NOTE ON A CONSTRUCTION FOR A (25,4,1) DESIGN

R. G. Stanton
Department of Computer Science
University of Manitoba
Winnipeg, Canada
R3T 2N2

When I was a graduate student, the Balanced Incomplete Block Design having parameters (25, 50, 8, 4, 1) had not yet been constructed. Of course, since that time, the fundamental work of Hanani has shown that all designs with block size four exist. Recently, I noticed that a design with these parameters could easily be constructed from the geometry PG(2,4); since I have not seen this construction before, it seemed that a short note on it might be justified.

Take the 21-point geometry PG(2,4) as being generated by the difference set (0, 1, 4, 14, 16), modulo 21; this gives twenty-one blocks of length 5. Delete the first element of each block; this leaves 21 quadruples generated cyclically (modulo 21) by the initial block (1, 4, 14, 16).

Deletion of the first elements of these 21 blocks has removed all pairs that are distance j from one another, where j = 1, 4, 7, or 5. These pairs can be generated by the two triples (0, 4, 5) and (0, 7, 14) and the 28 triples thus generated can be written down in the following array, in which the first set of triples is generated by (0, 7, 14) and the last three sets, generated from (0,4,5), are arranged according as the first element in each triple is congruent to 0, 1, or 2 (modulo 3).

0, 7, 14	0, 4, 5	1, 5, 6	2, 6, 7
1, 8, 15	3, 7, 8	4, 8, 9	5, 9, 10
2, 9, 16	6, 10, 11	7, 11, 12	8, 12, 13
3, 10, 17	9, 13, 14	10, 14, 15	11, 15, 16
4, 11, 18	12, 16, 17	13, 17, 18	14, 18, 19
5, 12, 19	15, 19, 20	16, 20, 0	17, 0, 1
6, 13, 20	18, 1, 2	19, 2, 3	20, 3, 4

It is now obvious that we can add an element 22 to the triples of the first column, 23 to the triples of the second column, 24 to the triples of the third column, and 25 to the triples of the fourth column, to give 28 quadruples. Also, we adjoin the block (22, 23, 24, 25); then we have constructed a total of 21 + 28 + 1 = 50 blocks, and the design is completed.

JCMCC 5(1989), pp. 61–62

All designs with parameters (25,50,8,4,1) that do not have a trivial automorphism group are listed or referenced in [2]; the total number of such designs is 16. If we record our design as a bipartite point-block graph on 75 vertices and then apply Kocay's "Groups & Graphs" package (see [1]) to the design, we immediately find that the design has an automorphism group of order 63 generated by the permutations

a = (1,4,16)(2,8,11)(3,12,6)(5,20,17)(9,15,18)(10,19,13)(23,24,25) and

b = (1,12,14)(2,16,9)(3,20,4)(5,7,15)(6,11,10)(8,19,21)(13,18,17)

with 22 an invariant point. The 25 points fall into three orbits of lengths 1, 3, and 21. Consequently this design is isomorphic to Design 2 in the Kramer, Magliveras, Tonchev list given in [2].

REFERENCES

[1]. W. L. Kocay, *Groups & Graphs, a Macintosh Application for Graph Theory*, J. of Combinatorial Mathematics and Combinatorial Computing 3 (1988), 195-206.

[2] Earl S. Kramer, S.S. Magliveras, and V.D. Tonchev, *On the Steiner Systems S(2,4,25) invariant under a group of order 9*, Annals of Discrete Mathematics 34 (1987), 307-314.