# On Super Edge-Magic Labelings of Unions of Star Graphs


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**Abstract**

Let $G = (V, E)$ be a graph with $|V| = p$ and $|E| = q$. The graph $G$ is *total edge-magic* if there exists a bijection $f : V \cup E \rightarrow \{1, 2, \ldots, p+q\}$ such that for all $e = (u, v) \in E$, $f(u) + f(e) + f(v)$ is constant throughout the graph. A total edge-magic graph is called *super edge-magic* if $f(V) = \{1, 2, \ldots, p\}$. Lee and Kong conjectured that for any odd positive integer $r$, the union of any $r$ star graphs is super edge-magic. In this paper, we supply substantial new evidence to support this conjecture for the case $r = 3$.

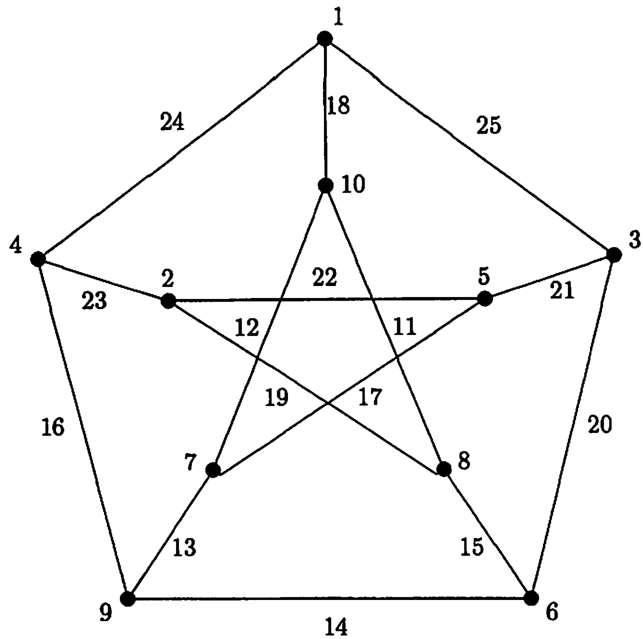

## 1 Preliminaries

Let $G = (V, E)$ be a graph with $|V| = p$ and $|E| = q$. The graph $G$ is *total edge-magic* if there exists a bijection $f : V \cup E \rightarrow \{1, 2, \ldots, p+q\}$ such that for all $e = (u, v) \in E$, $f(u) + f(e) + f(v)$ is constant throughout the graph. A total edge-magic graph is called *super edge-magic* if $f(V) = \{1, 2, \ldots, p\}$.

We illustrate the concept of a super edge-magic labeling with a labeling of the Petersen graph, as in [8]. Observe that the vertices are labeled with the integers $\{1, 2, \ldots, 10\}$, the edges are labeled with the integers

$\{11, 12, \ldots, 25\}$, and for each edge, the sum of the labels on the edge and its endpoints is 29.

**Example 1.** *Super edge-magic labeling of the Petersen graph*

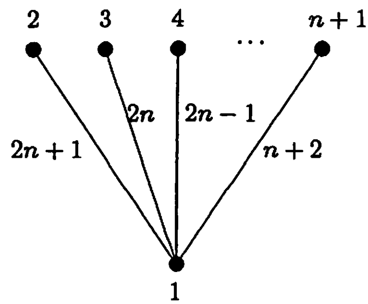



Super edge-magic graphs have been considered in several recent articles (for example, [2],[3],[4],[5],[6],[8],[9]). Unless otherwise indicated, we will use definitions as provided in [1]. For the sake of completeness, an *n-star* is a connected graph on $n+1$ vertices such that one vertex has degree $n$. The vertex of degree $n$ in an $n$-star is called the *central vertex*. Let $a_1, a_2, \ldots, a_r$ be a sequence of positive integers. As in [7], we let $St(a_1, a_2, \ldots, a_r)$ denote the union of the designated $a_i$-stars. This paper specifically address the super edge-magic labeling of unions of three star graphs. The motivation behind this work is the following conjecture by Sin-Min Lee and Man Kong [7]:

**Lee-Kong Conjecture:** *For any odd positive integer $r$, $St(a_1, a_2, \ldots, a_r)$ is super edge-magic.*

The super edge-magic labeling of a single star graph is trivial, and is included below in Example 2. Observe that the vertices are labeled with the $\{1, 2, \ldots, n+1\}$, the edges are labeled with $\{n+2, n+3, \ldots, 2n+1\}$, and that the sum of the labels of each edge and its endpoints is $2n+4$.

**Example 2.** *Super edge-magic labeling of a single n-star, $St(n)$*



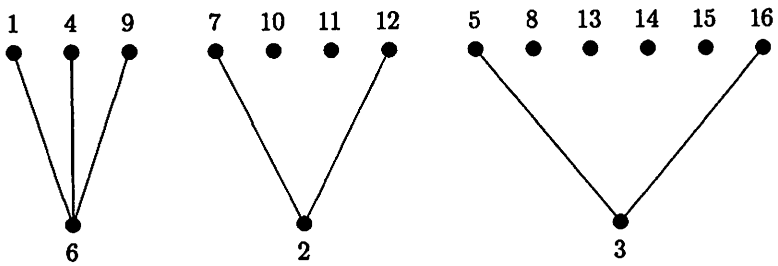

Let $G = (V, E)$ be a graph with $|V| = p$ and $|E| = q$. For each edge, we refer to the sum of the labels of its endpoints as its *vertex-sum*. Observe, as in [2], that to show a super edge-magic labeling of $G$, it suffices to demonstrate a labeling of the vertices with $\{1, 2, \ldots, p\}$ such that the set of vertex-sums form a set of consecutive integers. The completion of such a labeling of the vertices to a super edge-magic labeling of $G$ is unique — namely, assign $p+1$ to the edge with the greatest vertex-sum, assign $p+2$ to the edge with the second greatest vertex-sum, and so on until finally the label $p+q$ is assigned to the edge with the least vertex-sum.

In [7], Lee and Kong provided super edge-magic labelings for several infinite families of unions of star graphs – some for unions of two stars, some for unions of three stars, and some for unions of four stars. They showed that the following families of unions of three stars are super edge-magic: $St(1,1,n), n \geq 1$; $St(1,2,n), n \geq 2$; $St(1,n,n), n \geq 1$; $St(2,2,n), n \geq 2$; and $St(2,3,n), n \geq 3$. The labelings provided in this paper significantly extend these results, providing additional justification for the Lee-Kong Conjecture in the case of $r = 3$.

## 2 Strategies

In this paper, we affirm the Lee-Kong Conjecture for several infinite collections of infinite families of unions of three star graphs. We provide labeling schemes for the vertices which yield a set of consecutive vertex-sums. To simplify our diagrams, we present our star graphs with the central vertices aligned horizontally at the bottom and the pendant vertices aligned horizontally above. For $n$-stars with more than three pendant vertices, we include only the leftmost and rightmost edges in our presentation. All other edges in the star are implied. A labeling scheme for $St(3,4,6)$ is given in Example 3 below.

**Example 3.** *A super edge-magic labeling of* $St(3,4,6)$




The labels on the central vertices play a critical role in the super edge-magic labeling of our graphs. Each of these labels is used in tandem with more than on other label to obtain a vertex-sum. A simple counting argument gives a modular equation that the labels of the central vertices must satisfy. First, for $St(k,m,n)$, note that there are $k+m+n+3$ vertices and $k+m+n$ edges in the graph. Suppose there exists a super edge-magic labeling of $St(k,m,n)$ using the labels $a$, $b$, and $c$ on the central vertices, respectively. Then the sum of the labels of all the vertices, including repetition for the labels on the central vertices, is

$$\left(\sum_{i=1}^{k+m+n+3} i\right) + (k-1)\,a + (m-1)\,b + (n-1)\,c.$$

On the other hand, the vertex-sums must form a set of consecutive integers, one for each of the $k+m+n$ edges of the graph. One can easily verify that the average of any number of consecutive integers is the median. Said differently, for any integer $A$ and any positive integer $t$,

$$(A+1)+(A+2)+\cdots+(A+t) \equiv \begin{cases} 0 \pmod{t}, & \text{if } t \text{ is odd} \\ t/2 \pmod{t}, & \text{if } t \text{ is even} \end{cases}.$$

Using these two ideas, we create the following necessary modular equation for the labels $a$, $b$, and $c$, respectively, of the central vertices in a super edge-magic labeling of $St(k,m,n)$.

$$\left(\sum_{i=1}^{e+3} i\right) + (k-1)a + (m-1)b + (n-1)c \equiv \begin{cases} 0 \pmod{e}, & \text{if } e \text{ is odd} \\ e/2 \pmod{e}, & \text{if } e \text{ is even} \end{cases},$$

where $e = k+m+n$, the number of edges in the graph.

Returning to Example 3, for $St(3,4,6)$, the above equation becomes

$$\left(\sum_{i=1}^{16} i\right) + 2a + 3b + 5c = 136 + 2a + 3b + 5c \equiv 0(\text{mod } 13),$$

which simplifies to $2a+3b+5c \equiv 7(\text{mod } 13)$ for which $(a,b,c) = (6,2,3)$ is one possible solution.

We now place additional restrictions on the solutions of this modular equation that we want to consider as labels for the central vertices of our graphs, $St(k,m,n)$.

**Restriction 1:** Since this is a labeling, $a$, $b$, and $c$ must be distinct.

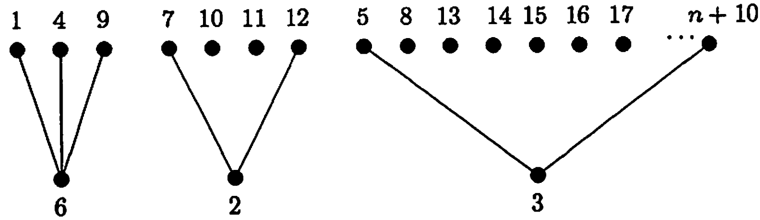
**Restriction 2:** Since $a$, $b$, and $c$ are the labels of vertices in $St(k,m,n)$,

$$\{a,b,c\} \subset \{1,2,\ldots,k+m+n+3\}.$$

**Restriction 3 (Stretching Condition):** In $St(k,m,n)$, use the greatest label, namely $k+m+n+3$, on a non-central vertex of the $n$-star to yield the greatest vertex-sum.

It is important to note why we impose the Stretching Condition on our labeling scheme. First, without some restrictions the list of possible choices for the labels of the central vertices is large, even for small examples. Second, if a labeling scheme satisfying this condition can be found for a specific graph, then we immediately have a labeling scheme for a related infinite family of graphs. We can extend a labeling scheme for $St(k,m,n)$ satisfying the Stretching Condition to any graph $St(k,m,j), j \geq n$ by labeling the $j-n$ additional vertices with the integers $k+m+n+4$ through $k+m+j+3$. Example 4, below, illustrates this extension of our labeling of $St(3,4,6)$ in Example 3, to $St(3,4,n), n \geq 6$.

**Example 4.** *Labeling of* $St(3,4,n), n \geq 6$




As an aside, note that the above scheme for $St(3,4,n)$ satisfies the Stretching Condition for any $n \geq 3$, and degeneratively provides a labeling scheme for $St(3,4,2)$.

Before proceding, it is appropriate here to introduce the notation we will use to denote our super edge-magic labelings. As this paper addresses only unions of star graphs, we will use a modified tuple notation to represent our labeling schemes.

We will use a modified $(t+1)$-tuple to denote the labeling of a single $t$-star as follows: The first entry in the tuple will be the label on the central vertex, followed by a semicolon to distinguish this label from the others. The remaining entries in the tuple will be the labels of the non-central vertices, listed in increasing order. A super edge-magic labeling of a union of three star graphs will be denoted by a set containing three of these modified tuples, of appropriate size. For example, the tuple notation for the super edge-magic labeling of $St(3,4,n), n \geq 6$ shown in Example 4 is

$$\{(6;1,4,9),(2;7,10,11,12),(3;5,8,13,14,\ldots,n+10)\},$$

where the ellipsis denotes consecutive integers.

Returning to our search for labels $(a,b,c)$ for the central vertices of $St(k,m,n)$, the Stretching Condition has several consequences which allow us to apply additional filters. The first filter is an obvious consequence.

**Filter 1:** The integer $k+m+n+3$ is not a label of a central vertex of $St(k,m,n)$.

By the Stretching Condition, the greatest vertex-sum is $c+k+m+n+3$. Since there are $k+m+n$ edges in $St(k,m,n)$ and the vertex-sums form a set of consecutive integers, the Stretching Condition implies this set is $S = \{c+4, c+5, \ldots, c+k+m+n+3\}$. Observe that in a super edge-magic labeling scheme for $St(k, m, n)$, if the sum of the labels of two of the central vertices is $s \in S$, then the vertex-sum $s$ must be attained as a vertex-sum on the remaining star. Filters 2 and 3 below exploit this observation.

**Filter 2:** If the sum of two labels of the central vertices is $s \in S$ and the label of the other central vertex is greater than or equal to $s$, then that choice of $(a, b, c)$ can be eliminated from consideration because it is impossible to achieve the vertex-sum $s$.

**Filter 3:** If the sum of two labels of the central vertices is $s \in S$ and the label of the other central vertex is exactly $s/2$, then that choice of $(a, b, c)$ can be eliminated from consideration because it is impossible to achieve the vertex-sum $s$.

We wrote a computer program to solve the modular equation and impose our three restrictions and our three filters. The Stretching Condition guarantees that a labeling scheme for any particular graph can be easily extended to a scheme for an infinite family of related graphs. There are two keys to finding labeling schemes for infinite collections of these infinite families of graphs: the first is to find a pattern in the computer-generated lists of possible values of $(a, b, c)$ for the central vertices for a related family of graphs, and the second is to create an easily extendable pattern for the labels of the non-central vertices.

# 3 Results

The results presented here significantly extend the known results on super-edge magic graphs. Each of our results shows a super edge-magic labeling scheme for an infinite collection of infinite families of graphs which are the union of three star graphs. The proof for each theorem is a general super edge-magic labeling scheme given in modified tuple notation, as described following Example 4. In our notation, an ellipsis denotes consecutive integers, unless otherwise specified. Theorems 1, 2, and 5 generalize the results of [7].

**Theorem 1.** *The graphs $St(1, m, n), m, n \geq 2$ are super edge-magic.*

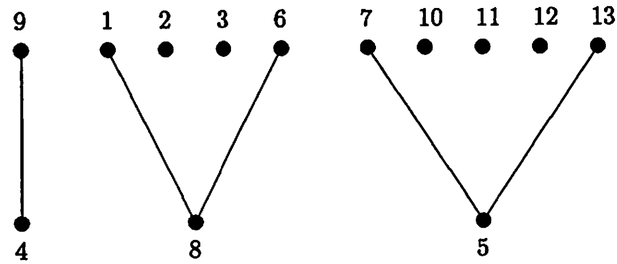
*Proof:* The following is a super edge-magic labeling of the graph.

$$\{(m; m+5), (m+4; 1, 2, \ldots, m-1, m+2),$$

$$(m+1; m+3, m+6, m+7, \ldots, m+n+4)\}$$

**Q.E.D.**

**Example 5.** *Illustration of Theorem 1 - $St(1,4,5)$*




**Theorem 2.** *The graphs $St(2,m,n), m,n \geq 3$ are super edge-magic.*

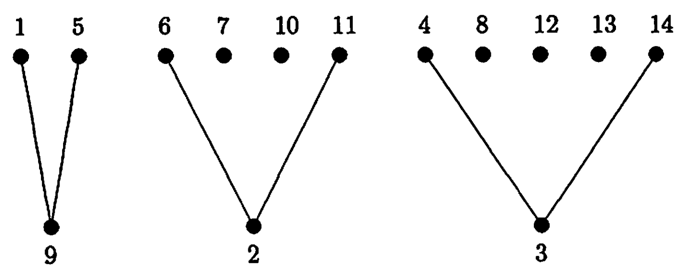
*Proof:* The following is a super edge-magic labeling of the graph.

$$\{(m+5; 1,5), (2; 6,7,\ldots,m+3, m+6, m+7),$$

$$(3; 4, m+4, m+8, m+9, \ldots, m+n+5)\}$$

**Q.E.D.**

**Example 6.** *Illustration of Theorem 2 - $St(2,4,5)$*



**Theorem 3.** *The graphs $St(3, m, n), m, n \geq 3$ are super edge-magic.*

*Proof:* We consider two cases: $m$ odd and $m$ even.

Case 1: The following is a super edge-magic labeling for $St(3, 2m+1, n)$.

$$\{(m+7; 1, 2, m+5),$$
$$(3; 5, 6, \ldots, m+4, m+8, m+9, \ldots, 2m+8),$$
$$(4; m+6, 2m+9, 2m+10, \ldots, 2m+n+7)\}$$

Case 2: The following is a super edge-magic labeling for $St(3, 2m, n)$.

$$\{(m+4; 1, 4, m+7),$$
$$(2; 5, 6, \ldots, m+2, m+5, m+8, m+9, \ldots, 2m+8),$$
$$(3; m+3, m+6, 2m+9, 2m+10, \ldots, 2m+n+6)\}$$

**Q.E.D.**

Theorem 4 is our first theorem with freedom in all three coordinates. It generalizes Theorem 1 and provides a labeling scheme for approximately 19/27 of all unions of three star graphs.

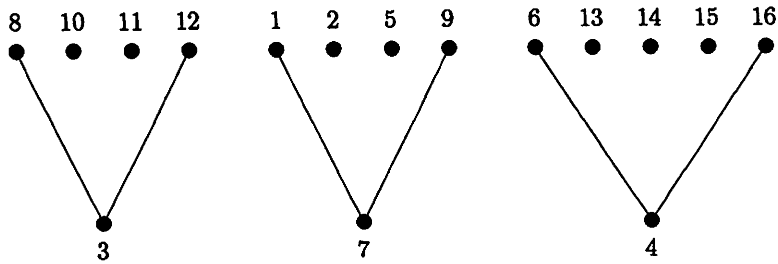
**Theorem 4.** *The graphs $St(3k+1, m, n), k \geq 0, m \geq k+2, n \geq 2$ are super edge-magic.*

*Proof:* The following is a super edge-magic labeling for $St(3k+1, m, n)$. Note: The leftmost ellipsis in the second star denotes consecutive labels as does the ellipsis in the third star. The ellipsis in the first star denotes a continuation of the pattern "label 3, skip 1" and the rightmost ellipsis in the second star denotes a continuation of the pattern "label 1, skip 3."

$$\{(m-k; m-k+5, m-k+7, m-k+8, m-k+9, \ldots,$$
$$m-k+(4k+3), m-k+(4k+4), m-k+(4k+5)),$$
$$(m-k+4; 1, 2, \ldots, m-k-1, m-k+2, m-k+6, m-k+10, \ldots, m-k+(4k+2)),$$
$$(m-k+1; m-k+3, m-k+(4k+6), m-k+(4k+7), \ldots, m-k+(4k+n+4))\}$$

**Q.E.D.**

**Example 7.** *Illustration of Theorem 4* - $St(4,4,5)$




The following theorem generalizes the result for $St(1,n,n)$ in [7]. Observe that the labeling scheme does not satisfy the Stretching Condition, but that the vertex-sums do form a set of consecutive integers. The second and third stars must be the same size, but there is no restriction on the relative sizes of $k$ and $m$.

**Theorem 5.** *The graphs* $St(k,m,m), k,m \geq 1$ *are super edge-magic.*

*Proof:* The following is a super edge-magic labeling for $St(k,m,m)$. Note: The ellipsis in the first star denotes consecutive labels, while the other two denote a continuation of the pattern of all odds or all evens, as appropriate, depending on the parity of $k$.

$$\{(2;4,5,\ldots,k+3),(3;k+4,k+6,\ldots,k+(2m+2)),$$
$$(1;k+5,k+7,\ldots,k+(2m+3))\}$$

**Q.E.D.**

**Theorem 6.** *The graphs* $St(2k,2k+3,n), k,n \geq 1$ *are super edge-magic.*

*Proof:* The following is a super edge-magic labeling for $St(2k,2k+3,n)$. Note: The ellipsis in the $n$-star denotes consecutive labels, while the ellipses in the $2k$- and $(2k+3)$-stars denote a continuation of the pattern of using the integers congruent to $-3$ and $-2 \pmod 4$ on the $2k$-star, and the integers congruent to $-1$ and $0 \pmod 4$ on the $2k+3$-star.

$$\{(4k+6;1,2,5,6,\ldots,4k-3,4k-2),$$

$$(4k+2; 3, 4, 7, 8, \ldots, 4k-1, 4k, 4k+3, 4k+4, 4k+5),$$
$$(4k+1; 4k+7, 4k+8, \ldots, 4k+n+6)\}$$

**Q.E.D.**

**Theorem 7.** *The graphs $St(5, m, n), m \geq 5, n \geq 2m-4$ are super edge-magic.*

*Proof:* The following is a super edge-magic labeling for $St(5, m, n)$.

$$\{(2m; 1, 2, 3, m+2, 2m+4),$$
$$(m; 2m+1, 2m+5, 2m+6, \ldots, 3m+3),$$
$$(m+1; 4, 5, \ldots, m-1, m+3, m+4, \ldots, 2m-1, 2m+2, 2m+3,$$
$$3m+4, 3m+5, \ldots, m+n+8)\}$$

**Q.E.D.**

Theorem 8 improves on Theorem 7 for the cases it covers by reducing the minimum size of $n$ for which the scheme works.

**Theorem 8.** *The graphs $St(5, 3m, n), m \geq 1, n \geq 2m+1$ are super edge-magic.*

*Proof:* The following is a super edge-magic labeling for $St(5, 3m, n)$. Note that the ellipsis in the second star denotes a continuation of the pattern using integers congruent to $-2$, $-1$, and 0 (mod 5), while the first ellipsis in the third star denotes a continuation of the use of the integers congruent to 1 and 2 (mod 5). The second ellipsis in the third star denotes consecutive integers.

$$\{(5m+5; 2, 3, 6, 5m+4, 5m+6),$$
$$(5m+8; 1, 4, 5, 8, 9, 10, 13, 14, 15, \ldots, 5m-2, 5m-1, 5m),$$
$$(5m+3; 7, 11, 12, 16, 17, \ldots, 5m+1, 5m+2, 5m+7,$$
$$5m+9, 5m+10, \ldots, 3m+n+8)\}$$

**Q.E.D.**

**Theorem 9.** *The graphs $St(k, 2k-1, n), k, n \geq 1$ are super edge-magic.*

*Proof:* The following is a super edge-magic labeling for $St(k, 2k-1, n)$. Note that the ellipsis in the first star denotes a continuation of the pattern of multiples of 3. The ellipsis in the second star denotes a continuation of the pattern of integers congruent to 1 and 2 (mod 3) beginning with 5. The ellipsis in the third star denotes the use of consecutive integers.

$$\{(4; 3, 6, 9, \ldots, 3k), (1; 5, 7, 8, 10, 11, \ldots, 3k+1, 3k+2),$$
$$(3k+3, 3k+4, \ldots, 3k+n+2)\}$$

**Q.E.D.**

**Theorem 10.** *The graphs $St(6, m, n), m \geq 6, n$ sufficiently large, are super edge-magic.*

*Proof:* We consider three cases.

Case 1: The following is a super edge-magic labeling for $St(6, 2m, n)$, $m \geq 3$, $n \geq m$.

$$\{(2m+5; 1, 2, 3, 4, m+5, 2m+7),$$
$$(m+3; 5, 6, \ldots, m+2, 2m+6, 2m+8, 2m+9, \ldots, 3m+8),$$
$$(m+4; m+6, m+7, \ldots, 2m+4, 3m+9, 3m+10, \ldots, 2m+n+9)\}$$

Case 2: The following is a super edge-magic labeling for $St(6, 2m+1, n)$, $m \geq 4$, $n \geq 2m-4$.

$$\{(2m+2; 1, 2, 3, m+3, 2m+4, 3m+5),$$
$$(m+1; 2m+3, 2m+5, 2m+6, \ldots, 3m+4, 3m+6, 3m+7, \ldots, 4m+5),$$
$$(m+2; 4, 5, \ldots, m, m+4, m+5, \ldots, 2m+1, 4m+6, 4m+7, \ldots, 2m+n+10)\}$$

Case 3: The following is a super edge-magic labeling for $St(6, 7, n), n \geq 2$.

$$\{(8; 1, 2, 3, 6, 10, 14), (4; 9, 11, 12, 13, 15, 16, 17), (5, 18, 19, \ldots, n+16)\}$$

**Q.E.D.**

The following is a generalization of Theorem 10, Case 2, and is our second result with freedom in all three parameters, $k$, $m$, and $n$.

**Theorem 11.** *The graphs* $St(3k, 2m+1, n), k \geq 2, m \geq 4, n \geq (m-3)k+2$ *are super edge-magic.*

*Proof:* The following is a super edge-magic labeling for $St(3k, 2m+1, n)$ and applies for all $k \geq 2, m \geq 4, n \geq (m-3)k+2$. The ellipsis in the first star represents a continuation of the pattern in which we use all integers congruent to 1, 2, and 3 (mod $m$). The ellipses in the second star represent the use of consecutive integers. The lower ellipses in the third star represent consecutive integers, whereas the centered ellipsis in the third star represents a continuation of the pattern of using integers congruent to $4, 5, \ldots, m-1$ and 0 (mod m) until reaching the value of $(k-1)m$.

$$\{(km+2; 1, 2, 3, m+1, m+2, m+3, \cdots, (k-2)m+1, (k-2)m+2, (k-2)m+3,$$
$$(k-1)m+3, km+4, (k+1)m+5),$$
$$((k-1)m+1; km+3, km+5, km+6, \ldots, (k+1)m+4,$$
$$(k+1)m+6, (k+1)m+7, \ldots, (k+2)m+5),$$
$$((k-1)m+2; 4, 5, \ldots, m, m+4, m+5, \ldots, 2m, \cdots, (k-2)m+4,$$
$$(k-2)m+5, \ldots, (k-1)m, (k-1)m+4, (k-1)m+5, \ldots, km+1,$$
$$(k+2)m+6, (k+2)m+7, \ldots, n+(2m+1)+3k+3)\}$$

**Q.E.D.**

**Theorem 12.** *The graphs* $St(9, 2m, n), m \geq 3, n \geq 2m-2$ *are super edge-magic.*

*Proof:* The following is a super edge-magic labeling for $St(3k, 2m+1, n)$. All ellipses represent consecutive integers.

$$\{(3m+6; 1, 2, 3, m+2, m+3, m+4, 2m+3, 2m+6, 3m+8),$$
$$(2m+4; 2m+7, 2m+8, \ldots, 3m+4, 3m+7, 3m+9, 3m+10, \ldots, 4m+9),$$
$$(2m+5; 4, 5, \ldots, m+1, m+5, m+6, \ldots, 2m+2, 3m+5,$$
$$4m+10, 4m+11, \ldots, 2m+n+12)\}$$

Q.E.D.

**Theorem 13.** *The graphs $St(12, 2m, n), m \geq 7, n \geq 4m - 21$ are super edge-magic.*

*Proof:* The following is a super edge-magic labeling for $St(12, 2m, n)$. All ellipses represent consecutive integers.

$$\{(4m-7; 1, 2, 3, m-1, m, m+1, 2m-4, 2m-2, 2m-1, 3m-3, 4m-4, 5m-5),$$

$$(3m-6; 2m-3, 3m-4, 4m-6, 4m-5, 4m-3, 4m-2, \ldots, 5m-6$$

$$, 5m-4, 5m-3, \ldots, 6m-7),$$

$$(3m-5; 4, 5, \ldots, m-2, m+2, m+3, \ldots, 2m-5, 2m, 2m+1, \ldots, 3m-7,$$

$$3m-2, 3m-1, \ldots, 4m-8, 6m-6, 6m-5, \ldots, 2m+n+15)\}$$

Q.E.D.

# 4 Conclusions

We have searched for one general labeling scheme that would show that every union of three stars is super edge-magic, thus proving the Lee-Kong Conjecture for the case $r = 3$. While we did not attain this goal, the results contained herein significantly extend the previously published results on super edge-magic unions of star graphs. The previously published results relating to the union of three star graphs are five infinite families of graphs contained in [7]. Each of our thirteen theorems is a labeling scheme for an infinite collection of such infinite families of unions of three star graphs. We believe that there exists a general labeling scheme that can be used for all unions of three star graphs, as a degenerate case of a more general scheme for labeling all unions of an odd numbers of star graphs. Such a general labeling scheme would prove the Lee-Kong Conjecture.

Note that there is no known general conjecture for the super edge-magic labeling of an even number of star graphs. Identifying a characterization for which of these graphs is super edge-magic is difficult as some infinite families of such graphs are known to be super edge-magic [for example: $St(1, 1, 1, 4n + 3)$] while other infinite families are known to be not super edge-magic [for example $St(1, 1, 1, 4n + 1)$].

## 5 Acknowledgments

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