$\mathbf{Z}$-cyclic whist tournaments for q^2 players

John W. Jones
Department of Mathematics and Statistics
Arizona State University, Tempe, AZ 85287-1804
jj@asu.edu

Philip A. Leonard
Department of Mathematics and Statistics
Arizona State University, Tempe, AZ 85287-1804
philip.leonard@asu.edu

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Abstract

Expanding upon a comment by P. A. Leonard [9], we exhibit $\mathbf{Z}$-cyclic patterned-starter based whist tournaments for q^2 players, where $q = 4k+3$ is prime; the cases $3 < q < 200$ are included herein, with data for $200 < q < 5{,}000$ available electronically.

1 Introduction

A *whist tournament*, Wh($4n+1$), for $4n+1$ players is a schedule of games, each involving two partners playing against two others, such that

1. the games are arranged in $4n+1$ rounds, each with n games,
2. each player plays in one game in all but one of the rounds,
3. each player partners every other player exactly once,
4. each player opposes every other player exactly twice.

The games are represented by quadruples (A, B, C, D), where partners A, C oppose partners B, D. A Wh($4n+1$) is said to be $\mathbf{Z}$-*cyclic* if A, B, C, D are in $\mathbf{Z}_{4n+1}$ and the games of round $j+1$ are obtained by adding 1 (mod $4n+1$) to all elements of the games in round j. We adopt the convention that the initial round, with $j = 0$, of a **Z**-cyclic Wh$(4n+1)$ is the one in which player 0 does not play.

It is convenient to adopt the description of **Z**-cyclic Wh$(4n+1)$ given in [1]. The pairs $\{A_1, B_1\}, \ldots, \{A_{2n}, B_{2n}\}$ of nonzero elements of $\mathbf{Z}_{4n+1}$ form a *starter* in $\mathbf{Z}_{4n+1}$ if

$$(\alpha) \bigcup_{i=1}^{2n} \{A_i, B_i\} = \mathbf{Z}_{4n+1} - \{0\}, \qquad (\beta) \bigcup_{i=1}^{2n} \{\pm(A_i - B_i)\} = \mathbf{Z}_{4n+1} - \{0\}.$$

Similarly, we shall say that the pairs $\{A_1, B_1\}, \ldots, \{A_{4n}, B_{4n}\}$ of $\mathbf{Z}_{4n+1}$ form a 2-*fold starter* if

(γ) the elements A_i, B_i are the elements of $\mathbf{Z}_{4n+1} - \{0\}$, each occurring twice,

(δ) the elements $\pm(A_i - B_i)$ are the elements of $\mathbf{Z}_{4n+1} - \{0\}$, each twice.

Taking into account our convention that the initial round of a **Z**-cyclic Wh$(4n+1)$ is the round in which 0 does not play, then the conditions for

$$(A_1, B_1, C_1, D_1), \ldots, (A_n, B_n, C_n, D_n)$$

to be the initial round for a **Z**-cyclic Wh$(4n+1)$ are precisely [1]

(ϵ) the pairs $\{A_i, C_i\}$, $\{B_i, D_i\}$, $1 \le i \le n$, form a starter.

(ζ) the pairs $\{A_i, B_i\}$, $\{B_i, C_i\}$, $\{C_i, D_i\}$, $\{D_i, A_i\}$, $1 \le i \le n$, form a 2-fold starter.

In this paper we revisit the special case when $4n + 1 = q^2$, where $q \equiv 3 \pmod 4$ is prime. The first **Z**-*cyclic* Wh$(4n+1)$ of this type was found by Finizio [5] for $q = 7$. In [9] examples were exhibited for $q = 11, 19, 23, 31$, and claimed for all $q < 500$. In several places in the literature, existence of **Z**-cyclic Wh(q^2) for $3 < q < 500$ has been noted because of the importance of these tournaments in connection with product theorems and other constructions for various types of whist tournaments. In fact, as noted in [9], each new value of q for which such a tournament exists gives rise to infinitely many new whist tournaments. Further, the tournaments under consideration are ZCPS (**Z**-cyclic patterned-starter) tournaments, having all their tables of the form $(A, B, -A, -B)$. Such tournaments are also directed whist tournaments [7] and ordered whist tournaments [8]. Thus the tournaments of the present article play a role in the study of whist tournaments of these special types.

The goal of this paper is to present examples for the cases with $43 < q < 200$. We shall briefly revisit the construction used in [9], the computations involved in extending the range of values, and refer the reader to `http://hobbes.la.asu.edu/whist.html` for data covering all $q < 5{,}000$.

2 Construction and Search

The method for finding the tournaments we shall present is found in [9]. It relies on arithmetical properties of $\mathbf{Z}/q^2\mathbf{Z}$, in particular, on utilizing the q-th and $2q$-th powers among the units of this ring. The tournaments have multiplicative symmetry based on these elements. Readers should consult [9] for details to amplify what is given here.

Let w denote a primitive root modulo q^2. The coset $C(j)$ is defined for $0 \le j \le q-1$ by $C(j) = \{w^{qk+j} | 0 \le k \le q-2\}$. The appearance of a quadruple $[a, b; u, v]$ indicates the presence of a table $(A, B, -A, -B)$ with $A \in C(a), B \in C(b), A+B \in C(u), A-B \in C(v)$. Computer searching was done for each prime with reference to an array with entries of the form $[a, b; u, v]$. Each lower case letter (or the special symbol ∞) refers to a coset of the q-th powers in the group of units of $\mathbf{Z}/q^2\mathbf{Z}$ (or to a multiple of q). For each entry, the corresponding tournament table can be retrieved. A successful outcome, where a, b and u, v represent all possible cosets (and the multiples of q), yields $(q+1)/2$ tables for the initial round of a whist tournament. The remaining tables are generated from these by successive multiplication by a primitive $2q$-th power, say w^{2q}.

For illustration, consider $q = 11$. A valid selection is shown by the underlined entries in the following array:

$[4,5;4,4]$	$[3,6;10,3]$	$[8,1;5,2]$	$[2,7;0,\infty]$	$[9,10;9,9]$	$[0,\infty;5,6]$
$[4,5;4,1]$	$[3,6;0,\infty]$	$[8,1;10,3]$	$[2,7;8,6]$	$[9,10;9,6]$	$[0,\infty;4,7]$
$[4,5;0,\infty]$	$[3,6;3,8]$	$[8,1;4,2]$	$[2,7;6,3]$	$[9,10;5,\infty]$	$[0,\infty;1,10]$
$[4,5;9,10]$	$[3,6;9,7]$	$[8,1;\infty,0]$	$[2,7;10,1]$	$[9,10;3,4]$	$[0,\infty;3,8]$
$[4,5;1,0]$	$[3,6;7,8]$	$[8,1;7,0]$	$[2,7;5,5]$	$[9,10;6,5]$	$[0,\infty;3,8]$

The underlined entries from this array yield, in order, the pairs $(A, B) =$ $(16, 17)$, $(8, 92)$, $(14, 119)$, $(4, 63)$, $(28, 47)$, $(3, 11)$ for corresponding tables $(A, B, -A, -B)$. The remaining tables for the first round are found by multiplying these six by the powers of $2^{22} = 81$.

In general, tournaments are found by searching arrays such as that for $q = 11$ and retrieving the pairs (A, B) from the underlying data. As reported in [9], the number of successes is 5 for $q = 11$, 183 for $q = 19$, 2,978 for $q = 23$, and 2,948,855 for $q = 31$. These values were determined by backtracking. The abundance of successes for these small values suggested using a hill-climbing algorithm to seek tournaments for subsequent cases. This was carried our for $q < 500$ as noted in [9], [2], but with only the quadruples $[a, b; u, v]$, and not the pairs (A, B) in play. Computations done for the present article extended the search and retrieved the pairs represented by the successful searches.

3 The Data

Data are given here for primes $q = 4k+3$ satisfying $3 < q < 200$. The data enable one to quickly construct a single **Z**-cyclic whist tournament on q^2 players for each prime considered. For each prime $q \equiv 3 \pmod 4$, we give g, an element of order $(q-1)/2$ in $(\mathbf{Z}/q^2\mathbf{Z})^*$, and a total of $m = (q+1)/2$ pairs (A_i, B_i).

Given this information, one constructs the initial round tables of the form $[g^j A_k, g^j B_k; -g^j A_k, -g^j B_k]$ where all computations are done in $\mathbf{Z}/q^2\mathbf{Z}$. Here, $1 \le k \le (q+1)/2$ and $0 \le j < (q-1)/2$. Finally, the tables for the remaining rounds of the tournament come about from successively adding 1 to the entries of the tables from the initial round, computations again done in $\mathbf{Z}/q^2\mathbf{Z}$. In a data file, available at `http://hobbes.la.asu.edu/whist.html`, similar information is provided for all $q < 5{,}000$.

q	g	Pairs
11	81	(9, 11) (16, 17) (28, 20) (8, 92) (14,103) (4, 83)
19	99	(68, 19) (256,317) (29,330) (128,296) (195,237) (64,185) (278,113)
		(32,101) (139,191) (16,202)
23	255	(334, 23) (75,405) (284,325) (15,400) (480,112) (3,413) (96,285)
		(318,423) (125,421) (362,522) (25, 29) (182,468)
31	846	(374, 31) (323,762) (576,653) (428, 75) (192, 57) (463, 67) (64, 39)
		(795,393) (662,767) (265,218) (541,958) (729,353) (821,116) (243,545)
		(594, 78) (81,615)

q	g	Pairs
43	1557	(367, 43) (1129,1628) (1014,1730) (1609,1755) (338, 945) (1769,1567)
		(729, 818) (1206, 552) (243,1765) (402, 589) (81, 139) (134,1831)
		(27, 667) (661,1195) (9, 762) (1453,1563) (3,1422) (1717,1022)
		(1193, 568) (1805,1124) (87, 866) (1218, 781)
47	2093	(1, 47) (2165, 515) (1219, 468) (433, 271) (2011, 873) (1412, 493)
		(844,1419) (1166,1230) (1936,1187) (675,1477) (829,2184) (135, 695)
		(1933,1531) (27, 828) (1712, 710) (889,1931) (1226, 722) (1945, 90)
		(687,1356) (389,2122) (1021,1231) (1845,2173) (646,1660) (369,1648)
59	299	(970, 59) (1622,2001) (1977,1740) (811,1620) (2729, 671) (2146,1749)
		(3105,2285) (1073, 171) (3293, 789) (2277, 132) (3387, 90) (2879,3325)
		(3434,2818) (3180, 699) (1717,2831) (1590,2736) (2599,2181) (795,2049)
		(3040,1175) (2138,1128) (1520,3316) (1069, 962) (760, 195) (2275, 225)
		(380,3311) (2878,2189) (190,3049) (1439, 897) (95, 344) (2460,2890)
67	875	(4210, 67) (4321,3474) (2580,2528) (4405, 840) (1290,2333) (4447, 586)
		(645,2387) (4468,3360) (2567,1287) (2234,1141) (3528, 94) (1117,4462)
		(1764,2307) (2803,2375) (882, 966) (3646, 821) (441, 250) (1823,4061)
		(2465, 870) (3156,4276) (3477,1000) (1578,3932) (3983,3210) (789,2059)
		(4236,1609) (2639,4320) (2118,2578) (3564,1565) (1059,1901) (1782,3882)
		(2774, 883) (891,2031) (1387,3299) (2690,2290)
71	1895	(577, 71) (393, 133) (175,4112) (4377,4036) (25,4138) (4226,4306)
		(2164,3155) (2044,4997) (4630,3929) (292,1674) (3542,2682) (1482,3427)
		(506, 150) (1652,3617) (3673, 832) (236,3022) (1965, 3) (1474, 990)

q	g	Pairs					
		(1721, 4508)	(2371, 2844)	(966, 2728)	(1779, 2399)	(138, 2563)	(4575, 1794)
		(1460, 2818)	(2814, 2476)	(2369, 961)	(402, 906)	(3219, 3039)	(2938, 346)
		(1180, 1109)	(1860, 3653)	(2329, 356)	(1706, 366)	(1773, 749)	(1684, 3213)
79	1905	(6094, 79)	(243, 677)	(5682, 1679)	(81, 3152)	(1894, 2508)	(27, 5948)
		(4792, 88)	(9, 960)	(5758, 1646)	(3, 5240)	(6080, 3751)	(2399, 3309)
		(4107, 2756)	(4267, 1263)	(1369, 887)	(5583, 4350)	(4617, 86)	(1861, 2347)
		(1539, 258)	(4781, 5177)	(513, 3767)	(3674, 2400)	(171, 5100)	(3305, 2371)
		(57, 5763)	(3182, 872)	(19, 1187)	(3141, 4633)	(4167, 777)	(1047, 2517)
		(1389, 2231)	(349, 4385)	(463, 5596)	(4277, 5178)	(4315, 1991)	(3506, 684)
		(5599, 4492)	(3249, 2124)	(6027, 3854)	(1083, 5689)		
83	4320	(1389, 83)	(2448, 5699)	(5437, 6478)	(1224, 6663)	(6163, 4659)	(612, 6613)
		(6526, 3643)	(306, 924)	(3263, 4809)	(153, 4739)	(5076, 6862)	(3521, 2602)
		(2538, 2683)	(5205, 2789)	(1269, 4027)	(6047, 252)	(4079, 6159)	(6468, 1253)
		(5484, 6227)	(3234, 4408)	(2742, 1741)	(1617, 6692)	(1371, 3923)	(4253, 6392)
		(4130, 1032)	(5571, 5302)	(2065, 3160)	(6230, 4593)	(4477, 3929)	(3115, 2314)
		(5683, 3880)	(5002, 1945)	(6286, 6012)	(2501, 2529)	(3143, 1599)	(4695, 3146)
		(5016, 2769)	(5792, 428)	(2508, 4185)	(2896, 36)	(1254, 1950)	(1448, 5206)

q	g	Pairs				
103	2703	(4355, 103)	(711, 4113)	(6594, 1317)	(2264, 510)	(9806, 4296)
		(8940, 10264)	(4083, 1474)	.(1788, 6777)	(7182, 7916)	(6723, 8313)
		(5680, 5721)	(7710, 2199)	(1136, 629)	(1143, 8881)	(2349, 1586)
		(4552, 66)	(8957, 5116)	(5154, 4105)	(6035, 8495)	(9518, 544)
		(1207, 9936)	(8269, 614)	(4485, 880)	(10141, 7341)	(897, 188)
		(4150, 2671)	(4423, 3717)	(830, 8711)	(7250, 3500)	(166, 3398)
		(1450, 1031)	(2155, 4996)	(290, 7695)	(431, 202)	(58, 4557)
		(2208, 1760)	(6377, 2895)	(6807, 1276)	(5519, 6727)	(5605, 4179)
		(9591, 816)	(1121, 9749)	(4040, 7980)	(2346, 5523)	(808, 3984)
		(2591, 7996)	(6527, 3639)	(2640, 5667)	(5549, 9435)	(528, 7117)
		(9597, 7080)	(6471, 7186)			
107	11132	(7952, 107)	(2514, 7677)	(696, 4986)	(1257, 5746)	(348, 7790)
		(6353, 8656)	(174, 2558)	(8901, 1004)	(87, 1212)	(10175, 7364)
		(5768, 5075)	(10812, 4470)	(2884, 658)	(5406, 3873)	(1442, 7977)
		(2703, 2440)	(721, 8849)	(7076, 8685)	(6085, 5190)	(3538, 918)
		(8767, 3593)	(1769, 7489)	(10108, 7035)	(6609, 5925)	(5054, 8174)
		(9029, 9832)	(2527, 3306)	(10239, 2389)	(6988, 2026)	(10844, 5520)
		(3494, 11303)	(5422, 6751)	(1747, 440)	(2711, 9745)	(6598, 880)
		(7080, 4363)	(3299, 5384)	(3540, 9538)	(7374, 11373)	(1770, 1634)
		(3687, 2807)	(885, 6807)	(7568, 2419)	(6167, 1321)	(3784, 9669)
		(8808, 10705)	(1892, 3902)	(4404, 432)	(946, 9373)	(2202, 8123)
		(473, 11446)	(1101, 266)	(5961, 4849)	(6275, 3070)	
127	8137	(6485, 127)	(8099, 13037)	(5617, 5388)	(8076, 13096)	(12625, 12817)
		(2692, 7963)	(14961, 3983)	(11650, 12863)	(4987, 11949)	(14636, 11102)
		(12415, 3589)	(10255, 10179)	(14891, 697)	(14171, 1075)	(10340, 8505)
		(10100, 4140)	(8823, 11999)	(8743, 9517)	(2941, 11160)	(13667, 164)
		(11733, 11350)	(9932, 12542)	(3911, 627)	(8687, 4894)	(6680, 10998)
		(8272, 2035)	(7603, 4905)	(13510, 10742)	(13287, 8510)	(15256, 2490)
		(4429, 12710)	(15838, 203)	(12229, 9399)	(16032, 9709)	(14829, 13043)
		(5344, 10446)	(4943, 2717)	(12534, 10401)	(7024, 2510)	(4178, 14415)
		(13094, 13452)	(6769, 12703)	(9741, 16009)	(13009, 6515)	(3247, 8609)
		(15089, 13796)	(11835, 12705)	(10406, 336)	(3945, 13509)	(8845, 1748)
		(1315, 3145)	(13701, 5855)	(11191, 8678)	(4567, 6044)	(14483, 4586)

q	g	Pairs				
		(12275, 3503)	(10204, 2435)	(9468, 13316)	(14154, 8110)	(3156, 4471)
		(4718, 1433)	(1052, 1728)	(6949, 10674)	(5727, 12519)	
131	659	(13286, 131)	(3733, 702)	(1114, 9097)	(10447, 764)	(557, 12288)
		(13804, 14371)	(8859, 11550)	(6902, 6679)	(13010, 5383)	(3451, 13249)
		(6505, 13957)	(10306, 305)	(11833, 4968)	(5153, 8455)	(14497, 7593)
		(11157, 6201)	(15829, 12290)	(14159, 2440)	(16495, 2313)	(15660, 7777)
		(16828, 6896)	(7830, 3647)	(8414, 5730)	(3915, 1804)	(4207, 12236)
		(10538, 1459)	(10684, 9562)	(5269, 8970)	(5342, 3548)	(11215, 12084)
		(2671, 16865)	(14188, 8781)	(9916, 14435)	(7094, 15364)	(4958, 7477)
		(3547, 236)	(2479, 16973)	(10354, 782)	(9820, 6718)	(5177, 944)
		(4910, 3017)	(11169, 4788)	(2455, 6525)	(14165, 15940)	(9808, 13595)
		(15663, 9865)	(4904, 1447)	(16412, 9957)	(2452, 12657)	(8206, 5000)
		(1226, 3425)	(4103, 11181)	(613, 11895)	(10632, 3739)	(8887, 3195)
		(5316, 13136)	(13024, 7675)	(2658, 1198)	(6512, 3615)	(1329, 2816)
		(3256, 6634)	(9245, 12343)	(1628, 7845)	(13203, 4297)	(814, 15690)
		(15182, 2388)				
139	14043	(8878, 139)	(18278, 4001)	(11746, 6057)	(9139, 17760)	(5873, 18583)
		(14230, 7322)	(12597, 14010)	(7115, 12691)	(15959, 1885)	(13218, 6833)
		(17640, 12520)	(6609, 9867)	(8820, 1178)	(12965, 17061)	(4410, 18314)
		(16143, 6726)	(2205, 14020)	(17732, 15277)	(10763, 10025)	(8866, 16330)
		(15042, 4714)	(4433, 7991)	(7521, 2326)	(11877, 9793)	(13421, 19188)
		(15599, 8333)	(16371, 7470)	(17460, 1119)	(17846, 10291)	(8730, 6550)
		(8923, 11356)	(4365, 18246)	(14122, 13941)	(11843, 18588)	(7061, 12166)
		(15582, 5969)	(13191, 12840)	(7791, 10668)	(16256, 18317)	(13556, 9503)
		(8128, 12567)	(6778, 74)	(4064, 11749)	(3389, 11017)	(2032, 9036)
		(11355, 11078)	(1016, 3761)	(15338, 17125)	(508, 17777)	(7669, 10513)
		(254, 17323)	(13495, 4047)	(127, 14823)	(16408, 6059)	(9724, 10759)
		(8204, 6820)	(4862, 14298)	(4102, 7730)	(2431, 7159)	(2051, 15334)
		(10876, 8709)	(10686, 14887)	(5438, 10900)	(5343, 8281)	(2719, 15831)
		(12332, 5235)	(11020, 12038)	(6166, 2073)	(5510, 9245)	(3083, 14304)
151	15589	(2866, 151)	(15143, 14066)	(2222, 18386)	(6324, 5042)	(15571, 7437)
		(1054, 22784)	(21596, 596)	(7776, 7721)	(18800, 13987)	(1296, 10012)
		(18334, 1684)	(216, 4278)	(10656, 7763)	(36, 4665)	(1776, 16365)
		(6, 20835)	(296, 357)	(4752, 15917)	(15250, 15839)	(15101, 5990)
		(10142, 22724)	(6317, 4762)	(16891, 6475)	(4853, 17958)	(21816, 6652)
		(4609, 11344)	(3636, 16813)	(19769, 10926)	(606, 20775)	(7095, 21067)
		(101, 66)	(12583, 3081)	(3817, 319)	(21098, 21673)	(19637, 18216)
		(18717, 7959)	(7073, 7962)	(14520, 2351)	(4979, 18224)	(2420, 21728)
		(4630, 8222)	(15604, 17403)	(8372, 20925)	(10201, 2518)	(16596, 13457)
		(20701, 9846)	(2766, 22346)	(22451, 6626)	(461, 8321)	(7542, 17876)
		(3877, 11282)	(1257, 19946)	(19647, 21679)	(11610, 1286)	(14675, 20969)
		(1935, 6001)	(6246, 18355)	(11723, 20023)	(1041, 17940)	(5754, 9018)
		(11574, 13385)	(959, 9893)	(1929, 1080)	(3960, 3808)	(11722, 20624)
		(660, 12930)	(9554, 8519)	(110, 5613)	(16793, 15313)	(15219, 1692)
		(6599, 2875)	(13937, 17096)	(4900, 19019)	(6123, 1495)	(8417, 4965)
		(12421, 7199)				
163	7665	(1, 163)	(17196, 6196)	(5461, 19136)	(8598, 4783)	(16015, 26439)
		(4299, 24168)	(21292, 13914)	(15434, 4731)	(10646, 15183)	(7717, 24938)
		(5323, 13646)	(17143, 11612)	(15946, 15368)	(21856, 13867)	(7973, 21972)
		(10928, 19879)	(17271, 3261)	(5464, 1053)	(21920, 20484)	(2732, 11500)
		(10960, 13481)	(1366, 10066)	(5480, 9321)	(683, 8424)	(2740, 20759)
		(13626, 13398)	(1370, 24367)	(6813, 5092)	(685, 21167)	(16691, 2384)
		(13627, 22724)	(21630, 17378)	(20098, 3637)	(10815, 20728)	(10049, 25152)

q	g	Pairs
		(18692, 13335) (18309, 26556) (9346, 20117) (22439, 22381) (4673, 10267)
		(24504, 24038) (15621, 3762) (12252, 12273) (21095, 4974) (6126, 5232)
		(23832, 22811) (3063, 18019) (11916, 3507) (14816, 21273) (5958, 21731)
		(7408, 14130) (2979, 23192) (3704, 22148) (14774, 14818) (1852, 21397)
		(7387, 19629) (926, 7044) (16978, 14654) (463, 23642) (8489, 6611)
		(13516, 26236) (17529, 7114) (6758, 4433) (22049, 11324) (3379, 16107)
		(24309, 22804) (14974, 16483) (25439, 17255) (7487, 13196) (26004, 4839)
		(17028, 3141) (13002, 21274) (8514, 25290) (6501, 1117) (4257, 24851)
		(16535, 18459) (15413, 25770) (21552, 13045) (20991, 8524) (10776, 6671)
		(23780, 16661) (5388, 5451)
167	7039	(15877, 167) (16526, 12974) (16874, 4344) (8883, 10681) (25686, 13932)
		(18510, 14700) (10715, 25182) (3702, 2755) (2143, 24900) (11896, 25294)
		(17162, 8324) (7957, 21006) (14588, 1072) (12747, 26597) (19651, 19134)
		(13705, 3076) (9508, 24775) (2741, 13205) (18635, 16343) (6126, 5251)
		(3727, 16030) (6803, 217) (11901, 3320) (18094, 14448) (7958, 27842)
		(25930, 12376) (18325, 1364) (5186, 5629) (3665, 7473) (6615, 26912)
		(733, 17219) (1323, 25759) (16880, 18400) (16998, 18418) (3376, 4661)
		(20133, 26833) (6253, 26186) (20760, 2446) (17984, 7709) (4152, 23573)
		(25908, 7493) (11986, 20896) (21915, 19359) (7975, 25526) (4383, 20253)
		(1595, 22744) (17610, 4924) (319, 1182) (3522, 12994) (22375, 18889)
		(11860, 21424) (4475, 17032) (2372, 21874) (895, 17927) (11630, 25491)
		(179, 9236) (2326, 27650) (22347, 12396) (6043, 7797) (15625, 27841)
		(17942, 16444) (3125, 21253) (14744, 8291) (625, 15605) (25260, 27027)
		(125, 20078) (5052, 16189) (25, 781) (12166, 26008) (5, 8825)
		(8011, 25066) (14273, 6838) (7180, 22209) (13528, 3697) (1436, 2633)
		(19439, 15540) (5865, 3186) (26199, 3225) (1173, 17818) (27551, 27237)
		(16968, 23317) (11088, 15666) (20127, 7600) (18951, 7623)
179	7522	(19650, 179) (29803, 31186) (20496, 31373) (30922, 8745) (10248, 18807)
		(15461, 14883) (5124, 28010) (23751, 26848) (2562, 31987) (27896, 30181)
		(1281, 22292) (13948, 25844) (16661, 26779) (6974, 10854) (24351, 14639)
		(3487, 12036) (28196, 20) (17764, 5660) (14098, 26522) (8882, 22463)
		(7049, 9302) (4441, 7170) (19545, 23475) (18241, 9812) (25793, 409)
		(25141, 2501) (28917, 15928) (28591, 16831) (30479, 13510) (30316, 6057)
		(31260, 23630) (15158, 28945) (15630, 9792) (7579, 525) (7815, 9262)
		(19810, 4647) (19928, 27612) (9905, 28233) (9964, 20311) (20973, 10806)
		(4982, 15681) (26507, 28234) (2491, 3700) (29274, 10565) (17266, 24180)
		(14637, 178) (8633, 16319) (23339, 30504) (20337, 4894) (27690, 25015)
		(26189, 7847) (13845, 6340) (29115, 19576) (22943, 19745) (30578, 26167)
		(27492, 6407) (15289, 30518) (13746, 7580) (23665, 6678) (6873, 13079)
		(27853, 22526) (19457, 341) (29947, 15243) (25749, 21268) (30994, 4278)
		(28895, 14768) (15497, 22140) (30468, 2478) (23769, 10298) (15234, 26404)
		(27905, 20596) (7617, 16563) (29973, 21619) (19829, 5979) (31007, 14470)
		(25935, 1474) (31524, 16714) (28988, 19565) (15762, 19700) (14494, 77)
		(7881, 23080) (7247, 12849) (19961, 19981) (19644, 22018) (26001, 25969)
		(9822, 494) (29021, 8803) (4911, 18289) (30531, 25106) (18476, 27133)
191	27674	(19102, 191) (3952, 7144) (6859, 9100) (208, 13643) (361, 17134)
		(1931, 36085) (19, 2670) (13542, 1011) (20878, 17198) (10313, 14934)
		(15854, 23690) (8223, 25112) (21955, 35287) (8113, 16289) (18436, 19043)
		(427, 33896) (25931, 17985) (19223, 16407) (9045, 31178) (10612, 24248)
		(35037, 30496) (17839, 19678) (36405, 3956) (4779, 13290) (36477, 3915)
		(17532, 8692) (7680, 14425) (10523, 33499) (29205, 33747) (6314, 29440)
		(34178, 13197) (25293, 1136) (7559, 33229) (30132, 18955) (6158, 8294)
		(5426, 14395) (32965, 5411) (15646, 2300) (1735, 6395) (20024, 32002)

q	g	Pairs
		(25052, 28705) (4894, 31172) (18599, 1788) (15618, 19848) (4819, 3612)
		(822, 36153) (13694, 9955) (26924, 12835) (10321, 17145) (35978, 29420)
		(29344, 29352) (17254, 19503) (22665, 13998) (33549, 10364) (5033, 32292)
		(11366, 33439) (4105, 15944) (29399, 7121) (34777, 7343) (26508, 16639)
		(24871, 1907) (32116, 11776) (1309, 5709) (26651, 23066) (3909, 31647)
		(12923, 11588) (9806, 23319) (31401, 12300) (33157, 18626) (13173, 3729)
		(34386, 22386) (25654, 30092) (9490, 34125) (30151, 10331) (19700, 28194)
		(5427, 26441) (6797, 7089) (13726, 6368) (9958, 30263) (21843, 10312)
		(33165, 30022) (14590, 10814) (19026, 27583) (4608, 11812) (24042, 4673)
		(17523, 32348) (24306, 26093) (27803, 10651) (28160, 22712) (26424, 1744)
		(34123, 1568) (10991, 17065) (3716, 29566) (19779, 9407) (15556, 32248)
		(1041, 2067)
199	15332	(24011, 199) (6561, 6361) (2593, 25248) (2187, 16440) (27265, 11521)
		(729, 20610) (35489, 17257) (243, 34752) (25030, 16045) (81, 38997)
		(34744, 36854) (27, 24807) (37982, 34383) (9, 18754) (25861, 29373)
		(3, 30032) (35021, 13399) (1302, 30077) (24874, 36058) (29453, 30131)
		(34692, 33932) (23018, 13228) (11564, 20503) (20873, 37082) (17055, 21425)
		(20158, 8346) (5685, 39202) (33120, 7114) (1895, 22115) (11040, 1266)
		(13832, 294) (3680, 23114) (17811, 31447) (14427, 37086) (5937, 25948)
		(4809, 5481) (1979, 5717) (1603, 14486) (13860, 22387) (26935, 33736)
		(4620, 33355) (35379, 37647) (1540, 28444) (11793, 1464) (26914, 36044)
		(3931, 37011) (35372, 9348) (27711, 18847) (24991, 16081) (9237, 16940)
		(34731, 1427) (3079, 21844) (11577, 33811) (27427, 3969) (3859, 4373)
		(35543, 22876) (27687, 38260) (25048, 22680) (9229, 6674) (34750, 7390)
		(29477, 2444) (37984, 33895) (23026, 10752) (39062, 9195) (34076, 11656)
		(26221, 27585) (24559, 23325) (35141, 23189) (34587, 26134) (24914, 4807)
		(11529, 31596) (21505, 35365) (3843, 21590) (33569, 35400) (1281, 28261)
		(24390, 8326) (427, 31956) (8130, 20377) (26543, 13126) (2710, 26086)
		(22048, 10397) (27304, 37043) (33750, 16455) (35502, 12490) (11250, 28573)
		(11834, 31548) (3750, 29738) (17145, 10136) (1250, 5184) (5715, 7900)
		(13617, 31608) (1905, 33058) (4539, 15585) (635, 28171) (1513, 32985)
		(13412, 6467) (26905, 29765) (17671, 32) (35369, 14352) (32291, 31718)

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