Bounds on a generalized total domination parameter

Michael A. Henning, University of Zululand
and
Henda C. Swart, University of Natal.

Abstract

If n is an integer, $n \geq 2$ and u and v are vertices of a graph G, then u and v are said to be K_n-adjacent vertices of G if there is a subgraph of G, isomorphic to K_n, containing u and v. A total K_n-dominating set of G is a set D of vertices such that every vertex of G is K_n-adjacent to a vertex of D. The total K_n-domination number $\gamma^t_{K_n}(G)$ of G is the minimum cardinality among the total K_n-dominating sets of vertices of G. It is shown that, for $n \in \{3,4,5\}$, if G is a graph with no K_n-isolated vertex, then $\gamma^t_{K_n}(G) \leq (2p)/n$. Further, K_n-connectivity is defined and it is shown that, for $n \in \{3,4\}$, if G is a K_n-connected graph of order $\geq n+1$, then $\gamma^t_{K_n}(G) \leq (2p)/(n+1)$. We establish that the upper bounds obtained are best possible.

The terminology and notation of [2] will be used throughout. In particular, G will denote a graph with vertex set V, edge set E, order p and size q. If n is an integer, $n \geq 2$ and u and v are distinct vertices of a graph G, then u and v are said to be K_n-adjacent vertices of G if there is a subgraph of G, isomorphic to K_n, containing u and v. Therefore, u and v are K_2-adjacent vertices of G if and only if u and v are adjacent vertices of G. The set of all vertices K_n-adjacent to a vertex v in G is denoted by $N^{K_n}(v)$ and $|N^{K_n}(v)|$ by $K_n - \deg v$. Furthermore, $\delta_{K_n}(G) = \min\{K_n - \deg v | v \in V(G)\}$ and $\triangle_{K_n}(G) = \max\{K_n - \deg v | v \in V(G)\}$. A vertex that is contained in no subgraph of G, isomorphic to K_n, is called a K_n-isolated vertex of G.

This definition of K_n-adjacency suggests a generalization of the concept of connectedness in graphs. Let u and v be vertices of a graph G. A $u - v$ K_n path of G is a finite, alternating sequence of vertices and subgraphs of G, isomorphic to K_n, beginning with u and ending with v, such that the vertices of the sequence are distinct, the subgraphs of the sequence are distinct and every subgraph of the sequence is immediately preceded and succeeded by a vertex that is contained in that subgraph. The vertex u is said to be K_n-connected to the vertex v in G if there exists a $u - v$

K_n-path in G. A graph G is K_n-connected if every two of its vertices are K_n-connected. A graph that is not K_n-connected is K_n-disconnected. A K_n-component of a graph G is a maximal K_n-connected subgraph of G.

We are now in a position to generalize the concept of total domination in graphs. Cockayne, Dawes and Hedetniemi [4] initiated a study of total dominating sets in graphs. Our definition of K_n-adjacency suggests a generalization of total domination in a graph. For $n \geq 2$, a total K_n-dominating set of G is a set D of vertices such that every vertex of G is K_n-adjacent to a vertex of D. The total K_n-domination number $\gamma^t_{K_n}(G)$ of G is the minimum cardinality among the total K_n-dominating sets of vertices of G. We note that this parameter is defined only for graphs with no K_n-isolated vertex.

It is our purpose to show that, for $n \in \{3,4,5\}$, if G is a graph with no K_n-isolated vertex, then $\gamma^t_{K_n}(G) \leq (2p)/n$. Further, we show that, for $n \in \{3,4\}$, if G is a K_n-connected graph of order $\geq n+1$, then $\gamma^t_{K_n}(G) \leq (2p)/(n+1)$. We establish that the upper bounds obtained are best possible.

These concepts find application in many situations and structures which give rise to graphs. Consider, for instance, the following example. Given that each member of a population serves on some committee of n members, find a smallest representative subcommittee in the population such that the following two conditions are satisfied: Firstly, every non-member serves on a committee (of n members) with at least one member of the subcommittee to enhance communication, and, secondly, every member serves on a committee (of n members) with at least one other member of the subcommittee to foster co-operation among the members of the subcommittee itself. It is then of interest to determine an upper bound on the minimum size of the subcommittee in terms of n and the size of the population. This situation may be represented in graph theoretic terms as follows: We associate the members in the population with the vertices of a graph G, two vertices of G being adjacent if and only if there is some committee (of n members) on which they both serve. Hence for each such committee in our population, there is an associated subgraph of G, isomorphic to K_n, the vertices of which correspond to the members of the committee. In graph theoretic terms, the problem is to find a minimum total K_n- dominating set of G or to determine bounds on its cardinality.

We begin our investigation of total K_n-dominating sets in graphs with the following elementary characterization of minimal such sets. This result is analogous to a generalization of the authors [5] to a classical result of Ore [6] concerning minimal dominating sets in graphs.

Lemma 1

For $n \geq 2$, let D be a total K_n-dominating set of a graph G. Then D is a minimal total K_n-dominating set of G if and only if each vertex $d \in D$ has at least one of the following two properties:

P(1,n): there exists a vertex $v \in$ V-D such that $N^{K_n}(v) \cap D = \{d\}$;
P(2,n): there exists a vertex $z \in$ D-$\{d\}$ such that $N^{K_n}(z) \cap D = \{d\}$.

The following two results relate the total K_n-domination number and the maximum K_n-degree $\Delta_{K_n}(G)$ of a graph G. These results generalize those of Cockayne, Hawes and Hedetniemi [4].

Theorem 1.1

For $n \geq 2$, if G is a graph with no K_n-isolated vertex, then $\gamma^t_{K_n}(G) \leq p - \Delta_{K_n}(G) + 1$.

PROOF

Let v be a vertex of G such that K_n- deg $v = \Delta_{K_n}(G)$ and let $X = V - (N^{K_n}(v) \cup \{v\})$. If $X = \emptyset$, then $\Delta_{K_n}(G) = p - 1$ and $\gamma^t_{K_n}(G) = 2 = p - \Delta_{K_n}(G) + 1$. Suppose then that $X \neq \emptyset$ and let S be the set of all K_n-isolated vertices of $\langle X \rangle$.

If $S = \emptyset$, then $D = X \cup \{v, w\}$, where $w \in N^{K_n}(v)$, is a total K_n-dominating set of G with $|D| = p - \Delta_{K_n}(G) + 1$. If $S \neq \emptyset$, then, since G contains no K_n-isolated vertex, each vertex s of S is K_n-adjacent to some vertex in $N^{K_n}(v)$; let $M(S)$ be a subset of $N^{K_n}(v)$ of smallest cardinality such that each vertex $s \in S$ is K_n-adjacent to some element of $M(S)$. We note that $|M(S)| \leq |S|$. Necessarily $D = (X - S) \cup M(S) \cup \{v\}$ is a total K_n-dominating set of G with $|D| = |X| - |S| + |M(S)| + 1 \leq |X| + 1 = p - \Delta_{K_n}(G)$. This completes the proof of the theorem. □.

Theorem 1.2

For $n \geq 2$, if G is a K_n-connected graph with $\Delta_{K_n}(G) < p - 1$, then $\gamma^t_{K_n}(G) \leq p - \Delta_{K_n}(G)$.

PROOF

Using the notation introduced in the proof of Theorem 1.1, we note that, since $\Delta_{K_n}(G) < p - 1$, $X \neq \emptyset$. If $S \neq \emptyset$, then we proceed in a manner identical to that used in the proof of Theorem 1.1 to show that $\gamma^t_{K_n}(G) \leq p - \Delta_{K_n}(G)$. Suppose then that $S = \emptyset$. Since G is K_n-connected, there exists a vertex x in X that is K_n-adjacent to some vertex $y \in N^{K_n}(v)$. Now let C be the vertex set of the K_n- component of $\langle X \rangle$ which contains x. For convenience, let Δ'_{K_n} denote the maximum K_n- degree of $\langle C \rangle$. Further, let Y be a total K_n- dominating set of $\langle C \rangle$, with $|Y| = \gamma^t_{K_n}(\langle C \rangle)$. Since $\langle C \rangle$ is a graph with no K_n-isolated vertex, it follows from Theorem 1.1 that $|Y| \leq |C| - \Delta'_{K_n} + 1$. We consider two possibilities.

If $\Delta'_{K_n} = n-1$, then $\langle C \rangle \cong K_n$. Necessarily $D = \{v, x, y\} \cup (X - C)$ is a total K_n-dominating set of G with $|D| = 3 + |X| - |C| = 3 + (p - \Delta_{K_n}(G) - 1) - n = p - \Delta_{K_n}(G) - (n-2) \leq p - \Delta_{K_n}(G)$.

If $\Delta'_{K_n} \geq n$, then $D = \{v, y\} \cup Y \cup (X - C)$ is necessarily a total K_n-dominating set of G with $|D| \leq 2 + (|C| - \Delta'_{K_n} + 1) + (p - \Delta_{K_n}(G) - 1) - |C| \leq p - \Delta_{K_n}(G) - (n-2) \leq p - \Delta_{K_n}(G)$.

This completes the proof of the theorem. □

For n a given integer, $n \geq 3$, let $C(1,n), C(2,n), C(3,n), C(4,n)$ and $C(5,n)$ denote the following conditions on a graph G:

- $C(1,n)$: G has no K_n-isolated vertex.
- $C(2,n)$: For each edge e of G, $G - e$ contains at least one K_n-isolated vertex.
- $C(3,n)$: For each edge e of G, $G - e$ is K_n- disconnected.
- $C(4,n)$: There exist two K_n-adjacent vertices u and v of G such that $G - \{u, v\}$ contains at least $n-1$ K_n-isolated vertices.
- $C(5,n)$: There exist two K_n-adjacent vertices u and v of G such that $G - \{u, v\}$ contains at least $n-2$ K_n- isolated vertices.

The following lemma will prove to be useful.

Lemma 2

Let G be a graph satisfying conditions $C(1,n)$ and $C(2,n)$. If there exists a vertex of G of degree at least n, then, for $n \in \{3,4\}$, G satisfies condition $C(4,n)$ and, for $n = 5$, G satisfies condition $C(5,5)$.

PROOF

Suppose there exists a vertex v of G with $\deg v > n-1$. Since G has no K_n-isolated vertex, there is some subgraph F of G, isomorphic to K_n, that contains v. Since $\deg v > n-1$, there exists an edge of G, e_1, say, incident with v and not contained in F. As G satisfies condition $C(2,n)$, $G - e_1$ contains some K_n-isolated vertex w_1, say. Since $F \subset G - e_1$, w_1 is distinct from v. Let F_1 denote a subgraph of G, isomorphic to K_n, containing w_1 and v. Since $\deg v > n-1$, there exists an edge of G, e_2, say, incident with v and not contained in F_1. Since G satisfies condition $C(2,n)$, $G - e_2$ contains some K_n-isolated vertex w_2, say. Since $F_1 \subset G - e_2$, w_2 is distinct from w_1 and v. Let F_2 denote a subgraph of G, isomorphic to K_n, containing w_2 and v. We consider two cases.

Case 1: Suppose that $n = 3$. Since $\deg v > 2$, there exists a neighbour of v, u say, distinct from w_1 and w_2. Then $G - \{u,v\}$ contains at least two K_3-isolated vertices, namely, the vertices w_1 and w_2. Hence G satisfies condition $C(4,3)$.

Case 2: Suppose that $n \geq 4$. Let u be a vertex of G, distinct from w_1 and w_2, in the closed neighbourhood of v. If there exists an edge e_3, say, of G incident with u that is not contained in F_1 or F_2, then, since G satisfies condition $C(2,n)$, $G - e_3$ contains some K_n- isolated vertex w_3, say. Since $F_1 \subset G - e_3$ and $F_2 \subset G - e_3$, $w_3 \notin V(F_1) \cup V(F_2)$. This implies, however, that (if $u \neq v$) $G - \{u,v\}$ or (if $u = v$) $G - \{u^*,v\}$, where u^* is any vertex in $N(v) - \{w_1,w_2,w_3\}$, contains at least three K_4-isolated vertices, namely, the vertices in $\{w_1,w_2,w_3\}$. In particular, G satisfies condition $C(4,4)$ if $n = 4$ and condition $C(5,5)$ if $n = 5$.

Suppose then that there is no such edge e_3 of G incident with some vertex $u \in N[v] - \{w_1,w_2\}$. This, together with the observation that every edge of G incident with w_1 or w_2 is contained in a $K_n \subset G$ with v, implies that every edge of G incident with w_1 or w_2 is contained in F_1 or F_2. Hence, necessarily, $\langle N[v] \rangle = \langle V(F_1) \cup V(F_2) \rangle$ is a K_n-component of the graph G

(possibly, $G = \langle N[v] \rangle$) and each edge of this K_n-component is contained in F_1 or F_2. This implies, however, that each vertex, distinct from v, in such a K_n-component of G is K_n-isolated in $G - v$. Hence, for any vertex $u \in N(v)$, $G - \{u, v\}$ contains $\deg v - 1 \geq n - 1$ K_n-isolated vertices. In particular, G satisfies condition $C(4,4)$ if $n = 4$ and condition $C(5,5)$ if $n = 5$.

This completes the proof of the lemma. □

Corollary 1

For $n \in \{3,4\}$, if G is a K_n-connected graph with $p(G) \geq n+1$ that satisfies condition $C(2,n)$, then G satisfies condition $C(4,n)$.

PROOF

For $n \in \{3,4\}$, since G is a K_n-connected graph of order $\geq n+1$, there exists a vertex v of G with $\deg v \geq n$. Hence, by Lemma 2, for $n \in \{3,4\}$, G satisfies condition $C(4,n)$. □

Corollary 2

For $n \in \{3,4,5\}$, if G is a graph with no K_n-isolated vertex that satisfies condition $C(2,n)$, then G satisfies condition $C(5,n)$.

PROOF

If $\deg_G v \leq n-1$ for every vertex v of G, then $G = mK_n (m \geq 1)$. This implies, however, that for any two K_n- adjacent vertices u and v of G, $G - \{u,v\}$ contains $n-2$ K_n-isolated vertices. Hence, in this case, G satisfies condition $C(5,n)$ for all values of $n \geq 3$ (in particular, for $n \in \{3,4,5\}$).

Suppose then that there exists a vertex v of G with $\deg v \geq n$. Then, by Lemma 2, for $n \in \{3,4\}$, G satisfies condition $C(4,n)$ (hence, certainly, condition $C(5,n)$) and, for $n = 5$, condition $C(5,5)$. □

The next result establishes an upper bound, for $n \in \{3,4,5\}$, on the total K_n-domination number of a graph G with no K_n-isolated vertex.

Theorem 2

For $n \in \{3,4,5\}$, if G is a graph with no K_n-isolated vertex, then $\gamma^t_{K_n}(G) \leq (2p)/n$.

PROOF

We proceed by induction on integers p, where $p \geq n$. If G is a graph of order $p = n$ such that G contains no K_n-isolated vertex, then $G \cong K_n$ and $\gamma^t_{K_n}(G) = (2p)/n$.

Assume for all graphs H of order less than p (where $p \geq n+1$ is an integer) that if H contains no K_n-isolated vertex, then $\gamma^t_{K_n}(H) \leq (2p(H))/n$.Let G be a graph of order p that contains no K_n-isolated vertex. We show that $\gamma^t_{K_n}(G) \leq (2p)/n$. Let G' be a graph obtained from G by the deletion (if necessary) of a set of edges of G such that G' satisfies conditions $C(1,n)$ and $C(2,n)$. Then by Corollary 2, for $n \in \{3,4,5\}$, G' satisfies condition $C(5,n)$. Hence there exist two K_n- adjacent vertices u and v of G' such that $G' - \{u,v\}$ contains at least $n-2$ K_n-isolated vertices (where $n \in \{3,4,5\}$); let S denote the set of all K_n-isolated vertices of $G' - \{u,v\}$ and consider the graph $G'' = G' - (S \cup \{u,v\})$, say.

By the manner in which G'' is constructed, G'' contains no K_n-isolated vertex. Thus, by the inductive hypothesis, $\gamma^t_{K_n}(G'') \leq (2p(G''))/n$. Now let D'' be a total K_n-dominating set of G'' with $|D''| = \gamma^t_{K_n}(G'')$ and consider the set $D = D'' \cup \{u,v\}$. Necessarily D is a total K_n-dominating set of G' with $|D| \leq (2p(G''))/n + 2 = (2(p - |S| - 2))/n + 2 \leq (2(p-n))/n + 2 = (2p)/n$. However, G' is a spanning subgraph of G, implying necessarily that, for $n \in \{3,4,5\}$, $\gamma^t_{K_n}(G) \leq \gamma^t_{K_n}(G') \leq (2p)/n$. □

Since there exist graphs (viz. $G = mK_n (m \geq 1)$) that contain no K_n-isolated vertex and such that $\gamma^t_{K_n}(G) = (2p)/n$, the upper bound $(2p)/n$ established in the result of Theorem 2 is best possible.

The next result establishes an upper bound, for $n \in \{3,4\}$, on the total K_n-domination number of a K_n-connected graph G of order $\geq n+1$. This result generalizes that of Cockayne, Dawes and Hedetniemi [4].

Theorem 3

For $n \in \{2,3,4\}$, if G is a K_n-connected graph of order $\geq n+1$, then $\gamma^t_{K_n}(G) \leq (2p)/(n+1)$.

PROOF

We may assume $n = 3$ or 4 since the case $n = 2$ is the result of Cockayne, Dawes and Hedetniemi. We proceed by induction on p, where $p \geq n+1$. If G is a K_n-connected graph of order $p = n+1$, then $G = K_{n+1}$ or $G = K_{n+1} - e$ and, in each case, G has total K_n-domination number $2 = (2p)/(n+1)$.

Assume for all K_n-connected graphs H with $n+1 \leq p(H) < p$ that $\gamma^t_{K_n}(H) \leq 2p(H)/(n+1)$. Let G be a K_n-connected graph of order p. We show that $\gamma^t_{K_n}(G) \leq (2p)/(n+1)$.

Let G' be the K_n-connected graph obtained from G by the deletion (if necessary) of a set of edges of G such that G' satisfies condition $C(3,n)$.

Before proceeding further with the proof of Theorem 3, we prove the following lemmata.

Lemma 3

If G' satisfies condition $C(4,n)$, then $\gamma^t_{K_n}(G') \leq (2p)/(n+1)$.

PROOF

Suppose G' satisfies condition $C(4,n)$. Then there exist two K_n-adjacent vertices u and v of G' such that $G' - \{u,v\}$ contains at least $n-1$ K_n-isolated vertices; let S denote the set of all K_n-isolated vertices of $G' - \{u,v\}$ and consider the graph $G'' = G' - (S \cup \{u,v\})$, say.

By the manner in which G'' is constructed, G'' contains no K_n-isolated vertex. Hence each K_n-component of G'' is of order at least n. Let $G_1, ..., G_k$ denote the K_n- components (if any) of G'' of order n and let $G_{k+1}, ..., G_m (m \geq k+1)$ denote the remaining K_n- components of G''. Since G' is K_n-connected and since each vertex of S is K_n-isolated in $G' - \{u,v\}$, there is necessarily for each i with $1 \leq i \leq k$, a vertex of G_i that is K_n-adjacent to u or v; let w_i denote such a vertex of G_i.

By the inductive hypothesis, $\gamma^t_{K_n}(G_i) \leq (2p(G_i))/(n+1)$ for each i with $k+1 \leq i \leq m$; let D_i denote a total K_n-dominating set of $G_i (k+1 \leq i \leq m)$ with $|D_i| = \gamma^t_{K_n}(G_i)$ and consider the set $D = \{u, v\} \cup (\cup_{i=1}^{k}\{w_i\}) \cup (\cup_{i=k+1}^{m} D_i)$. Necessarily, D is a total K_n-dominating set of G' with

$$\begin{aligned} |D| &\leq 2 + k + \textstyle\sum_{i=k+1}^{m} 2p(G_i)/(n+1) \\ &= 2 + k + (2(p - 2 - |S| - kn))/(n+1) \\ &\leq 2 + k + (2(p - (n+1) - kn))/(n+1) \\ &= (2p)/(n+1) - 2(kn)/(n+1) + k \\ &\leq (2p)/(n+1); \end{aligned}$$

consequently, $\gamma^t_{K_n}(G') \leq |D| \leq (2p)/(n+1)$. This completes the proof of Lemma 3. □

Lemma 4

If G' does not satisfy condition C(2,n), then $\gamma^t_{K_n}(G') \leq (2p)/(n+1)$.

PROOF

Since G' does not satisfy condition $C(2, n)$, there exists an edge e of G' such that $G' - e$ contains no K_n-isolated vertex; let $e = uv$ denote such an edge of G'. However G' satisfies condition $C(3, n)$ and so $G' - e$ is K_n-disconnected. We consider two cases.

Case 1: Suppose that there exists a K_n-component of $G' - e$ of order n. We show that G' satisfies condition $C(4, n)$. If an end vertex of $e = uv$ is contained in some K_n-component of order n of $G' - e$, then each vertex, distinct from u or v, of such a K_n-component is K_n- isolated in $G' - \{u, v\}$. This would imply, however, that G' satisfies condition $C(4, n)$.

Suppose then that no K_n-component of order n of $G' - e$ contains an end vertex of e. The K_n-connectivity of G' implies, necessarily, that there is a vertex in each K_n- component of order n of $G' - e$ that is contained in a subgraph of G', isomorphic to K_n, that contains the edge e (and hence each of u and v); let w denote such a vertex in some K_n-component of order n of $G' - e$. Then, necessarily, each vertex, distinct from w, of the K_n-component of $G' - e$ containing w, is K_n-isolated in $G' - \{w, v\}$ (and in $G' - \{w, u\}$). This would imply, however, that G' satisfies condition $C(4, n)$.

Hence G' satisfies condition $C(4, n)$. Thus, by Lemma 3, $\gamma^t_{K_n}(G') \leq (2p)/(n+1)$.

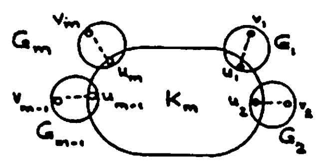
Case 2: Suppose that each K_n-component of $G' - e$ is of order at least $n+1$. Let $G_1, G_2, ..., G_m (m \geq 2)$ denote the K_n-components of $G' - e$. By the inductive hypothesis, $\gamma^t_{K_n}(G_i) \leq 2p(G_i)/(n+1)$ for each i with $1 \leq i \leq m$; let D_i denote a total K_n-dominating set of G_i with $|D_i| = \gamma^t_{K_n}(G_i)$ and consider the set $D = \cup_{i=1}^m D_i$. Necessarily, D is a total K_n- dominating set of G' with $|D| = \sum_{i=1}^m |D_i| \leq \sum_{i=1}^m (2p(G_i))/(n+1) = (2p)/(n+1)$. Hence $\gamma^t_{K_n}(G') \leq |D| \leq (2p)/(n+1)$. □

We now continue with our proof of Theorem 3. For $n \in \{3, 4\}$, if G' satisfies condition $C(2, n)$, then Corollary 1 and Lemma 3 imply that $\gamma^t_{K_n}(G') \leq (2p)/(n+1)$. If, however, G' does not satisfy condition $C(2, n)$ (with $n \in \{3, 4\}$), then, by Lemma 4, $\gamma^t_{K_n}(G') \leq (2p)/(n+1)$. Hence in both cases $\gamma^t_{K_n}(G') \leq (2p)/(n+1)$. However G' is a spanning subgraph of G, implying, necessarily, that $\gamma^t_{K_n}(G) \leq \gamma^t_{K_n}(G') \leq (2p)/(n+1)$ where $n \in \{3, 4\}$. This completes the proof of Theorem 3 □

For $m \geq n \geq 3$, let H be a graph obtained by the removal of a single edge uv from K_{n+1} and let $G_1, G_2, ..., G_m$ be m disjoint copies of H and u_i the vertex in G_i corresponding to $u (i = 1, 2, ..., m)$. Let G be the graph obtained from $\cup_{i=1}^m G_i$ by inserting an edge between every pair of vertices u_i, u_j with $i \neq j (i, j = 1, ..., m)$. (The graph G is shown in Figure 1.) Then G is a K_n-connected graph with $p(G) \geq n+1$ and $\gamma^t_{K_n}(G) = (2p)/(n+1)$.

Figure 1 A K_n-connected graph G with $p(G) \geq n+1$ and $\gamma^t_{K_n}(G) = (2p)/(n+1)$.

G :



Hence, for every integer $n \geq 3$, there exist K_n-connected graphs G with $p(G) \geq n+1$ and $\gamma^t_{K_n}(G) = (2p)/(n+1)$; consequently, the upper bound established in the result of Theorem 3 is best possible.

In view of the results of Theorem 2 and Theorem 3, we pose the following two questions: Firstly, is it true for all values of $n \geq 3$, that if G is a graph with no K_n-isolated vertex, then $\gamma^t_{K_n}(G) \leq (2p)/n$? Secondly, is it true for all values of $n \geq 3$, that if G is a K_n-connected graph of order $\geq n+1$, then $\gamma^t_{K_n}(G) \leq (2p)/(n+1)$? It is perhaps relevant to note that, if $\gamma_{K_n}(G)$ denotes the K_n-domination number of G, defined to be the cardinality of a smallest set D of vertices of G such that every vertex in $V - D$ is K_n-adjacent to a vertex in D, then $\gamma_{K_n}(G) \leq p/n$ for $n = 2,3,4$, but *not* for $n \geq 5$ (cf. [5]).

The authors gratefully acknowledge the support of the Foundation For Research Development of the C.S.I.R.

References

[1] C. Berge, *Graphs and Hypergraphs*, North-Holland, London, 1973.

[2] G. Chartrand and L. Lesniak, *Graphs & Digraphs*, 2nd ed., Wadsworth, Monterey, 1986.

[3] E.J. Cockayne, *Domination in undirected graphs - a survey*, Theory and Applications of Graphs in America's Bicentennial year, Y. Alavi and D.R. Lick (Eds.) Springer-Verlag, Berlin, 1978, 141-147.

[4] E.J. Cockayne, R.M. Dawes and S.T. Hedetniemi, *Total domination in graphs*, Networks, 10 (1980), 211- 219.

[5] M.A. Henning and Henda C. Swart, *Bounds on a generalized domination parameter*. Submitted for publication.

[6] O. Ore, *Theory of Graphs*, Amer. Math. Soc. Publ., Providence, 1962.