, the largest automorphism group belongs to the code associated with the geometric design $PG(2,3)$, which pocesses the largest automorphism group among the 80 designs.
- For all of the 80 designs, rank$\mathcal{D}_1 = 14$. On the other hand, the 3-rank of the trivial design, namely 2-$(15,3,13)$ is also equal to 14 [15]. This means that there is a unique C_1^T for all these 80 designs, whereas in the binary case, the block by point matrices produce 5 non-isomorphic codes [14].

4. Ternary codes from 2-$(9,3,4)$ designs

There exist 332 non-isomorphic simple 2-$(9,3,3)$ designs [4]. In [10], an algorithm to produce the whole family of 2-$(9,3,3)$ designs (including designs with repeated blocks) which amounts to a total of 22521 non-isomorphic designs is presented. Employing this algorithm, we have reproduced all the simple designs of this family [3].We studied the ternary codes $C_2^\perp$ arising from $\mathcal{D}_2^\perp$ associated with the simple 2-$(9,3,4)$ designs. We note that each of these codes is an $[48, k]$ code, for $k = 13$ or 14. The results are reported in Table 2. There are some noteworthy points here to be mentioned too:

- Here again, the weight distributions of these codes provide a very efficient design isomorphism invariant, and it suffices to go up to W_{16}.

- The code #284 with $|\text{Aut } C|/|\text{Aut } D| = 48$, has the largest automorphism group among these codes.
- The code #277 has the largest minimum weight with $d = 14$.
- For the case $k = 13$, the codes C_2^T, are all isomorphic. This simply follows from Wilson's p-rank theorem as in the previous section. Contrary to this, for $k = 14$, there exist 3 non-isomorphic C_2^T codes. The weight distribution of these codes $C_2^{T\perp}$ are listed in Table 4.
- The designs #277, 302, 317, 318, 326, and 327 are the only designs which can be partitioned into 4 copies of 2-$(9, 3, 1)$ designs and can also be extended to large sets of designs.There are only 2 non-isomorphic large sets in this family of designs [9].

5. Binary codes from 2-$(9, 3, 4)$ designs

We consider binary codes $C_2^\perp$ arising from $\mathcal{D}_2^\perp$ associated with the family of 2-$(9, 3, 4)$ designs.

- Again, the weight distributions of the codes furnish a good design isomorphism invariant, and it suffices to go up to W_{12}. See Table 3.
- The code #326 has the largest automorphism group.
- The codes C_2^T are all isomorphic.

6. Some codes of designs of small order

In Table 5, we demonstrate the parameters of some designs of small orders and the results of the weight distributions of some related codes serving as isomorphism invariant. As it is seen, in only two cases the results are disappointing.

Table 1. Ternary Codes from 2-(15, 3, 1) designs.

D^*	k^*	$\|AutD\|$	$\|AutC\|$	W_7	W_9	W_{10}	W_{12}	W_{13}
1	14	20160	20160	30	0	0	1680	3150
2	14	192	192	30	0	48	1056	2670
3	14	96	96	30	0	72	744	2430
4	14	8	8	30	0	64	728	2462
5	14	32	32	30	0	48	912	2662
6	14	24	24	30	0	48	840	2598
7	14	288	288	30	0	0	1248	3126
8	14	4	4	30	0	64	656	2458
9	14	2	2	30	0	60	592	2438
10	14	2	2	30	0	52	684	2538
11	14	2	2	30	4	48	668	2570
12	14	3	3	30	2	66	574	2422
13	14	8	8	30	0	60	648	2474
14	14	12	12	30	0	72	564	2358
15	14	4	4	30	0	44	740	2606
16	14	168	168	30	0	84	588	2310
17	14	24	24	30	0	36	816	2706
18	14	4	4	30	0	52	648	2506
19	14	12	12	30	8	24	856	2818
20	14	3	3	30	2	54	574	2470
21	14	3	3	30	2	54	580	2494
22	14	3	3	30	8	48	598	2542
23	14	1	1	30	0	44	582	2584
24	14	1	1	30	2	54	546	2502
25	14	1	1	30	2	54	566	2500
26	14	1	1	30	2	60	552	2460
27	14	1	1	30	2	38	620	2650
28	14	1	1	30	4	48	584	2568
29	14	3	3	30	2	54	538	2482
30	14	2	2	30	4	44	614	2628
31	14	4	4	30	0	40	656	2618
32	14	1	1	30	4	40	600	2672
33	14	1	1	30	4	46	564	2578
34	14	1	1	30	4	42	570	2620
35	14	3	3	30	2	36	556	2632
36	14	4	4	30	0	40	624	2682
37	14	12	12	30	0	24	696	2826
38	14	1	1	30	4	24	636	2774
39	14	1	1	30	2	40	596	2672
40	14	1	1	30	2	48	570	2590
41	14	1	1	30	2	40	572	2608
42	14	2	2	30	8	36	694	2860
43	14	6	6	30	0	36	678	2772
44	14	2	2	30	0	28	606	2732
45	14	1	1	30	2	30	624	2766
46	14	1	1	30	4	28	616	2792
47	14	1	1	30	2	36	584	2678
48	14	1	1	30	4	36	594	2714
49	14	1	1	30	4	32	608	2750
50	14	1	1	30	4	16	708	2884
51	14	1	1	30	6	38	602	2720
52	14	1	1	30	4	36	556	2664
53	14	1	1	30	4	34	562	2652
54	14	1	1	30	8	46	586	2654
55	14	1	1	30	6	38	608	2732
56	14	1	1	30	4	32	580	2708
57	14	1	1	30	4	24	658	2842
58	14	1	1	30	4	30	616	2734
59	14	3	3	30	8	54	562	2626
60	14	1	1	30	10	34	670	2812
61	14	21	21	30	14	42	616	2590
62	14	3	3	30	2	24	544	2686
63	14	3	3	30	2	12	700	2908
64	14	3	3	30	8	42	586	2638
65	14	1	1	30	4	28	618	2792
66	14	1	1	30	4	20	638	2850
67	14	1	1	30	4	16	664	2910
68	14	1	1	30	2	22	592	2780
69	14	1	1	30	6	18	628	2830
70	14	1	1	30	4	36	596	2688
71	14	1	1	30	2	18	602	2788
72	14	1	1	30	6	22	648	2856
73	14	4	4	30	8	24	768	2970
74	14	4	4	30	0	32	640	2734
75	14	3	3	30	8	36	694	2826
76	14	5	5	30	8	40	570	2570
77	14	3	3	30	2	6	592	2904
78	14	4	4	30	8	24	728	2858
79	14	36	36	30	0	0	960	3138
80	14	60	60	30	0	0	840	3150

* D denotes the design and k is the dimension of the code.

Table 2. Ternary Codes from 2-(9,3,4) designs.

D^*	k^*	$\|AutD\|$	$\|AutC\|$	W_8	W_{12}	W_{14}	W_{15}	W_{16}
1	14	1	1	6	30	38	24	254
2	13	1	2	0	16	26	14	114
3	13	1	4	4	22	26	14	132
4	13	1	4	6	24	24	18	120
5	13	1	2	4	16	18	12	126
6	13	1	4	8	28	20	18	140
7	13	1	2	10	34	18	8	168
8	13	1	4	8	22	24	14	166
9	13	1	4	6	18	28	10	150
10	13	1	2	6	22	18	10	128
11	13	1	4	4	26	18	14	128
12	13	1	4	4	30	28	10	168
13	13	1	2	6	26	22	10	116
14	13	1	4	2	18	28	16	108
15	13	1	2	4	16	22	26	138
16	13	1	2	6	16	18	18	100
17	13	1	4	6	28	26	12	152
18	13	1	2	8	34	26	6	144
19	13	1	4	6	24	18	22	154
20	13	1	2	8	20	24	10	124
21	13	1	2	4	26	26	6	138
22	13	1	2	8	30	40	12	156
23	13	1	2	4	14	42	12	94
24	13	1	2	8	26	20	10	124
25	13	1	2	10	26	28	4	130
26	13	1	2	4	26	22	14	126
27	13	1	4	8	26	38	24	150
28	13	1	2	6	22	36	22	138
29	13	1	4	8	16	22	20	116
30	13	1	4	6	22	30	10	118
31	13	1	4	4	20	22	8	114
32	13	1	2	10	32	24	6	138
33	13	1	2	10	26	28	10	152
34	13	1	4	10	30	22	8	186
35	13	1	2	4	18	18	24	100
36	13	1	4	4	22	24	12	126
37	13	1	2	4	22	20	22	112
38	13	1	2	6	30	24	12	88
39	13	1	4	6	20	16	26	122
40	13	1	2	4	24	12	10	88
41	13	1	2	4	30	28	10	122
42	13	1	4	8	28	18	10	142
43	13	1	2	4	30	32	12	108
44	13	1	4	4	30	14	12	136
45	13	1	4	4	22	26	18	120
46	13	1	4	2	20	26	8	150
47	13	1	2	4	34	22	6	126
48	13	1	4	4	22	26	4	118
49	13	1	4	6	18	28	14	144
50	13	1	2	4	28	10	16	126
51	13	1	2	4	38	34	4	122
52	13	1	4	8	26	26	6	174
53	13	1	2	2	20	28	18	110
54	13	1	2	4	14	34	24	114
55	13	1	4	4	22	28	12	134
56	13	1	4	4	14	30	10	134
57	13	1	4	4	22	24	8	150
58	13	1	2	4	16	22	10	126
59	13	1	4	4	14	28	28	146
60	13	1	2	2	20	24	26	96
61	13	1	2	4	14	22	8	106
62	13	1	4	4	18	22	14	116
63	13	1	2	6	18	18	14	128
64	13	1	2	4	26	16	8	130
65	13	1	4	6	18	28	4	130
66	13	1	4	4	12	20	12	112
67	13	1	4	6	16	16	8	130
68	13	1	12	10	28	50	24	176
69	13	1	4	2	16	40	14	102
70	13	1	2	4	20	22	22	94
71	13	1	2	0	18	52	4	128
72	13	1	4	2	18	34	8	104
73	13	1	2	12	20	26	10	128
74	13	1	4	2	12	40	6	118
75	13	1	2	4	22	10	12	116
76	13	1	2	10	26	38	20	126
77	13	1	2	12	14	26	10	170
78	13	1	2	8	18	28	18	110
79	13	1	2	12	24	26	20	152
80	13	1	4	8	22	24	4	156
81	13	1	2	4	16	20	6	110
82	13	1	2	4	12	28	10	124
83	13	1	2	4	20	26	16	132

* D denotes the design and k is the dimension of the code.

Table 2. Continued.

D	k	$\|AutD\|$	$\|AutC\|$	W_8	W_{12}	W_{14}	W_{15}	W_{16}
84	13	1	2	2	22	26	16	118
85	13	1	2	2	18	24	14	96
86	13	1	2	2	28	38	6	118
87	13	1	2	2	30	32	8	146
88	13	1	2	2	20	22	6	110
89	13	1	2	2	14	32	20	78
90	13	1	2	6	22	32	16	124
91	13	1	2	4	18	32	14	108
92	13	1	4	6	30	30	12	138
93	13	1	2	6	24	38	10	120
94	13	1	2	8	16	32	6	130
95	13	1	2	4	18	36	16	110
96	13	1	2	10	24	20	10	150
97	13	1	4	8	30	44	12	164
98	13	1	4	4	30	42	12	126
99	13	1	4	0	18	44	8	108
100	13	1	2	4	20	32	6	118
101	13	1	8	4	18	32	12	128
102	13	1	4	8	20	16	20	96
103	13	1	2	4	18	48	8	118
104	13	1	2	0	24	38	18	124
105	13	1	4	6	18	16	14	110
106	13	1	4	6	12	28	2	162
107	13	1	4	2	26	32	14	170
108	13	1	2	8	26	28	16	114
109	13	1	2	4	20	30	8	110
110	13	1	2	6	32	38	6	126
111	13	1	2	4	14	24	4	108
112	13	1	2	6	28	16	8	128
113	13	1	2	4	20	24	12	106
114	13	1	2	12	26	24	8	162
115	13	1	4	4	26	24	12	132
116	13	1	2	6	14	28	26	86
117	13	1	4	10	16	38	16	128
118	13	1	2	4	8	24	26	94
119	13	1	8	4	20	32	16	138
120	13	1	2	2	18	32	6	96
121	13	1	2	4	30	28	14	122
122	13	1	4	4	14	26	22	122
123	13	1	2	2	20	28	22	122
124	13	1	2	2	18	26	18	96
125	13	1	2	2	18	34	12	116
126	13	1	2	2	26	24	12	94
127	13	1	4	4	36	38	16	176
128	13	1	2	8	26	12	14	118
129	13	1	2	6	32	18	10	132
130	13	1	6	4	20	28	10	174
131	13	1	4	6	26	42	14	142
132	13	1	2	8	32	14	2	158
133	13	1	2	0	18	28	20	90
134	13	1	2	6	26	10	6	130
135	13	1	2	4	34	12	6	116
136	13	1	2	4	26	26	12	96
137	13	1	2	6	16	26	12	122
138	13	1	4	8	30	26	4	136
139	13	1	2	6	22	28	10	110
140	13	1	2	2	20	36	10	134
141	13	1	2	2	20	28	12	120
142	13	1	1	8	20	22	14	102
143	13	1	2	8	20	32	14	126
144	13	1	1	8	24	22	20	106
145	13	1	2	4	12	34	20	114
146	13	1	2	4	26	20	12	124
147	13	1	1	4	18	20	18	88
148	13	1	2	6	20	26	16	96
149	13	1	1	4	20	34	10	82
150	13	1	1	8	18	18	16	116
151	13	1	2	10	42	58	8	202
152	13	1	2	4	20	14	10	88
153	13	1	1	6	20	18	10	92
154	13	1	2	10	24	24	16	124
155	13	1	2	6	22	22	6	120
156	13	1	2	4	12	36	16	102
157	13	1	1	4	18	44	4	114
158	13	1	2	2	18	26	10	100
159	13	1	1	4	20	26	4	72
160	13	1	2	2	20	30	4	110
161	13	1	1	6	12	14	12	106
162	13	1	2	4	18	28	12	96
163	13	1	1	4	26	18	12	70
164	13	1	2	0	18	30	18	128
165	13	1	2	0	18	24	24	122
166	13	1	2	4	24	24	14	84

Table 2. Continued.

D	k	$\|AutD\|$	$\|AutC\|$	W_8	W_{12}	W_{14}	W_{15}	W_{16}
167	13	1	1	8	24	34	8	104
168	13	1	1	2	26	18	14	108
169	13	1	2	2	24	42	18	102
170	13	1	2	2	14	34	22	84
171	13	1	2	4	16	26	10	98
172	13	1	2	4	12	30	10	114
173	13	1	2	2	20	40	10	82
174	13	1	2	2	22	30	8	118
175	13	1	2	0	16	24	20	116
176	13	1	2	0	20	20	18	106
177	13	1	2	2	18	14	18	88
178	13	1	1	2	26	26	18	120
179	13	1	2	2	20	22	14	98
180	13	1	1	4	18	30	8	58
181	13	1	4	8	24	36	10	172
182	13	1	4	6	16	32	8	140
183	13	1	2	2	32	24	10	116
184	13	1	2	2	28	36	0	120
185	13	1	2	4	26	40	6	110
186	13	1	4	4	16	24	10	116
187	13	1	1	10	20	28	6	138
188	13	1	2	8	24	26	12	134
189	13	1	2	8	16	24	12	130
190	13	1	1	10	18	38	10	108
191	13	1	2	10	26	32	16	158
192	13	1	1	8	24	28	6	94
193	13	1	1	10	8	24	14	104
194	13	1	2	4	22	28	10	102
195	13	1	2	8	32	22	14	156
196	13	1	2	6	30	20	6	110
197	13	1	1	6	14	16	6	90
198	13	1	2	6	18	20	4	96
199	13	1	2	6	16	28	4	82
200	13	1	1	6	16	14	4	74
201	13	1	1	8	20	30	12	108
202	13	1	2	8	32	30	8	146
203	13	1	1	6	34	24	6	94
204	13	1	1	6	24	20	8	92
205	13	1	2	8	48	58	6	184
206	13	1	2	6	18	42	10	100
207	13	1	1	10	18	18	2	106
208	13	1	1	8	36	34	4	140
209	13	1	2	10	26	22	6	138
210	13	1	1	14	38	40	8	162
211	13	1	1	6	34	40	4	96
212	13	1	2	6	28	20	6	100
213	13	1	1	6	20	40	18	84
214	13	1	2	2	28	26	8	106
215	13	1	2	10	22	26	8	148
216	13	1	2	4	18	42	8	92
217	13	1	1	0	18	28	10	86
218	13	1	1	4	14	34	8	76
219	13	1	2	6	16	22	6	118
220	13	1	2	0	24	38	14	142
221	13	1	1	2	16	24	8	96
222	13	1	2	4	18	24	10	124
223	13	1	1	4	20	38	8	92
224	13	1	2	6	28	20	14	102
225	13	1	2	2	18	24	10	100
226	13	1	4	8	20	22	6	142
227	13	1	2	12	20	34	4	150
228	13	1	2	8	14	24	6	142
229	13	1	2	12	28	20	2	178
230	13	1	2	0	22	26	16	130
231	13	1	4	4	22	18	8	132
232	13	1	2	8	22	12	12	130
233	13	1	2	2	18	20	10	124
234	13	1	4	8	16	14	8	130
235	13	1	2	10	16	30	4	136
236	13	1	2	4	22	24	4	114
237	13	1	2	6	38	18	6	166
238	13	1	2	6	16	14	14	142
239	13	1	2	2	30	32	10	118
240	13	1	2	4	30	14	10	146
241	13	1	1	10	34	18	2	136
242	13	1	1	12	32	22	12	128
243	13	1	2	8	26	18	12	148
244	13	1	1	12	22	24	2	136
245	13	1	1	8	30	12	0	144
246	13	1	1	0	34	4	10	102
247	13	1	1	10	44	12	0	146
248	13	1	2	8	34	12	10	144
249	13	1	2	10	36	24	0	174

Table 2. Continued.

D	k	$\lvert AutD\rvert$	$\lvert AutC\rvert$	W_8	W_{12}	W_{14}	W_{15}	W_{16}
250	13	1	1	4	24	20	6	116
251	13	1	2	2	22	32	6	122
252	13	1	2	10	36	14	4	152
253	13	1	2	8	20	26	6	118
254	13	1	2	6	30	16	10	104
255	13	1	1	0	24	12	10	92
256	13	1	1	4	16	16	8	92
257	13	1	1	10	32	14	10	102
258	13	1	2	8	28	10	12	136
259	13	1	4	4	38	36	6	146
260	13	1	2	2	12	24	10	106
261	13	1	2	6	32	16	6	150
262	13	1	2	2	18	34	16	106
263	13	1	2	2	16	20	10	112
264	13	1	2	2	16	20	10	124
265	13	1	2	6	18	20	14	106
266	13	1	1	2	18	26	14	110
267	13	1	1	4	22	26	2	98
268	13	1	1	4	16	38	2	110
269	13	1	1	2	14	14	10	90
270	13	1	1	0	12	26	20	102
271	13	1	1	6	18	22	4	90
272	13	1	1	0	10	30	14	90
273	13	1	1	4	22	6	10	76
274	13	1	1	6	22	22	10	124
275	13	1	1	2	14	28	8	64
276	14	16	16	16	40	40	64	418
277	13	18	18	0	0	18	36	144
278	13	2	8	4	28	32	10	158
279	13	2	8	4	24	26	6	122
280	13	2	4	12	28	34	28	166
281	13	2	8	4	14	44	30	170
282	13	2	8	6	20	28	16	128
283	13	2	4	8	32	2	18	146
284	13	2	96	4	30	24	16	152
285	13	2	2	8	14	28	24	118
286	13	2	2	12	44	26	8	216
287	13	2	2	0	20	44	18	76
288	13	2	8	6	18	26	16	154
289	13	2	4	6	12	22	8	128
290	13	2	4	0	28	26	12	148
291	13	2	2	10	6	26	12	114
292	13	2	4	6	20	30	14	122
293	13	2	8	8	22	26	8	122
294	13	2	2	10	20	14	18	118
295	13	2	2	4	26	24	6	120
296	13	2	2	8	22	30	8	132
297	13	2	2	4	38	32	8	136
298	13	2	4	2	16	42	8	92
299	13	2	2	0	24	54	8	82
300	13	2	4	8	36	16	4	176
301	13	2	2	6	22	14	12	84
302	13	2	8	8	18	26	8	180
303	13	2	4	10	24	18	2	146
304	13	2	8	6	30	18	8	130
305	13	2	8	6	42	14	4	158
306	13	2	2	2	16	26	12	78
307	13	2	2	14	46	26	8	206
308	13	2	2	12	36	52	0	234
309	13	2	4	4	24	26	8	140
310	13	2	4	0	24	38	12	112
311	13	2	2	0	20	38	20	120
312	13	2	2	6	26	16	8	102
313	13	2	2	8	44	46	6	198
314	13	2	2	6	46	50	4	168
315	14	24	24	12	48	40	0	426
316	14	3	3	8	36	46	12	252
317	14	3	3	0	28	72	24	222
318	13	3	24	4	12	38	2	138
319	13	3	3	10	24	44	8	132
320	13	3	3	4	30	2	2	72
321	13	3	3	6	18	18	12	108
322	14	4	4	8	40	36	24	348
323	14	4	4	4	40	48	8	230
324	14	4	4	2	28	60	32	242
325	14	4	4	10	44	48	8	328
326	14	54	54	0	72	108	54	108
327	13	6	6	18	26	48	0	264
328	13	6	6	0	30	18	12	132
329	13	6	6	0	18	6	12	84
330	13	6	6	12	50	0	0	252
331	14	6	6	12	70	132	18	372
332	13	9	9	6	0	6	24	72

Table 3. Binary Codes from 2-(9, 3, 4) designs.

D^*	k^*	$\|AutD\|$	$\|AutC\|$	W_4	W_6	W_8	W_{10}	W_{12}
1	20	1	1	2	14	73	302	1084
2	20	1	1	2	14	73	290	1128
3	20	1	1	2	13	79	282	1117
4	20	1	1	2	13	77	291	1112
5	20	1	1	4	13	67	298	1162
6	20	1	1	3	10	82	279	1128
7	20	1	1	4	9	81	285	1151
8	20	1	1	3	15	71	284	1160
9	20	1	1	3	16	64	302	1139
10	20	1	1	2	17	63	294	1127
11	20	1	1	2	13	78	287	1108
12	20	1	1	3	12	74	287	1135
13	20	1	1	3	15	64	292	1136
14	20	1	1	2	17	59	316	1104
15	20	1	1	2	15	68	305	1090
16	20	1	1	1	18	65	298	1116
17	20	1	1	4	9	76	294	1116
18	20	1	1	4	9	79	296	1133
19	20	1	1	3	15	63	318	1119
20	20	1	1	4	10	78	305	1140
21	20	1	1	3	12	73	291	1127
22	20	1	1	4	9	76	295	1097
23	20	1	1	5	11	65	286	1186
24	20	1	1	4	8	78	296	1114
25	20	1	1	4	14	66	297	1133
26	20	1	1	3	12	73	298	1101
27	20	1	1	4	11	68	303	1127
28	20	1	1	3	13	68	304	1103
29	20	1	1	3	14	66	317	1104
30	20	1	1	4	13	63	317	1124
31	20	1	1	3	16	60	311	1146
32	20	1	1	4	13	65	301	1097
33	20	1	1	5	9	74	293	1159
34	20	1	1	4	11	75	311	1127
35	20	1	1	4	11	72	295	1169
36	20	1	1	4	12	70	293	1183
37	20	1	1	3	12	70	305	1117
38	20	1	1	3	10	80	288	1121
39	20	1	1	2	13	78	287	1108
40	20	1	1	2	14	71	298	1110
41	20	1	1	3	11	75	294	1123
42	20	1	1	4	14	66	308	1165
43	20	1	1	3	12	75	285	1140
44	20	1	1	3	14	63	309	1121
45	20	1	1	2	16	64	305	1115
46	20	1	1	2	15	68	302	1109
47	20	1	1	3	14	64	298	1148
48	20	1	1	2	18	63	297	1157
49	20	1	1	3	12	74	289	1137
50	20	1	1	2	15	69	290	1141
51	20	1	1	4	9	73	299	1146
52	20	1	1	4	10	76	297	1136
53	20	1	1	3	15	69	289	1171
54	20	1	1	3	15	63	296	1151
55	20	1	1	2	15	73	278	1152
56	20	1	1	3	14	68	307	1138
57	20	1	1	3	16	65	293	1170
58	20	1	1	3	13	69	290	1156
59	20	1	1	2	16	63	312	1105
60	20	1	1	2	14	73	294	1119
61	20	1	1	3	12	71	296	1140
62	20	1	1	2	17	60	307	1131
63	20	1	1	2	15	69	295	1129
64	20	1	1	3	13	71	301	1150
65	20	1	1	3	13	71	305	1132
66	20	1	1	3	16	58	320	1148
67	20	1	1	3	17	63	306	1175
68	20	1	1	4	11	74	277	1166
69	20	1	1	3	17	58	293	1159
70	20	1	1	2	17	61	299	1145
71	20	1	1	3	15	64	292	1143
72	20	1	1	4	14	60	298	1161
73	20	1	1	4	11	74	306	1157
74	20	1	1	2	16	63	307	1115
75	20	1	1	2	15	68	293	1138
76	20	1	1	4	9	79	280	1144
77	20	1	1	5	11	76	291	1157
78	20	1	1	4	10	75	286	1136
79	20	1	1	4	9	81	288	1138
80	20	1	1	4	11	72	301	1146
81	20	1	1	3	14	65	297	1137
82	20	1	1	4	14	66	299	1140
83	20	1	1	3	13	70	294	1129

* D denotes the design and k is the dimension of the code.

Table 3. Continued.

D	k	$\|AutD\|$	$\|AutC\|$	W_4	W_6	W_8	W_{10}	W_{12}
84	20	1	1	2	16	65	296	1148
85	20	1	1	2	16	64	299	1133
86	20	1	1	3	12	70	304	1107
87	20	1	1	3	13	72	279	1163
88	20	1	1	2	14	73	291	1122
89	20	1	1	4	10	73	289	1162
90	20	1	1	4	10	71	299	1132
91	20	1	1	5	11	65	293	1155
92	20	1	1	4	14	60	303	1136
93	20	1	1	4	10	76	276	1161
94	20	1	1	5	10	71	294	1164
95	20	1	1	5	14	61	282	1176
96	20	1	1	4	13	66	302	1149
97	20	1	1	4	12	72	281	1150
98	20	1	1	4	17	60	277	1177
99	20	1	1	4	14	58	308	1137
100	20	1	1	5	12	65	276	1190
101	20	1	1	4	19	53	296	1171
102	20	1	1	3	13	69	317	1095
103	20	1	1	4	12	69	285	1156
104	20	1	1	2	15	67	306	1101
105	20	1	1	2	18	60	312	1142
106	20	1	1	3	14	68	304	1143
107	20	1	1	2	16	70	297	1142
108	20	1	1	4	13	66	287	1156
109	20	1	1	3	15	65	291	1139
110	20	1	1	4	9	78	280	1151
111	20	1	1	4	10	69	308	1128
112	20	1	1	3	13	70	285	1151
113	20	1	1	3	14	64	303	1123
114	20	1	1	3	14	67	302	1150
115	20	1	1	2	16	66	294	1128
116	20	1	1	3	11	75	295	1130
117	20	1	1	3	12	76	300	1115
118	20	1	1	1	17	68	304	1091
119	20	1	1	3	18	54	309	1179
120	20	1	1	4	13	64	299	1146
121	20	1	1	3	14	68	291	1149
122	20	1	1	2	17	63	310	1142
123	20	1	1	2	15	67	308	1091
124	20	1	1	2	14	73	292	1123
125	20	1	1	3	14	69	287	1140
126	20	1	1	2	14	74	285	1139
127	20	1	1	3	15	65	289	1156
128	20	1	1	4	12	68	288	1154
129	20	1	1	4	9	75	291	1139
130	20	1	1	4	14	70	297	1161
131	20	1	1	4	11	70	292	1135
132	20	1	1	5	8	77	281	1189
133	20	1	1	3	12	75	301	1133
134	20	1	1	2	16	64	296	1129
135	20	1	1	3	14	63	310	1105
136	20	1	1	3	11	75	297	1110
137	20	1	1	4	10	70	302	1128
138	20	1	1	4	15	62	299	1137
139	20	1	1	5	8	72	292	1169
140	20	1	1	4	13	71	282	1175
141	20	1	1	3	15	66	301	1144
142	20	1	1	4	11	69	294	1141
143	20	1	1	4	10	73	289	1148
144	20	1	1	4	12	62	304	1148
145	20	1	1	2	15	70	291	1120
146	20	1	1	2	15	68	305	1095
147	20	1	1	2	16	63	310	1105
148	20	1	1	3	10	82	279	1133
149	20	1	1	3	13	71	290	1124
150	20	1	1	4	11	72	292	1176
151	20	1	1	5	8	78	282	1124
152	20	1	1	2	15	66	306	1110
153	20	1	1	4	10	75	300	1135
154	20	1	1	4	8	80	285	1140
155	20	1	1	4	10	75	302	1133
156	20	1	1	4	13	68	295	1158
157	20	1	1	5	11	67	287	1131
158	20	1	1	3	12	72	293	1134
159	20	1	1	3	10	79	288	1127
160	20	1	1	3	12	73	291	1129
161	20	1	1	3	13	71	290	1127
162	20	1	1	4	12	66	299	1135
163	20	1	1	3	10	78	297	1094
164	20	1	1	1	19	63	299	1104
165	20	1	1	1	19	63	296	1126
166	20	1	1	3	14	66	292	1151

Table 3. Continued.

D	k	$\|AutD\|$	$\|AutC\|$	W_4	W_6	W_8	W_{10}	W_{12}
167	20	1	1	5	7	83	259	1187
168	20	1	1	4	10	70	296	1161
169	20	1	1	4	13	62	300	1147
170	20	1	1	3	13	68	304	1109
171	20	1	1	3	12	72	295	1124
172	20	1	1	4	13	66	289	1151
173	20	1	1	5	11	71	269	1177
174	20	1	1	3	15	64	287	1159
175	20	1	1	1	16	74	290	1104
176	20	1	1	1	18	66	294	1134
177	20	1	1	1	18	64	302	1108
178	20	1	1	3	15	60	309	1121
179	20	1	1	2	17	62	297	1142
180	20	1	1	4	10	72	297	1125
181	20	1	1	4	12	75	280	1171
182	20	1	1	5	14	68	268	1200
183	20	1	1	4	12	66	295	1147
184	20	1	1	4	12	64	301	1149
185	20	1	1	4	10	74	285	1161
186	20	1	1	4	13	61	320	1144
187	20	1	1	5	8	77	294	1141
188	20	1	1	6	12	68	264	1205
189	20	1	1	5	11	70	300	1182
190	20	1	1	6	9	72	271	1167
191	20	1	1	6	7	73	286	1150
192	20	1	1	5	8	73	292	1143
193	20	1	1	6	9	69	287	1146
194	20	1	1	5	11	68	280	1169
195	20	1	1	5	11	69	289	1185
196	20	1	1	3	10	81	280	1134
197	20	1	1	4	11	72	298	1149
198	20	1	1	4	10	77	289	1158
199	20	1	1	4	9	74	299	1130
200	20	1	1	4	8	78	295	1125
201	20	1	1	5	8	70	296	1160
202	20	1	1	5	12	71	284	1144
203	20	1	1	4	8	77	293	1145
204	20	1	1	5	7	78	274	1189
205	20	1	1	5	9	77	272	1157
206	20	1	1	5	10	68	288	1163
207	20	1	1	5	7	84	272	1186
208	20	1	1	5	7	74	295	1140
209	20	1	1	5	10	67	309	1146
210	20	1	1	5	9	73	295	1146
211	20	1	1	5	7	80	266	1194
212	20	1	1	4	12	66	304	1170
213	20	1	1	7	9	65	282	1178
214	20	1	1	3	15	61	300	1145
215	20	1	1	4	13	66	303	1139
216	20	1	1	5	13	66	269	1185
217	20	1	1	3	11	76	287	1140
218	20	1	1	5	9	71	288	1166
219	20	1	1	3	16	58	315	1155
220	20	1	1	2	18	59	297	1153
221	20	1	1	4	13	67	298	1171
222	20	1	1	3	13	71	303	1141
223	20	1	1	4	11	70	291	1144
224	20	1	1	4	15	60	306	1142
225	20	1	1	3	12	70	302	1130
226	20	1	1	3	16	67	298	1178
227	20	1	1	4	12	75	292	1174
228	20	1	1	4	11	77	295	1177
229	20	1	1	5	9	78	301	1201
230	20	1	1	2	18	62	303	1141
231	20	1	1	3	17	65	296	1188
232	20	1	1	2	16	68	298	1148
233	20	1	1	2	17	65	304	1135
234	20	1	1	4	13	65	326	1127
235	20	1	1	4	12	79	281	1177
236	20	1	1	3	14	75	277	1175
237	20	1	1	3	15	67	297	1135
238	20	1	1	3	14	73	298	1183
239	20	1	1	3	14	73	289	1154
240	20	1	1	3	13	74	296	1135
241	20	1	1	4	11	73	287	1171
242	20	1	1	5	9	70	294	1194
243	20	1	1	3	14	65	311	1141
244	20	1	1	5	7	82	303	1146
245	20	1	1	5	9	72	292	1164
246	20	1	1	3	16	59	310	1166
247	20	1	1	4	10	72	301	1161
248	20	1	1	3	13	71	297	1152
249	20	1	1	4	12	72	281	1174

Table 3. Continued.

D	k	$\|AutD\|$	$\|AutC\|$	W_4	W_6	W_8	W_{10}	W_{12}
250	20	1	1	4	12	71	291	1157
251	20	1	1	3	15	68	291	1162
252	20	1	1	3	12	77	286	1142
253	20	1	1	4	11	78	273	1189
254	20	1	1	3	14	71	290	1139
255	20	1	1	2	17	64	309	1135
256	20	1	1	3	13	72	300	1144
257	20	1	1	4	10	72	304	1148
258	20	1	1	2	15	67	303	1093
259	20	1	1	2	18	65	277	1148
260	20	1	1	1	18	64	302	1111
261	20	1	1	3	14	64	300	1126
262	20	1	1	2	17	61	304	1118
263	20	1	1	1	18	64	301	1116
264	20	1	1	1	18	67	290	1132
265	20	1	1	2	13	78	279	1140
266	20	1	1	2	18	59	298	1147
267	20	1	1	3	12	74	283	1147
268	20	1	1	4	10	77	272	1161
269	20	1	1	3	12	70	302	1125
270	20	1	1	1	21	54	305	1142
271	20	1	1	4	9	77	285	1149
272	20	1	1	1	18	64	304	1103
273	20	1	1	2	14	70	300	1107
274	20	1	1	4	11	67	305	1122
275	20	1	1	4	11	68	297	1153
276	20	16	16	2	12	85	264	1158
277	20	18	18	0	27	36	336	1080
278	20	2	2	2	15	71	288	1138
279	20	2	2	2	19	54	309	1112
280	20	2	2	5	11	63	312	1092
281	20	2	2	2	17	61	308	1110
282	20	2	2	4	17	54	284	1180
283	20	2	2	2	13	73	307	1065
284	20	2	2	4	13	68	307	1169
285	20	2	2	4	11	67	300	1138
286	20	2	2	4	9	75	291	1130
287	20	2	2	4	13	62	299	1151
288	20	2	2	6	16	65	270	1149
289	20	2	2	4	13	71	298	1183
290	20	2	2	2	17	61	301	1138
291	20	2	2	6	10	75	264	1201
292	20	2	2	4	13	66	282	1186
293	20	2	2	5	17	59	266	1206
294	20	2	2	6	11	65	285	1129
295	20	2	2	4	9	77	284	1156
296	20	2	2	6	11	67	282	1144
297	20	2	2	6	9	69	270	1204
298	20	2	2	6	15	61	267	1175
299	20	2	2	6	11	67	278	1150
300	20	2	2	4	11	67	301	1129
301	20	2	2	4	11	65	304	1141
302	20	2	2	4	14	73	297	1201
303	20	2	2	4	11	77	297	1151
304	20	2	2	3	17	56	312	1157
305	20	2	2	3	15	67	286	1182
306	20	2	2	4	10	74	304	1138
307	20	2	2	5	10	72	295	1145
308	20	2	2	6	8	85	247	1201
309	20	2	2	2	17	66	290	1099
310	20	2	2	2	17	61	303	1120
311	20	2	2	2	19	52	314	1130
312	20	2	2	4	11	65	307	1127
313	20	2	2	4	13	69	287	1112
314	20	2	2	5	14	68	279	1105
315	20	24	24	6	16	65	276	1226
316	20	3	3	3	13	69	322	1092
317	20	3	3	3	16	59	324	1122
318	20	3	3	3	12	74	292	1129
319	20	3	3	6	6	86	235	1235
320	20	3	3	3	9	83	283	1129
321	20	3	3	6	9	60	315	1145
322	20	4	4	2	16	67	302	1094
323	20	4	4	3	16	51	344	1067
324	20	4	4	4	18	57	314	1148
325	20	4	4	4	14	63	320	1148
326	20	54	54	9	9	54	327	1116
327	20	6	6	6	6	84	312	1154
328	20	6	6	3	15	60	309	1126
329	20	6	6	0	21	60	312	1086
330	20	6	6	3	12	63	318	1111
331	20	6	6	6	9	84	252	1160
332	20	9	9	0	18	75	288	1089

Table 4. Ternary codes $C_2^{T\perp}$ from 2-(9,3,4) designs

D^*	k^*	W_8	W_{18}	W_{24}	W_{27}	W_{32}	W_{36}
1	14	2				4	2
276	14	2				4	2
315	14	2				4	2
316	14	2				4	2
317	14	2				4	2
322	14	2				4	2
323	14	2				4	2
324	14	2				4	2
325	14	2				4	2
326	14			6			2
331	14		2		4		2

$^*D$ denotes the design and k is the dimension of the code.

Table 5. Some codes of designs of small orders designs

			C_1	C_1^T	C_1	C_1^T
design	m^*	n^*	binary		Ternary	
2-(8,4,3)	4	14	+	+	+	-
2-(9,3,2)	13	24	-	-	+	-
2-(9,4,3)	11	18	-	-	-	-
2-(10,4,2)	3	15	+	+	-	-
2-(13,3,1)	2	26	+	-	+	-
2-(15,7,3)	5	15	-	+	-	-
2-(16,6,2)	3	16	+	+	-	-
2-(19,9,4)	6	19	-	-	-	-

$^*m$ is the number of non-isomorphic designs and n is the size of codes. '+' indicates that the weight distribution of given codes can separate the designs.

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