Minimal complete arcs in $PG(2,q)$, $q \leq 29$

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Abstract

In this paper it has been verified, by a computer-based proof, that the smallest size of a complete arc is 12 in $PG(2,27)$ and 13 in $PG(2,29)$. Also the spectrum of the sizes of the complete arcs of $PG(2,27)$ has been found. The classification of the smallest complete arcs of $PG(2,27)$ is given: there are seven non-equivalent 12−arcs and for each of them the automorphism group and some geometrical properties are presented. Some examples of complete 13−arcs of $PG(2,29)$ are also described.

1 Introduction

In the projective plane $PG(2,q)$ over the Galois field $GF(q)$ an n−arc is a set of n points no 3 of which are collinear. An n-arc is called complete if it is not contained in an $(n+1)$-arc of the same projective plane. For a detailed description of the most important properties of these geometric structures, we refer the reader to [7]. In [9] the close relationship between the theory of complete n−arcs, coding theory and mathematical statistics is presented. In particular arcs and linear maximum distance separable codes (MDS codes) are equivalent objects [21], [22], [23]. Partly because of this fact, in recent years, the problem of determining the spectrum of values of n for which a complete arc exists has been intensively investigated. This article concerns the minimal complete arcs in $PG(2,q)$ for $q \leq 29$. The minimal size of a complete n-arc of $PG(2,q)$ is indicated by $t(2,q)$. General lower bounds on $t(2,q)$ are given in the following table:

q	$t(2,q) >$	References
q	$\sqrt{2q}+1$	[20]
$q=p^h$, p prime, $h=1,2,3$	$\sqrt{3q}+1/2$	[2],[3],[18]

Table 1.1: Lower bounds for $t(2,q)$

The values of $t(2,q), q \leq 25$ are stated in the following table:

q	$t(2,q)$	Number of classes up to $PGL(3,q)$	Number of classes up to $P\Gamma L(3,q)$	References
2	4	1		[7]
3	4	1		[7]
4	6	1	1	[7]
5	6	1		[7]
7	6	2		[7]
8	6	3	1	[7],[14]
9	6	1	1	[7]
11	7	1		[8],[6]
13	8	2		[1],[6]
16	9	6	2	[11],[14],[17]
17	10	560		[5],[17]
19	10	29		[5],[17]
23	10	1		[5]
25	12		606	[15]

Table 1.2: $t(2,q), \quad q \leq 25$

In this article it is demonstrated by a computer-based proof that

$$t_2(2,27) = 12, \qquad t_2(2,29) = 13,$$

and that the spectrum of the sizes of the complete arcs in $PG(2,27)$ is:

$$12, 13, 14, 15, 16, 17, 18, 19, 22, 28.$$

This result has been obtained by an exhaustive computer search that has been feasible because projective properties among arcs have been exploited. In fact in writing effective computer search programs in projective spaces, some strategy has to be adopted to avoid producing too many isomorphic copies of the same arc and searching through parts of the search space isomorphic to previously searched portions [19].

The authors also classify the smallest complete arcs in $PG(2,27)$ finding seven classes of 12–arcs up to $P\Gamma L(3,q)$. The smallest complete arcs of

$PG(2,29)$ have not been classified yet, but 254 non-equivalent examples of 13–arcs have been found. The automorphism group is D_3 for all the seven complete 12–arcs of $PG(2,27)$, while in $PG(2,29)$ 251 13-arcs have a trivial automorphism group and the remaining three 13–arcs are stabilized by Z_3. In both spaces some geometrical properties of the smallest complete arcs have been studied.

The plan of the paper is the following: in the second section the obtained results concerning the values of $t_2(2,27)$ and of $t_2(2,29)$ and the spectrum of the sizes of the complete arcs in $PG(2,27)$ are presented; in the third some geometrical properties of the smallest complete arcs in $PG(2,27)$ are given; in the fourth some smallest complete arcs in $PG(2,29)$ are described.

2 The determination of $t_2(2,27)$, $t_2(2,29)$ and of the spectrum of the sizes of complete arcs in $PG(2,27)$

The results presented in this paper have been obtained by an exhaustive computer search. The exhaustive search has been feasible because projective properties among arcs have been exploited to avoid obtaining too many isomorphic copies of the same solution arc and to avoid searching through parts of the search space isomorphic to previously searched portions. The algorithm used starts constructing a tree structure containing a representative of each class of non-equivalent arcs of size less than or equal to a fixed threshold h. If the threshold h were equal to the actual size of the sought arcs, the algorithm would be orderly, that is capable of constructing each goal configuration exactly once [19]. However, in the present case, the construction of the tree with the threshold h equal to the size of the sought arcs would have been too space and time consuming. For this reason a hybrid approach has been adopted. The tree representing the non-equivalent arcs of size less than or equal to eight has been constructed and then every non-equivalent 8–arc has been extended using a backtracking algorithm trying to obtain complete arcs of the desired size. In the backtracking phase, the information obtained during the classification of the arcs has been further exploited to prune the search tree. In fact the points that would have given arcs equivalent to already obtained ones have been excluded from the backtracking steps. The algorithm is described in detail in [15].

When studying the value of $t_2(2,27)$, during the classification, up to $P\Gamma L(3,27)$, of the arcs of $PG(2,27)$ of size less than or equal to 8, 4 non-equivalent arcs of size 5, 174 non-equivalent arcs of size 6, 8261 non-equivalent arcs of size 7 and 311313 non-equivalent arcs of size 8 have been found. Each 8-arc has been extended to obtain complete arcs of size less than or equal to 12 and twelve examples of complete 12–arcs have been found, so $t_2(2,27) = 12$. The search for the 12–arcs lasted about 7 days on a 200 MHz PC. The algorithm used guarantees that any other 12-arc is projectively equivalent to one element of this set. The 12–arcs found have been classified using MAGMA, a system for symbolic computation developed at the University of Sydney. The number of non-equivalent arcs up to $P\Gamma L(3,27)$ results is seven. In the next section the description of a representative of each equivalence class is presented.

When studying the value of $t_2(2,29)$, during the classification, up to $PGL(3,29)$, of the arcs of $PG(2,29)$ of size less than or equal to 8, 10 non-equivalent arcs of size 5, 682 non-equivalent arcs of size 6, 41301 non-equivalent arcs of size 7 and 1933469 non-equivalent arcs of size 8 have been found. In this case it has not been possible to look for complete arcs of size thirteen because the execution of the program would have lasted too long, hence each 8-arc has been extended to obtain complete arcs of size less than or equal to 12. No examples of complete 12–arcs have been found, so $t_2(2,29) = 13$. The search for the 12–arcs lasted about 40 days on a 300 MHz PC. In section 4 the description of some examples of complete arcs of size thirteen of $PG(2,29)$ is presented.

In $PG(2,27)$ examples of arcs of size k, $12 \leq k \leq 19$, are known ([10], [12], [13], [16]) and the second largest cardinality, $m'_2(2,25)$, is 22 ([4]). So, to determine the spectrum of the sizes of complete arcs, complete arcs of size twenty and twenty-one have been sought. In this case the algorithm presented above has been modified stopping the extension of the current 8–arc when the number of points available becomes too small to obtain a complete arc of the desired length. No complete arcs of size 20 and 21 have been found, so the spectrum of the sizes of complete arcs in $PG(2,27)$ is:

$$12, 13, 14, 15, 16, 17, 18, 19, 22, 28.$$

The execution of the program lasted about 35 days on a SUN Enterprise 450 with a 400 MHz processor.

3 Classification of the smallest complete arcs in PG(2,27)

The size of the smallest complete arcs of $PG(2,27)$ is twelve and the number of non-equivalent, up to $P\Gamma L(3,27)$, 12–arcs is seven. Six complete 12–arcs have the dihedral group D_3 as automorphism group, while the seventh has S_4. This and all the other properties presented in this section and in the next have been determined using MAGMA.

Each of these seven arcs contains the following set of points

$$R = \{(0,0,1),(0,1,0),(1,0,0),(1,1,1)\}$$

and the field $GF(27)$ has been constructed using the primitive polynomial $\xi^3 + 2\xi^2 + 1$.

The list of these arcs is the following:

$K_1 = R \cup \{(1,\xi^{12},\xi^4),\ (1,\xi^{24},\xi^{19}),\ (1,\xi,\xi^{11}),\ (1,2,\xi),\ (1,\xi^{16},\xi^{20}),\ (1,\xi^9,\xi^{22}),\ (1,\xi^{17},\xi^3),\ (1,\xi^{23},\xi^{21})\}$;

$K_2 = R \cup \{(1,\xi^2,\xi^6),\ (1,\xi,\xi^{11}),\ (1,\xi^{14},\xi^{25}),\ (1,2,\xi^7),\ (1,\xi^9,\xi^{23}),\ (1,\xi^{25},\xi^5),\ (1,\xi^{19},\xi^{22}),\ (1,\xi^{23},\xi^{21})\}$;

$K_3 = R \cup \{(1,\xi,\xi^{11}),\ (1,\xi^{21},\xi^6),\ (1,\xi^{19},\xi^{22}),\ (1,\xi^{15},\xi^{21}),\ (1,2,\xi^{25}),\ (1,\xi^{11},\xi^{20}),\ (1,\xi^9,\xi^4),\ (1,\xi^{22},\xi^9)\}$;

$K_4 = R \cup \{(1,\xi^2,\xi^{16}),\ (1,\xi^{10},\xi^6),\ (1,\xi^1,\xi^{11}),\ (1,\xi^{22},\xi^{15}),\ (1,\xi^{14},2),\ (1,\xi^{19},\xi^{22}),\ (1,\xi^7,\xi^{20}),\ (1,\xi^{24},\xi^{10})\}$;

$K_5 = R \cup \{(1,\xi^{16},\xi^4),\ (1,\xi^{17},\xi^{25}),\ (1,\xi^1,\xi^{11}),\ (1,\xi^{25},\xi^8),\ (1,\xi^{24},\xi^{23}),\ (1,\xi^{20},2),\ (1,\xi^{21},\xi^{17}),\ (1,\xi^{19},\xi^{22})\}$;

$K_6 = R \cup \{(1,\xi^{10},\xi^{19}),\ (1,\xi^{22},\xi^{23}),\ (1,\xi^1,\xi^{11}),\ (1,\xi^5,\xi^{24}),\ (1,\xi^2,\xi^{20}),\ (1,\xi^{21},\xi^{17}),\ (1,\xi^{19},\xi^{22}),\ (1,\xi^9,\xi^4)\}$;

$K_7 = R \cup \{(1,\xi^2,\xi^{17}),\ (1,\xi^{18},\xi^{21}),\ (1,\xi^{24},\xi^{20}),\ (1,\xi^1,\xi^{11}),\ (1,\xi^{15},\xi^5),\ (1,\xi^3,\xi^4),\ (1,\xi^{23},\xi^{25}),\ (1,\xi^{20},\xi^3)\}$.

Some geometrical properties of these arcs have been investigated; the following theorem summarizes some of the obtained results:

Theorem 1 *Let C be the set of all irreducible conics of $PG(2,27)$. Then the following hold:*
(a) $|K_i \cap G| \leq 8$, for $i = 1,...,7$ and for all $G \in C$;
(b) the value 8 is reached only for K_6, and K_7; for each of them there exist three conics in C intersecting them in 8 points.
(c) K_7 has a unique orbit (cardinality 12) while each K_i, $i = 1,...,6$, has two orbits of cardinality 3 and one orbit of cardinality 6.

In the following table we describe the conics intersecting K_6, and K_7 in 8 points:

	$G : \|K \cap G\| = 8$
K_6	$xy + \xi^{10}xz + \xi^{17}yz = 0$ $\xi^2 xy + \xi^{11}xz + y^2 + \xi^6 yz = 0$ $x^2 + \xi^{12}xy + \xi^{11}xz + \xi^{22}yz + \xi^{23}z^2 = 0$
K_7	$\xi^{12}xy + \xi^2 xz + y^2 + \xi^{25}yz + \xi^{20}z^2 = 0$ $xy + \xi^{23}xz + \xi^5 yz = 0$ $x^2 + \xi^{21}xy + \xi^{25}xz + \xi^{16}yz = 0$

Table 3.1: Conics intersecting K_6 and K_7 in 8 points

The orbits of K_i, $i = 1, ..., 6$, are described in the following table:

	$\|O\| = 3$	$\|O\| = 6$
K_1	$\{(0,1,0),(1,\xi^{16},\xi^{20}),(1,\xi^{23},\xi^{21})\}$; $\{(0,0,1),(1,\xi^{24},\xi^7),(1,\xi^{19},\xi^{22})\}$	$\{(1,\xi^{12},\xi^4),(1,0,0),$ $(1,\xi^{17},\xi^3),(1,1,1),$ $(1,2,\xi),(1,\xi,\xi^{11})\}$
K_2	$\{(1,0,0),(1,\xi^9,\xi^{23}),(1,\xi^{23},\xi^{21})\}$; $\{(0,0,1),(1,2,\xi^7),(1,\xi^{19},\xi^{22})\}$	$\{(1,\xi^2,\xi^6),(1,\xi,\xi^{11}),$ $(1,\xi^{25},\xi^5),(0,1,0),$ $(1,1,1),(1,\xi^{14},\xi^{25})\}$
K_3	$\{(1,0,0),(1,\xi^{11},\xi^{20}),(1,2,\xi^{25})\}$; $\{(0,1,0),(1,\xi^{15},\xi^{21}),(1,\xi^{22},\xi^9)\}$	$\{(1,\xi^{21},\xi^6),(1,\xi,\xi^{11}),$ $(1,\xi^9,\xi^4),(0,0,1),$ $(1,1,1),(1,\xi^{19},\xi^{22})\}$
K_4	$\{(0,1,0),(1,\xi^{19},\xi^{22}),(1,\xi^{24},\xi^{10})\}$; $\{(0,0,1),(1,\xi^{22},\xi^{15}),(1,\xi^{14},2)\}$	$\{(1,\xi^2,\xi^{16}),(1,0,0),$ $(1,\xi,\xi^{11}),(1,\xi^7,\xi^{20}),$ $(1,1,1),(1,\xi^{10},\xi^6)\}$
K_5	$\{(1,\xi^{25},\xi^8),(1,\xi^{16},\xi^4),(1,\xi^{17},\xi^{25})\}$; $\{(1,1,1),(1,\xi,\xi^{11}),(1,\xi^{19},\xi^{22})\}$	$\{(1,\xi^{24},\xi^{23}),(0,0,1),$ $(1,\xi^{21},\xi^{17}),(1,0,0),$ $(0,1,0),(1,\xi^{20},2)\}$
K_6	$\{(0,0,1),(1,\xi^2,\xi^{20}),(1,\xi^{21},\xi^{17})\}$; $\{(1,\xi^{19},\xi^{22}),(1,\xi^{22},\xi^{23}),(1,\xi^5,\xi^{24})\}$	$\{(1,\xi^{10},\xi^{19}),(1,1,1),$ $(1,\xi^9,\xi^4),(0,1,0),$ $(1,0,0),(1,\xi,\xi^{11})\}$

Table 3.2: Orbits of K_i $i = 1, ...6$

In the following table we describe the classes in relation to the existence of $G \in C$ such that $|K_i \cap G| = 6$, $i = 1, ..., 7$:

	$\|G\| : \| K_i \cap G \| = 6$
K_1	28
K_2	37
K_3	37
K_4	49
K_5	31
K_6	28
K_7	40

Table 3.3: Number of conics intersecting K_i, i=1, ..., 7, in 6 points

There are no $G \in C$ such that $|K_i \cap G| = 7$ for $i = 4, 6, 7$ while for the remaining arcs a description is given in the following table:

	$\|G\|:\|K \cap G\|=7$	*Equations*
K_1	6	$x^2 + \xi^{11}xy + \xi^{12}xz + \xi^{19}yz = 0$ $xy + \xi^{10}xz + \xi^{17}yz = 0$ $\xi^{17}xy + \xi^{16}xz + y^2 + \xi^{11}yz = 0$ $x^2 + \xi^{22}xy + \xi^{7}xz + \xi^{12}yz = 0$ $\xi^{19}xy + \xi^{12}xz + \xi^{10}yz + z^2 = 0$ $x^2 + \xi^{22}xy + \xi^{19}xz + \xi^{19}y^2 + \xi^{20}yz + \xi^3z^2 = 0$
K_2	3	$xy + \xi^{21}xz + \xi^{22}yz = 0$ $2xy + \xi^{11}xz + y^2 + \xi^{20}yz = 0$ $x^2 + \xi^{24}xy + \xi^{6}xz + \xi^{23}y^2 + +\xi^{14}yz = 0$
K_3	3	$xy + \xi^{20}xz + \xi^{3}yz + z^2 = 0$ $x^2 + \xi^{8}xy + \xi^{10}xz + \xi^{23}yz = 0$ $\xi^{21}xy + \xi^{10}xz + \xi^{21}yz + z^2 = 0$
K_5	3	$x^2 + xy + \xi^{14}xz + \xi^{18}yz = 0$ $\xi^{16}xy + 2xz + y^2 + \xi^{14}yz = 0$ $\xi xy + \xi^{8}xz + \xi^{25}yz + z^2 = 0$

Table 3.4: Conics intersecting K_i, i=1, 2, 3, 5, in 7 points

4 The smallest complete arcs in PG(2,29)

The smallest cardinality of the complete arcs in $PG(2,29)$ is 13. The classification of the complete 13−arcs has not been finished; in this section the description of the 254 non-equivalent 13−arcs found so far is given. Let $\Gamma = \{K_i\ ,\ i = 1, ..., 254\}$ be the set of these complete 13−arcs. It has been verified using MAGMA that 251 of them have a trivial automorphism group, while Z_3 is the stabilizer of the remaining three. The three arcs stabilized by Z_3 have 5 orbits, one consisting of 1 point and the others consisting of 3 points, as one may see in the following table:

	$\lvert O\rvert = 1$	$\lvert O\rvert = 3$
K'	$\{(1,1,1)\}$	$\{(1,6,4),\ (1,4,25),\ (1,12,9)\}$; $\{(1,14,20),\ (1,3,7),\ (1,18,24)\}$; $\{(0,0,1),\ (1,11,15),\ (1,27,23)\}$; $\{(1,13,21),\ (1,0,0),\ (0,1,0)\}$
K''	$\{(1,28,16)\}$	$\{(1,2,14),\ (1,0,0),\ (1,20,12)\}$; $\{(0,0,1),\ (1,8,19),\ (0,1,0)\}$; $\{(1,9,11),\ (1,1,1),\ (1,21,17)\}$; $\{(1,7,2),\ (1,13,21),\ (1,3,7)\}$
K'''	$\{(0,0,1)\}$	$\{(1,7,6),\ (1,5,3),\ (1,22,12)\}$; $\{(1,6,4),\ (1,0,0),\ (0,1,0)\}$; $\{(1,13,21),\ (1,28,27),\ (1,1,1)\}$; $\{(1,23,19),\ (1,3,7),\ (1,12,23)\}$

Table 4.1: Orbits of $K \in \Gamma$, having Z_3 as stabilizers

For each complete 13−arc the intersections with the irreducible conics have been computed. The following theorem summarizes the obtained results.

Theorem 2 *Let C be the set of all irreducible conics of $PG(2,29)$. Then the following hold:*
(a) $|K_i \cap G| \leq 8$, for $i = 1, ..., 254$ and for all $G \in C$;
(b) there are thirty complete 13-arcs such that for each of them there exist exactly one conic in C, G_i, with $|K_i \cap G_i| = 8$, $i = 1, ..., 30$ and there is exactly one element of Γ , namely $K_{31} = \{(0,0,1),\ (0,1,0),\ (1,0,0),\ (1,1,1),\ (1,19,20),\ (1,26,24),\ (1,6,28),\ (1,13,2),\ (1,3,7),\ (1,25,13),\ (1,10,15),\ (1,24,23),\ (1,28,26)\}$, for which there exist two conics $G_1, G_2 \in C$, $G_1 : 20xy + 26xz + 11yz + z^2 = 0$, $G_2 : xy + 25xz + 3yz + z^2 = 0$, such that $|K_{31} \cap G_1| = 8 = |K_{31} \cap G_2|$.
(c) $|K_i \cap G_j| \leq 7$ $i = 32, ..., 254$, $\forall G_j \in C$.

The following table gives the number of complete 13-arcs for which there exist j irreducible conics having seven intersections with them.

j	$\|\{K \in \Gamma : \|\{G \in \mathcal{C} : \|K \cap G\| = 7\}\| = j\}\|$
0	12
1	36
2	75
3	67
4	45
5	13
6	5
7	1

Table 4.2: Numbers of 13-arcs in Γ for which there exist j conics intersecting them in 7 points

The following remark states, for $j \geq 3$, the number of 13-arcs such that the conics having seven intersections with the arc have the maximum number of common points belonging to the arc. Also the 13-arc with the maximum number of conics that intersect it in seven points is given.

Remark 1 *1) If $j = 3$ there exists three 13-arcs such that for each of them*

$$|K \cap (\bigcap_{i=1}^{3} G_i)| = 3;$$

2) If $j = 4$ there exists four 13-arcs such that for each of them

$$|K \cap (\bigcap_{i=1}^{4} G_i)| = 2;$$

3) If $j = 5$ there exists three 13-arcs such that for each of them

$$|K \cap (\bigcap_{i=1}^{5} G_i)| = 1.$$

4) There is only one class, namely $K = \{(0,0,1), (0,1,0), (1,0,0), (1,1,1), (1,13,21), (1,28,19), (1,3,7), (1,5,8), (1,25,22), (1,2,26), (1,7,20), (1,14,15), (1,16,10)\}$, for which exist 7 conics G_i $i = 1, ..., 7$

$$\begin{aligned}
&G_1 : \ xy + 9xz + 20yz = 0\ ,\\
&G_2 : \ 9xy + 27xz + 15yz + z^2 = 0\ ,\\
&G_3 : \ 22xy + 4xz + 2yz + z^2 = 0,\\
&G_4 : 9xy + 16xz + y^2 + 11yz = 0,\\
&G_5 : x^2 + 3xy + 7xz + 18y^2 = 0,\\
&G_6 : x^2 + 6xy + xz + 10y^2 + 5yz + z^2 = 0,\\
&G_7 : x^2 + 7xy + 20xz + 18yz + 2z^2 = 0,
\end{aligned}$$

such that $|K \cap G_i| = 7$.

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