A Note On Some Ramsey Numbers $R(C_p, C_q, C_r)$

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Abstract

The Ramsey number $R(C_p, C_q, C_r)$ is the smallest positive integer m such that no matter how one colors the edges of the K_m in red, white and blue, there must be a red C_p, a white C_q or a blue C_r. In this work, we verified some known $R(C_p, C_q, C_r)$'s and compute some new $R(C_p, C_q, C_r)$'s. The results are based on computer algorithms.

1 Introduction

Let F, G and H be graphs, a $(F, G, H; n)$-graph is a coloring of the edges of the complete graph K_n in red, white and blue that does not contain a red F, a white G and a blue H. The Ramsey number $R(F, G, H)$ is defined to be the least integer $n > 0$ such that there is no $(F, G, H; n)$-graph. A regularly updated survey of the most recent results on the values and best known bounds of the Ramsey numbers $R(F, G, H)$ can be found in [6].

In this paper we consider the case where F, G and H are cycles of various orders. In Section 2, we present results for Ramsey numbers $R(C_p, C_q, C_r)$. Section 3 lists the statistics of $R(C_p, C_q, C_r)$. The graphs themselves are available from the author.

A computer program called *nauty*, written by Brendan McKay [5], was used extensively in this work for testing isomorphism of edge colorings.

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2 Results

In Table I below, we give a list of old and new $R(C_p, C_q, C_r)$'s.

p	q	r	$R(C_p, C_q, C_r)$	Reference
3	3	3	17	[4]
3	3	4	17	[2]
3	4	4	12	[8]
4	4	4	11	[1]
5	5	5	17	[10]
6	6	6	12	[11]
7	7	7	25	[3]
3	4	5	13	
3	4	6	13	
3	4	7	≥ 15	
4	4	5	12	
4	4	6	12	
4	4	7	12	
4	5	5	13	
4	5	6	13	
4	5	7	≥ 15	
4	6	6	11	
4	6	7	13	
3	3	5	≥ 21	
3	5	5	≥ 17	

Table I. Known $R(C_p, C_q, C_r)$.

3 Enumerations of $R(C_p, C_q, C_r)$

The algorithm used is an extension of the gluing algorithm to compute the Ramsey number $R(G, H)$ in [7] and [9]. We use the same definitions and notations as in [7].

A brief description to generate all $(C_p, C_q, C_r; n)$-graphs follows: Let v be a vertex, F be a $(P_{p-1}, C_q, C_r; s)$-graph, G be a $(C_p, P_{q-1}, C_r; t)$-graph and H be a $(C_p, C_q, P_{r-1}; u)$-graph, where $s + t + u + 1 = n$. We join every vertex in F to v by a red edge, every vertex in G to v by a white edge and every vertex in H to v by a blue edge. We then attach feasible cones in F to G to produce a coloring of K_{s+t+1} without a red C_p, a white C_q and a blue C_r. Next we attach feasible cones in H to $K_{s+t+1} \setminus \{v\}$ to produce a coloring of $K_{s+t+u+1}$ without a red C_p, a white C_q and a blue C_r.

It is computationally infeasible to generate all (C_p, C_q, C_r)-graphs. We only enumerate (C_p, C_q, C_r)-graphs on $R(C_p, C_q, C_r) - 1$ vertices, and their statistics are presented in Table II.

p	q	r	m	$(C_p, C_q, C_r; m)$ graphs
3	3	3	16	2
3	4	4	11	18812
3	4	5	12	21102901
3	4	6	12	7192853
3	4	7	14	$\geq$ 9966020
4	4	4	10	5880
4	4	5	11	62
4	4	6	11	644
4	4	7	11	4322
4	5	5	12	9054101
4	5	6	12	7168406
4	5	7	14	$\geq$ 14707004
4	6	6	10	4853
4	6	7	12	7168406
3	3	5	20	$\geq$ 949
3	5	5	16	$\geq$ 48780050

Table II. Number of $(C_p, C_q, C_r; m)$-graphs.

References

[1] A. Bialostocki and J. Schönheim, *On Some Turán and Ramsey Numbers for* C_4, Graph Theory and Combinatorics (ed B. Bollobás), Academic Press, London, (1984) 29-33.

[2] G. Exoo and D. F. Reynolds, *Ramsey Numbers Based on* C_5*-Decompositions*, Discrete Mathematics, 71 (1988) 119-127.

[3] R. J. Faudree, A. Schelten and I. Schiermeyer, *The Ramsey Number* $r(C_7, C_7, C_7)$, Discussiones Mathematicae Graph Theory, 23 (2003) 141-158.

[4] R. E. Greewood and A. M. Gleason, *Combinatorial Relations and Chromatic Graphs*, Canadian Journal of Mathematics, 7 (1955) 1-7.

[5] B. D. McKay, nauty users' guide (version 2.2), Technical Report TR-CS-90-02, Computer Science Department, Australian National University, 1990, source code at `http://cs.anu.edu.au/~bdm/nauty`.

[6] S. P. Radziszowski, *Small Ramsey numbers*, Electronic Journal of Combinatorics, Dynamic Survey 1, revision #10, 2004, `http://www.combinatorics.org`.

[7] S. P. Radziszowski and K. K. Tse, *A Computational Approach for the Ramsey Numbers* $R(C_4, K_n)$, The Journal of Combinatorial Mathematics and Combinatorial Computing, (2002) 195–207.

[8] C. U. Schulte, *Ramsey-Zalen für Bäume und Kreise*, Ph.D. thesis, Heinrich-Heine-Universität Düsseldorf, (1992).

[9] K. K. Tse, *On the Ramsey Number of the quadrilateral versus the book and the wheel*, Australasian Journal of Combinatorics, 27, (2002) 163–167.

[10] Y. Yang and P. Rowlinson, *On the Third Ramsey Numbers of Graphs with Five Edges*, Journal of Combinatorial Mathematics and Combinatorial Computing, 11 (1992) 213-222.

[11] Y. Yang and P. Rowlinson, *On Graphs without 6-Cycles and Related Ramsey Numbers*, Utilitas Mathematica, 44 (1993) 192-196.