A Nordhaus-Gaddum-type result for the 2-domination number

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Abstract

A vertex set $S \subseteq V(G)$ of a graph G is a 2-dominating set of G if $|N(v) \cap S| \geq 2$ for every vertex $v \in (V(G) - S)$, where $N(v)$ is the neighborhood of v. The 2-domination number $\gamma_2(G)$ of graph G is the minimum cardinality among the 2-dominating sets of G. In this paper we present the following Nordhaus-Gaddum-type result for the 2-domination number. If G is a graph of order n, and $\bar{G}$ is the complement of G, then

$$\gamma_2(G) + \gamma_2(\bar{G}) \leq n + 2,$$

and this bound is best possible in some sense.

Keywords: Domination; 2-domination number; Nordhaus-Gaddum-type result; complementary graph

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1. Terminology and introduction

We consider finite, undirected, and simple graphs G with vertex set $V(G)$ and edge set $E(G)$. The number of vertices $|V(G)|$ of a graph G is called the *order* of G and is denoted by $n = n(G)$. The *open neighborhood* $N_G(v)$ of a vertex v consists of the vertices adjacent to v and $d_G(v) = |N_G(v)|$ is the *degree* of v. The *closed neighborhood* of a vertex v is defined by $N_G[v] = N_G(v) \cup \{v\}$. By $\delta = \delta(G)$, we denote the *minimum degree* of the graph G. A vertex of degree one is called a *leaf* and its neighbor is called a *support vertex*. For a subset $S \subseteq V(G)$, let $G[S]$ be the subgraph induced by S. We write K_n for the complete graph of order n.

Let p be a positive integer. A subset $S \subseteq V(G)$ is a *p-dominating set* of the graph G, if $|N_G(v) \cap S| \geq p$ for every $v \in (V(G) - S)$. The *p-domination number* $\gamma_p(G)$ is the minimum cardinality among the p-dominating sets of G. Note that the 1-domination number $\gamma_1(G)$ is the usual *domination number* $\gamma(G)$. In [3] and [4], Fink and Jacobson introduced the concept of p-domination. For a comprehensive treatment of domination in graphs, see the monographs by Haynes, Hedetniemi, and Slater [5], [6].

In their now classical 1956 paper [9], Nordhaus and Gaddum established the inequality $\chi(G) + \chi(\bar{G}) \leq n(G) + 1$, where χ is the chromatic number. In 1972, Jaeger and Payan [7] published the first Nordhaus-Gaddum-type result involving domination, namely $\gamma(G) + \gamma(\bar{G}) \leq n(G) + 1$ for any graph G. Improvements and generalizations of this inequality can be found in Section 9.1 of the monograph [5] by Haynes, Hedetniemi, and Slater.

In this paper we prove

$$\gamma_2(G) + \gamma_2(\bar{G}) \leq n(G) + 2$$

for each graph G, and the proof will show that this bound is best possible.

2. Preliminary results

The following known results play an important role in our investigations.

Theorem 2.1 (Ore [8] 1962) If G is a graph with $\delta(G) \geq 1$, then

$$\gamma(G) = \gamma_1(G) \leq \frac{n(G)}{2}.$$

In 1985, Cockayne, Gamble, and Shepherd [2] gave the following extension of Ore's bound.

Theorem 2.2 (Cockayne, Gamble, Shepherd [2] 1985) Let p be a positive integer. If G is a graph of minimum degree $\delta(G) \geq p$, then

$$\gamma_p(G) \leq \frac{p \cdot n(G)}{p+1}.$$

As a generalization of Theorem 2.2, Caro and Roditty [1] presented in 1990 the following result.

Theorem 2.3 (Caro, Roditty [1] 1990) Let G be a graph and let p and j be positive integers such that $\delta(G) \geq (p(j+1))/(j) - 1$. Then

$$\gamma_p(G) \leq \frac{j \cdot n(G)}{j+1}.$$

An explicit proof of Theorem 2.3 can be found in the book of Volkmann [11], pp. 233-234. If we choose $p = 2$ and $j = 1$ in Theorem 2.3, then we obtain

$$\gamma_2(G) \leq \frac{n(G)}{2} \quad \text{if } \delta(G) \geq 3. \tag{1}$$

Also the next result contains (1) as a special case for $p = 2$.

Theorem 2.4 (Stracke, Volkmann [10] 1993) Let G be a graph and let p be a positive integer. If there are m vertices $x \in V(G)$ with $d_G(x) \geq 2p-1$, then

$$\gamma_p(G) \leq \frac{2n(G) - m}{2}.$$

Theorem 2.5 (Stracke, Volkmann [10] 1993) Let G be a graph and let p be a positive integer. If there are m vertices $x \in V(G)$ with $d_G(x) \geq p$, then

$$\gamma_p(G) \leq \frac{(p+1)n(G) - m}{p+1}.$$

3. Main result

Theorem 3.1 If G is a graph of order n, then

$$\gamma_2(G) + \gamma_2(\bar{G}) \leq n + 2. \tag{2}$$

Proof. If $1 \leq n \leq 5$, then it is straightforward to verify inequality (2). In particular, we have $\gamma_2(G) + \gamma_2(\bar{G}) = n + 2$ for $2 \leq n \leq 4$. Let now $n \geq 6$.

Case 1: Assume that $\delta(G) = 0$ or $\delta(\bar{G}) = 0$, say $\delta(G) = 0$. Because of $\delta(G) = 0$, we observe that $\delta(\bar{G}) \geq 1$. If there are least two vertices of degree 0 in G, then $\gamma_2(\bar{G}) = 2$ and we obtain $\gamma_2(G) + \gamma_2(\bar{G}) \leq n+2$. Thus we assume now that there exists exactly one vertex of degree 0 in G, say $d_G(u) = 0$.

Subcase 1.1: There exists a vertex $v \in V(G)$ with $d_G(v) = 1$. This implies that $N_G[v] \cup \{u\}$ is a 2-dominating set of $\bar{G}$ and thus $\gamma_2(\bar{G}) \leq 3$. If there exists a vertex w with $d_G(w) \geq 2$, then $V(G) - \{w\}$ is a 2-dominating set of G, and it follows that $\gamma_2(G) + \gamma_2(\bar{G}) \leq n - 1 + 3 = n + 2$. In the remaining case that $G - u$ is 1-regular, we deduce that $\gamma_2(\bar{G}) = 2$, and this immediately leads to (2).

Subcase 1.2: Assume that $\delta(G-u)=2$. If v is a vertex with $d_G(v)=2$, then $N_G[v]\cup\{u\}$ is a 2-dominating set of $\bar{G}$ and thus $\gamma_2(\bar{G})\leq 4$. Furthermore, Theorem 2.2 shows with $p=2$ that $\gamma_2(G-u)\leq 2(n-1)/3$ and so we arrive at

$$\gamma_2(G)+\gamma_2(\bar{G})\leq 1+\frac{2(n-1)}{3}+4.$$

Since $n\geq 6$, this easily leads to (2).

Subcase 1.3: Assume that $\delta(G-u)\geq 3$. According to Theorem 2.4 with $p=2$, we obtain

$$\gamma_2(G)\leq 1+\frac{2(n-1)-(n-1)}{2}=\frac{n+1}{2}. \tag{3}$$

Subcase 1.3.1: Assume that $\delta(\bar{G})\geq 2$. Let D be a minimum dominating set of $\bar{G}-u$. Since $\delta(\bar{G}-u)\geq 1$, Theorem 2.1 yields $|D|\leq(n-1)/2$. Now we observe that $D\cup\{u\}$ is a 2-dominating set of $\bar{G}$, and in view of (3), we obtain

$$\gamma_2(G)+\gamma_2(\bar{G})\leq\frac{n+1}{2}+\frac{n-1}{2}+1=n+1\leq n+2.$$

Subcase 1.3.2: Assume that there exists exactly one vertex in $\bar{G}$, say x, such that $d_{\bar{G}}(x)=1$. Let D be a minimum dominating set of $\bar{G}-\{u,x\}$. Since by the assumption $\delta(\bar{G}-\{u,x\})\geq 1$, Theorem 2.1 yields $|D|\leq(n-2)/2$. Now $D\cup\{u,x\}$ is a 2-dominating set of $\bar{G}$ and hence (3) yields

$$\gamma_2(G)+\gamma_2(\bar{G})\leq\frac{n+1}{2}+\frac{n-2}{2}+2\leq n+2.$$

Subcase 1.3.3: Assume that there exist two vertices in $\bar{G}$, say x,y, such that $d_{\bar{G}}(x)=d_{\bar{G}}(y)=1$. In this case, we see that $\{u,x,y\}$ is a 2-dominating set of G, and $V(G)-\{u\}$ is a 2-dominating set of $\bar{G}$ and thus $\gamma_2(G)+\gamma_2(\bar{G})\leq 3+n-1=n+2$.

Case 2: Assume that $\delta(G)=1$ or $\delta(\bar{G})=1$, say $\delta(G)=1$. Because of Case 1, we only have to discuss the case that $\delta(\bar{G})\geq 1$. Let $u\in V(G)$ with $d_G(u)=1$, and let v its unique neighbor in G.

Subcase 2.1: Assume that $d_G(x)\geq 2$ for $x\in(V(G)-\{u\})$. In view of Theorem 2.5 with $p=2$, we obtain

$$\gamma_2(G)\leq\frac{3n-(n-1)}{3}=\frac{2n+1}{3}. \tag{4}$$

Subcase 2.1.1: Assume that $d_G(x)\geq 3$ for $x\in(V(G)-\{u\})$. According to Theorem 2.4, we conclude that

$$\gamma_2(G)\leq\frac{2n-(n-1)}{2}=\frac{n+1}{2}. \tag{5}$$

Subcase 2.1.1.1: Assume that $\delta(\bar{G}-u) \geq 1$. Let D be a minimum dominating set of $\bar{G}-u$. Then Theorem 2.1 yields $|D| \leq (n-1)/2$. Since $D \cup \{u, v\}$ is a 2-dominating set of $\bar{G}$, it follows from (5) that

$$\gamma_2(G) + \gamma_2(\bar{G}) \leq \frac{n+1}{2} + \frac{n-1}{2} + 2 = n + 2.$$

Subcase 2.1.1.2: Assume that there exists exactly one vertex y in $\bar{G}$ such that $d_{\bar{G}}(y) = 1$. If $y = v$, then $\delta(\bar{G}-u) \geq 1$ and we are done by Subcase 2.1.1.1. Let now $y \neq v$. We observe that $\delta(\bar{G} - \{u, y\}) \geq 1$. Let D be a minimum dominating set of $\bar{G} - \{u, y\}$. Then Theorem 2.1 yields $|D| \leq (n-2)/2$. Now $D \cup \{u, v, y\}$ is a 2-dominating set of $\bar{G}$ and thus

$$\gamma_2(\bar{G}) \leq 3 + \frac{n-2}{2} = \frac{n+4}{2}. \tag{6}$$

If we distinguish the two cases n even and n odd, then (5) and (6) easily lead to (2).

Subcase 2.1.1.3: Assume that there exist exactly two vertices x, y in $\bar{G}$ such that $d_{\bar{G}}(x) = d_{\bar{G}}(y) = 1$ and $x = v$. This implies $\delta(\bar{G} - \{u, y\}) \geq 1$, and we obtain the desired result analogously to Subcase 2.1.1.2.

Subcase 2.1.1.4: Assume that there exist at least two vertices $x, y \neq v$ in $\bar{G}$ such that $d_{\bar{G}}(x) = d_{\bar{G}}(y) = 1$. This condition yields $N_{\bar{G}}(x) \cap N_{\bar{G}}(y) = \{u\}$ and hence $\{u, x, y\}$ is a 2-dominating set of G. In addition, $V(G) - \{u\}$ is a 2-dominating set of $\bar{G}$ and thus $\gamma_2(G) + \gamma_2(\bar{G}) \leq 3 + n - 1 = n + 2$.

Subcase 2.1.2: Assume that there exists a vertex y with $d_G(y) = 2$. We observe that $N_G[u] \cup N_G[y]$ is a 2-dominating set of $\bar{G}$ and thus $\gamma_2(\bar{G}) \leq 5$. It follows from (4) that

$$\gamma_2(G) + \gamma_2(\bar{G}) \leq \frac{2n+1}{3} + 5,$$

and this yields (2) when $n \geq 8$.

Let now $6 \leq n \leq 7$. If $N_G(u) \cap N_G(y) \neq \emptyset$, then $\gamma_2(\bar{G}) \leq 4$, and we arrive at (2) as above. In the case that $N_G(u) \cap N_G(y) = \emptyset$, let $N_G(y) = \{y_1, y_2\}$. If vy_1 or vy_2 or y_1y_2 is an edge of $\bar{G}$, then again $\gamma_2(\bar{G}) \leq 4$ and we are done. Thus assume in the following that vy_1, vy_2, and y_1y_2 are edges of G.

Subcase 2.1.2.1: Assume that $n = 6$ and $V(G) = \{u, v, x, y, y_1, y_2\}$. If $\{y_1, y_2\} \subseteq N_G(x)$, then $\{u, y_1, y_2\}$ is a 2-dominating set of G, and we obtain $\gamma_2(G) + \gamma_2(\bar{G}) \leq 3 + 5 = n + 2$. In the remaining case we assume, without loss of generality, that the edge xy_2 belongs to $\bar{G}$. This implies that $\{u, v, x, y_1\}$ is a 2-dominating set of $\bar{G}$ and hence (4) yields the desired result.

Subcase 2.1.2.2: Assume that $n = 7$ and $V(G) = \{u, v, x_1, x_2, y, y_1, y_2\}$. If $\{y_1, y_2\} \subseteq N_G(x_i)$ for any $i = 1, 2$, then $\{u, y_1, y_2, x_{3-i}\}$ is a 2-dominating set of G, and we obtain $\gamma_2(G) + \gamma_2(\bar{G}) \leq 4 + 5 = n + 2$. If $d_G(x_i) \geq 3$ for $i = 1$ and $i = 2$, then Theorem 2.4 with $p = 2$ leads to $\gamma_2(G) \leq 4$ and we are done.

Therefore assume, without loss of generality, that $d_G(x_1) = 2$. If $N_G(x_1) = \{v, y_i\}$ for any $i = 1, 2$, say $i = 1$, then $\{u, v, x_1, y_1\}$ is a 2-dominating set of $\bar{G}$ and so (4) yields the desired result. It remains the case that $x_2 \in N_G(x_1)$. If $N_G(x_1) = \{v, x_2\}$, then $\{u, v, x_1, y\}$ is a 2-dominating set of $\bar{G}$ and (4) yields the desired result. Finally, let $N_G(x_1) = \{x_2, y_i\}$ for any $i = 1, 2$, then $\{u, x_1, y, y_i\}$ is a 2-dominating set of $\bar{G}$ and we are done.

Subcase 2.2: There is a second vertex $w \neq u$ in G such that $d_G(w) = 1$. In this case $N_G[u] \cup N_G[w]$ is a 2-dominating set of $\bar{G}$ and thus $\gamma_2(\bar{G}) \leq 4$.

Subcase 2.2.1: There are at least 4 vertices of degree at least two in G. In view of Theorem 2.5 with $p = 2$, we deduce that $\gamma_2(G) \leq (3n-4)/3$ and hence $\gamma_2(G) + \gamma_2(\bar{G}) \leq n - 2 + 4 = n + 2$.

Subcase 2.2.2: Assume that G has two non-adjacent vertices, say x and y, of degree at least two. Then $V(G) - \{x, y\}$ is a 2-dominating set of G, and we obtain $\gamma_2(G) + \gamma_2(\bar{G}) \leq n - 2 + 4 = n + 2$.

Subcase 2.2.3: Assume that G has exactly three pairwise adjacent vertices, say x, y, z, of degree at least two. If a complete graph K_2 is a component of G, then $\gamma_2(\bar{G}) \leq 2$ and we are done. Otherwise, G is connected. If $n \geq 7$, then some vertex of $\{x, y, z\}$ has degree at least four, say $d_G(x) \geq 4$. If x_1, x_2 are two leaves attached at x, then $\{x, x_1, x_2\}$ and $V(G) - \{x\}$ is a 2-dominating set of $\bar{G}$ and G, respectively. This implies immediately (2). In the remaining case that $n = 6$, it is a simple matter to obtain the desired result.

Subcase 2.2.4: Assume that G has exactly two adjacent vertices, say x, y, of degree at least two. If K_2 is a component of G, then $\gamma_2(\bar{G}) \leq 2$ and we are done. Otherwise, G is connected such that, without loss of generality, $d_G(x) \geq 3$. If x_1, x_2 are two leaves attached at x, then $\{x, x_1, x_2\}$ and $V(G) - \{x\}$ is a 2-dominating set of $\bar{G}$ and G, respectively, and this implies (2).

Subcase 2.2.5: Assume that G has exactly one vertex x of degree at least two. If K_2 is a component of G, then $\gamma_2(\bar{G}) \leq 2$ and we are done. If not, then G is a star $K_{1,n-1}$, a contradiction to $\delta(\bar{G}) \geq 1$.

Subcase 2.2.6: Assume that G is 1-regular. It follows that $\gamma_2(\bar{G}) = 2$ and $\gamma(G) = n$ and we are done.

Case 3: Assume that $\delta(G) = 2$ or $\delta(\bar{G}) = 2$, say $\delta(G) = 2$. Because of the Cases 1 and 2, it remains to discuss the case that $\delta(\bar{G}) \geq 2$. According to Theorem 2.2, we have

$$\gamma_2(G) \leq \frac{2n}{3} \tag{7}$$

as well as $\gamma_2(\bar{G}) \leq 2n/3$. If there is at most one vertex in G and at most one vertex in $\bar{G}$ of degree two, then Theorem 2.4 leads to

$$\gamma_2(G) + \gamma_2(\bar{G}) \leq \frac{n+1}{2} + \frac{n+1}{2} = n + 1 \leq n + 2.$$

By reason of symmetry it remains the case that there are at least two vertices in G, say u and v, with $d_G(u) = d_G(v) = 2$. This implies that $N_G[u] \cup N_G[v]$ is a 2-dominating set of $\bar{G}$ and thus $\gamma_2(\bar{G}) \leq 6$. Hence it follows from (7) that

$$\gamma_2(G) + \gamma_2(\bar{G}) \leq \frac{2n}{3} + 6,$$

and this yields (2) when $n \geq 10$. In the following let $N_G(u) = \{u_1, u_2\}$ and $N_G(v) = \{v_1, v_2\}$.

Subcase 3.1: Assume that $n = 6$. If $N_G(u) = N_G(v)$, then $\gamma_2(\bar{G}) \leq 4$ and (7) leads to (2). If $N_G(u) \cap N_G(v) = \emptyset$, then $\{u_1, u_2, v_1, v_2\}$ is a 2-dominating set of G as well as of $\bar{G}$ and so $\gamma_2(G) + \gamma_2(\bar{G}) \leq 4 + 4 = n + 2$. It remains the case that $N_G(u)$ and $N_G(v)$ have one vertex in common, say $u_2 = v_2$. It follows that $\gamma_2(\bar{G}) \leq 5$. Let $V(G) = \{u, u_1, u_2, v, v_1, x\}$. If $\{u_1, v_1\} \subseteq N_G(x)$, then $\{u, v, x\}$ is a 2-dominating set of G, and we obtain $\gamma_2(G) + \gamma_2(\bar{G}) \leq 3 + 5 = n + 2$. If $\{u_1, u_2\} \subseteq N_G(x)$ or $\{v_1, u_2\} \subseteq N_G(x)$, say $\{u_1, u_2\} \subseteq N_G(x)$, then $\{u_1, u_2, v_1\}$ is a 2-dominating set of G and we are done.

Subcase 3.2: Assume that $7 \leq n \leq 9$. If $N_G(u) \cap N_G(v) \neq \emptyset$, then $\gamma_2(\bar{G}) \leq 5$, and we arrive at (2) as above. Let now $N_G(u) \cap N_G(v) = \emptyset$. If u_1u_2, v_1v_2, u_1v_1, u_1v_2, u_2v_1, or u_2v_2 is an edge of $\bar{G}$, then again $\gamma_2(\bar{G}) \leq 5$ and we are done. Thus assume in the following that $G[\{u_1, u_2, v_1, v_2\}]$ is a complete subgraph of G.

Subcase 3.2.1: Assume that $n = 7$ and $V(G) = \{u, u_1, u_2, v, v_1, v_2, x\}$. If $d_G(x) \leq 3$, then assume, without loss of generality, that $xu_1 \in E(\bar{G})$. We deduce that $\{u, u_2, v, v_1, v_2\}$ is a 2-dominating set of $\bar{G}$ and we are done. In the remaining case that $d_G(x) = 4$, we observe that $\{u, v, x\}$ is a 2-dominating set of G and hence $\gamma_2(G) + \gamma_2(\bar{G}) \leq 3 + 6 = n + 2$.

Subcase 3.2.2: Let $n = 8$ and $V(G) = \{u, u_1, u_2, v, v_1, v_2, x_1, x_2\}$. If $\{u_1, u_2, v_1, v_2\} \subseteq N_G(x_i)$ for any $i = 1, 2$, then $\{u, v, x_1, x_2\}$ is a 2-dominating set of G, and we obtain $\gamma_2(G) + \gamma_2(\bar{G}) \leq 4 + 6 = n + 2$. Otherwise, assume, without loss of generality, that $x_1u_1 \in E(\bar{G})$. If $x_2u_1 \in E(\bar{G})$, then $\{u_2, v_1, v_2, x_1, x_2\}$ is a 2-dominating set of $\bar{G}$ and (7) yields the desired result. If $x_2u_2 \in E(\bar{G})$, then $\{v, v_1, v_2, x_1, x_2\}$ is a 2-dominating set of $\bar{G}$ and (7) yields the desired result. If $x_2v_1 \in E(\bar{G})$ or $x_2v_2 \in E(\bar{G})$, say $x_2v_1 \in E(\bar{G})$, then $\{u, u_2, v_1, v_2, x_1\}$ is a 2-dominating set of $\bar{G}$ and we are done.

Subcase 3.2.3: Let $n = 9$ and $V(G) = \{u, u_1, u_2, v, v_1, v_2, x_1, x_2, x_3\}$. If $d_G(x_i) \geq 3$ for each $i \in \{1, 2, 3\}$, then it follows from Theorem 2.4 that

$\gamma_2(G) \leq 5$ and thus $\gamma_2(G) + \gamma_2(\bar{G}) \leq 5 + 6 = n + 2$. Hence assume in the following that, without loss of generality, $d_G(x_1) = 2$. If $N_G(u) \cap N_G(x_1) \neq \emptyset$ or $N_G(v) \cap N_G(x_1) \neq \emptyset$, then $\gamma_2(\bar{G}) \leq 5$ and (7) leads to the desired result. It remains the case that $N_G(x_1) = \{x_2, x_3\}$. However, now $\{u, v, x_1, x_2, x_3\}$ is a 2-dominating set of $\bar{G}$ and we are done.

Case 4: Assume that $\delta(G) \geq 3$ and $\delta(\bar{G}) \geq 3$. Applying (1), we arrive at $\gamma_2(G) + \gamma_2(\bar{G}) \leq n/2 + n/2 = n \leq n + 2$, and the proof of Theorem 3.1 is complete. □

Remark 3.2 Jaeger and Payan [7] have proved that $\gamma_1(G) + \gamma_1(\bar{G}) \leq n(G) + 1$, and Theorem 3.1 says that $\gamma_2(G) + \gamma_2(\bar{G}) \leq n(G) + 2$ for any graph G. So one could mean that $\gamma_p(G) + \gamma_p(\bar{G}) \leq n(G) + p$ for $p \geq 3$.

However, the following examples will show that this is not valid in general.

Let C_5 be a cycle of length 5. Then $\bar{C}_5$ is also a cycle of length 5, but we obtain $\gamma_3(C_5) + \gamma_3(\bar{C}_5) = 10 > 5 + 3 = 8 = n + p$.

More general, let t be a positive integer, and let G be a $2t$-regular graph of order $n = 4t + 1$. Then $\bar{G}$ is also a $2t$-regular graph, and we see that

$$\gamma_{2t+1}(G) + \gamma_{2t+1}(\bar{G}) = 2n = n + 4t + 1 > n + 2t + 1.$$

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