More large sets of KTS(v)

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Abstract

A large set of KTS(v), denoted by LKTS(v), is a collection of $(v-2)$ pairwise disjoint KTS(v) on the same set. In this paper, it is proved that there exists an LKTS($3^n.91$) for any integer $n \geq 1$.

Keywords: Steiner triple system, Kirkman triple system, Large set of Kirkman triple system, Transitive Kirkman triple system.

1 Introduction

A Steiner triple system of order v (briefly STS(v)) is a pair $(\mathcal{X}, \mathcal{B})$, where $\mathcal{X}$ is a set containing v-elements and $\mathcal{B}$ is a collection of 3-subsets (called *triple*) of $\mathcal{X}$, such that every unordered pair of $\mathcal{X}$ appears in exactly one triple. For $\mathcal{P} \subset \mathcal{B}$ and any $x \in \mathcal{X}$, if x appears in exactly one triple of $\mathcal{P}$, we call $\mathcal{P}$ *a parallel class* of the STS(v). If $\mathcal{B}$ can be partitioned into disjoint parallel classes, we call the STS(v) a *Kirkman triple systems*, which is denoted by KTS(v).

A large set of KTS(v), denoted by LKTS(v), is a collection of $(v-2)$ pairwise disjoint KTS(v) on the same set. The necessary condition for the existence of LKTS(v) is $v \equiv 3 \ (mod\ 6)$. So far, knowledge about the existence of LKTS(v) is very limited, see [1], [2], [3], [4], [8], [10], [11]. The known results can be summarized as follows.

Theorem 1.1 *(1) There exists an LKTS($3^n m$) for any positive integer n and $m \in$ {1, 5, 11, 17, 25, 35, 43, 67}.*
(2) There exists an LKTS($3^n.41$) for any integer $n \geq 2$.

*Researcher supported in part by YNSFC Grant 10001026

In this paper, it is proved that there exists an LKTS(3^n.91) for any integer $n \geq 1$.

2 Constructions

A Kirkman triple system $(\mathcal{X}, \mathcal{B})$ of order v is called *transitive*, denoted by TKTS(v), if there exists a transitive automorphism group G of order v of $(\mathcal{X}, \mathcal{B})$.

The known results on the existence of TKTS(v) can be summarized as follows.

Lemma 2.1 *(1) ([5], [6]) There exists a TKTS($3^k 5^l 11^m 17^n q_1 q_2 \cdots q_t$), where $k \geq 1$, l, m, $n \in \{0, 1\}$ and q_i is a prime power and $q_i \equiv 1$ (mod 6) for $1 \leq i \leq t$.*
(2) ([11]) There exists a TKTS(3^n.41) for any integer $n \geq 1$.

Denniston [4] gave a recursive construction for LKTS(v) using TKTS(v), which is shown below.

Lemma 2.2 *If there exists a TKTS(v) and an LKTS(v), then there exists an LKTS(3v).*

Let GF(q) be a finite field containing q elements, where q is a prime power and $q \equiv 7$ (*mod* 24). Let g be a primitive element of GF(q) and $-2 = g^\theta$. For any $x \in Z_{q-1}$, denote $< x > \equiv x$ (*mod* $\frac{q-1}{2}$).

Let $\{\lambda_i, \mu_i\}$ ($i = 1, 2, \cdots, \frac{q-7}{6}$) be a sequence of unordered pairs on $Z^*_{q-1} = Z_{q-1} \backslash \{0\}$ with the following properties:

(1) $\lambda_i \neq \mu_i$.

(2) $g^{\lambda_i} + g^{\mu_i} = -1$.

(3) $\{\lambda_i, \mu_i\} \subset Z^*_{q-1} \backslash \{\theta, q-1-\theta, \frac{q-1}{2}, \frac{q-1}{3}, \frac{2(q-1)}{3}\}$.

(4) $|\bigcup_{i=1}^{\frac{q-7}{6}} (\{\lambda_i, \mu_i\} \bigcup \{-\mu_i, \lambda_i - \mu_i\} \bigcup \{\mu_i - \lambda_i, -\lambda_i\})| = \frac{q-7}{2}$.

The following lemma is a restatement of Corollary 3.3 of [1], which is in fact a modification of the Y-Z partition construction of Wilson [9] and Schreiber [7].

Lemma 2.3 *Let GF(q) be a finite field and $q \equiv 7$ (mod 24). If there exist $\frac{q-7}{6}$ elements x_i and an element y in Z^*_{q-1} such that:*

(5) $\bigcup_{i=1}^{\frac{q-7}{6}} \{< x_i >, < x_i + \lambda_i >, < x_i + \mu_i >\} = Z^*_{\frac{q-1}{2}} \backslash \{< \theta >, < y >\}$,

then there exists an LKTS($q+2$).

3 Main result

Lemma 3.1 *There exists an LKTS($q+2$) for $q = 271$.*

Proof. Apply Lemma 2.3 with $q = 271$, $g = 6$ and $\theta = 19$, we should only find the suitable triples $\{\lambda_i, \mu_i, x_i\}$ and y, which are listed below.
$\bigcup_{i=1}^{\frac{q-7}{6}} \{\{\lambda_i, \mu_i, x_i\}\}$ = {{70, 184, 156}, {16, 123, 190}, {36, 225, 48}, {49, 216, 256}, {23, 201, 106}, {1, 7, 144}, {29, 141, 185}, {2, 13, 28}, {44, 121, 74}, {17, 211, 61}, {46, 222, 5}, {24, 87, 7}, {67, 168, 179}, {12, 227, 20}, {3, 235, 233}, {21, 127, 182}, {15, 174, 119}, {14, 89, 268}, {26, 152, 199}, {39, 205, 57}, {34, 134, 83}, {31, 110, 42}, {8, 204, 26}, {30, 155, 123}, {5, 88, 244}, {53, 210, 1}, {51, 131, 188}, {10, 50, 245}, {9, 73, 107}, {57, 142, 58}, {18, 172, 22}, {61, 132, 89}, {25, 72, 27}, {33, 91, 215}, {56, 153, 75}, {62, 186, 97}, {4, 109, 128}, {27, 229, 146}, {28, 148, 72}, {22, 130, 243}, {42, 137, 257}, {37, 188, 88}, {20, 238, 46}, {78, 177, 126}}.
$y = 4$. □

Theorem 3.2 *There exists an LKTS($3^n.91$) for any integer $n \geq 1$.*

Proof. Combine Lemmas 2.1 and 3.1, the conclusion then follows. □

References

[1] Y. Chang and G. Ge, *Some new large sets of KTS(v)*, Ars Combin. **51** (1999), 306-312.

[2] R. H. F. Denniston, *Doubly resolvability of some complete 3-designs*, Manuscripta Math. **12** (1974), 105-112.

[3] R. H. F. Denniston, *Four doubly resolvable complete 3-designs*, Ars Combin. **7** (1979), 265-272.

[4] R. H. F. Denniston, *Further case of double resolvability*, J. Combin. Theory (A) **26** (1979), 298-303.

[5] J. Lei, *Further results on transitive Kirkman triple systems*, J. Statist. Plan. Infer. **51** (1996), 189-194.

[6] J. Lei and Y. Chang, *Transitive Kirkman triple systems*, Journal of Hebei Science Academy, No. **2** (1990), 1-7.

[7] S. Schreiber, *Covering all triples on n marks by disjoint Steiner systems*, J. Combin. Theory (A) **15** (1973), 347-350.

[8] S. Schreiber, quoted by H. Hanani, *Resolvable designs*, Colloquio Internazionale Sulle Teorie Combinatorie (Atti dei Convegni Lincei, 17), Rome (1976), Tomo II, 249-252.

[9] R. M. Wilson, *Some partitions of all triples into Steiner triple systems*, Lecture Notes in Math. 411 (1974), 267-277.

[10] L. Wu, *On large set of KTS(v)*, in: W. D. Wallis etal. eds., Combinatorial Designs and Applications (Marcel Dekker, New York) (1990), 175-178.

[11] L. Wu, *Large sets of* $KTS(3^n.41)$, J. Suzhou Univ. Vol. 14, N. 4 (1998), 1-2.