Branches and Joints in the study of self switching of graphs

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Abstract

A vertex $v \in V(G)$ is said to be a ***self vertex switching*** of G if G is isomorphic to G^v, where G^v is the graph obtained from G by deleting all edges of G incident to v and adding all edges incident to v which are not in G. Two vertices u and v in G are said to be ***interchange similar*** if there exists an automorphism α of G such that $\alpha(u) = v$ and $\alpha(v) = u$. In this paper, we give a characterization for a cut vertex in G to be a self vertex switching where G is a connected graph such that any two self vertex switchings if exist, are interchange similar.

Key words: Switching, Self vertex switching, Interchange similar, Branches and Joints.

1 Introduction

For a finite undirected graph $G(V, E)$ with $|V(G)| = p$ and a set $\sigma \subseteq V$, Seidel [6] defined the switching of G by σ as the graph $G^\sigma(V, E')$, which is obtained from G by removing all edges between σ and its compliment V-σ and adding as edges all non edges between σ and V-σ. For more details on switching classes of graphs we refer to Hage [1], Hertz [2], Seidel [6] and Seidel and Taylor [7]. When $\sigma = \{v\} \subset V$, we call the corresponding switching $G^{\{v\}}$ as ***vertex switching*** and denoted it as G^v [8]. A subset σ of $V(G)$ to be a ***self switching*** of G if $G \cong G^\sigma$. The set ***of all self switchings of*** G with cardinality k is denoted by $SS_k(G)$ and its cardinality by $ss_k(G)$[9]. If $k = 1$, then we call the corresponding self switching as ***self vertex switching*** [3].

Let X be a block or a vertex of G. A maximal connected subgraph B of G, whose intersection with X is a single vertex v such that v is not a cut vertex of B, is called a ***limb*** at X rooted at v. If X is a vertex, then a ***branch*** at X is same as a limb at X [4]. Two vertices u and v in a graph G are said to be ***interchange similar*** if there exists an automorphism α of G such that $\alpha(u) = v$ and $\alpha(v) = u$ [5]. Since $V(G) = V(G^v)$, an isomorphism between G andG^v, in this paper we consider the isomorphism as a permutation on V.

Let G be a graph such that any two self vertex switchings, if exist, are interchange similar. Characterization of a vertex in G to be a self vertex switching was given in [9](See Theorem 1.2). In this paper, we give another characterization for a cut vertex in a connected graph to be a self vertex switching using the branches idea. Using this characterization, we characterize the connected graph which has more than one cut vertices as self vertex switchings and proofs are given only to theorems and few lemmas.

Now consider the following results, which are required in the subsequent sections. We consider simple finite graphs only unless otherwise it is mentioned specifically.

Theorem 1.1.[3] If v is a self vertex switching of a graph G of order p, then $d_G(v) = (p-1)/2$. ■

Theorem 1.2.[9] Let G be a graph such that any two self vertex switchings, if exist, are interchange similar. A vertex v in G is a self vertex switching if and only if G-v has an automorphism which maps the elements of $N(v)$ onto $[N(v)]^c$. ■

Theorem 1.3.[9] If $\sigma \subset V(G)$ is a self switching of a graph G, then the number of edges between the vertices of σ and V-σ in G is $k(p-k)/2$ where $k = |\sigma|$. ■

2 Branches and Joints

Let $\sigma \subset V$ and $B^* = G[\sigma]$, the induced subgraph induced by σ. Throughout this paper we consider $B^* = G[\sigma]$ **is connected.** Let $D_1, D_2, \ldots, D_r$ be the components of G-σ. If we consider the switching of G by σ, then the edges between the vertices of B^* and D_i in G are deleted and the non edges are added, $1 \leq i \leq r$. Here, we define joints at σ in G and study more about switching of graphs.

Definition 2.1. Let G_1 and G_2 be any two graphs and σ be a subset of both $V(G_1)$ and $V(G_2)$. An isomorphism f between G_1 and G_2 is said to be a σ-***preserving*** isomorphism if $f(\sigma) = \sigma$.

Definition 2.2. Let $G(V, E)$ be a graph. A subgraph B of G which contains B^* is called a ***joint*** at σ if B-σ is connected and maximal.

If B is connected, then we call B as ***c-joint*** otherwise ***d-joint.*** If $\sigma = \{v\}$, then the c-joints are the ***branches*** at v and a d-joint is the union of v and a component of G not containing v.

Example 2.3. Consider the graph G given in figure 2.1. Let $\sigma = \{1, 2, 3\}$. The c-joints B_1, B_2 and the d-joint D_1 at σ are given in figures 2.2, 2.3 and 2.4 respectively.

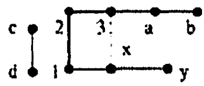

Fig.2.1: G

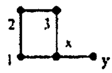

Fig.2.2: B_1

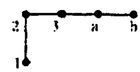

Fig.2.3: B_2

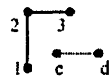

Fig.2.4: D_1

Definition 2.4. A joint B at σ in G is said to be ***total joint*** if B is the join of B^* and B-σ.

Definition 2.5. A joint B at σ in G is said to be ***self switching joint*** at σ if $B \cong B^{\sigma}$.

If $\sigma = \{v\}$, then we call the self switching joint B at σ as ***self switching branch*** at v.

Definition 2.6. In G two joints B_1 and B_2 at σ are said to be ***complementary switching joints*** at σ if there exist σ-preserving isomorphisms f_1 between B_1 and B_2^{σ} and f_2 between B_2 and B_1^{σ} such that $f_1|\sigma = f_2|\sigma$.

In the above case if $\sigma = \{v\}$ and B_1 and B_2 are branches at v in G, then we call B_1 and B_2 as ***complementary switching branches*** at v.

Definition 2.7. In G two joints B_1 and B_2 at σ are said to be ***similar*** if there exists a σ-preserving isomorphism between B_1 and B_2. Two joints B_1 and B_2 at σ are said to be the ***same*** if there is an isomorphism between B_1 and B_2 fixing all elements of σ.

Definition 2.8. For $k \geq 3$, a sequence $B_1, B_2, ..., B_k$ of joints at σ in G is said to be a ***cycle switching*** at σ if there exist σ-preserving isomorphisms f_i between B_i and B_{i+1}^{σ} under subscript modulo k such that $f_i|\sigma = f_j|\sigma$, $1 \leq i, j \leq k$ and it is denoted as $(B_1, B_2, ..., B_k)$. A cycle switching $(B_1, B_2, ..., B_k)$ is said to be ***proper*** if for $i \neq j$, B_i and B_j are not the same joints, $1 \leq i, j \leq k$.

Theorem 2.9. B is a joint at σ in a graph G if and only if B^{σ} is a joint at σ in G^{σ}. ■

Corollary 2.10. Let v be any vertex of a connected graph G such that G^{v} is connected. Then B is a branch at v in G if and only if B^{v} is a branch at v in G^{v}. ■

Corollary 2.11. If B_1, B_2, ..., B_k are the joints at σ in G such that $G = \bigcup_{i=1}^{k} B_i$, $k \geq 2$, then $G^{\sigma} = \bigcup_{i=1}^{k} B_i^{\sigma}$. ■

3 Sufficient condition for $\sigma \subset V(G)$ to be self switching

In this section, we prove that any cyclic switching at $\sigma \subset V(G)$ can be expressed as a product of proper cycle switchings at σ. In [9], we gave a sufficient condition for σ is a self switching of G. Here we give another sufficient condition for σ to be self switching where $G[\sigma]$ is connected.

Lemma 3.1. Let G be graph and B_1 and B_2 be any two joints at σ in G. If f is a σ-preserving isomorphism between B_1 and B_2^{σ}, then f is also a σ-preserving isomorphism between B_1^{σ} and B_2. ■

Lemma 3.2. In a graph G, a cycle switching at σ can be expressed as a product of proper cycle switchings at σ.

Proof. Let $C = (B_1, B_2, ..., B_k)$ be a cycle switching at σ in G. If B_i is not the same to B_j, for $i \neq j$ and $1 \leq i, j \leq k$, then C itself is proper. If not, then there exist some i and j such that $1 \leq i, j \leq k$ and B_i is the same to B_j. This implies, there exists an isomorphism α between B_i and B_j fixing all elements of σ. Let f_i be a σ-preserving isomorphism between B_i and B_{i+1}^{σ} under subscript modulo k and $f_1|\sigma = f_2|\sigma = \ldots = f_k|\sigma$. This implies, $B_1 \cong B_2^{\sigma}$, $B_2 \cong B_3^{\sigma}$, ..., $B_k \cong B_1^{\sigma}$ and $B_i \cong B_j$. This implies,

$$B_1 \cong B_2^{\sigma}, B_2 \cong B_3^{\sigma}, \ldots B_{i-1} \cong B_i^{\sigma} \cong B_j^{\sigma}; \tag{1}$$

$$B_j \cong B_i \cong B_{i+1}^{\sigma}, B_{i+1} \cong B_{i+2}^{\sigma}, \ldots, B_{j-1} \cong B_j^{\sigma} \cong B_i^{\sigma}; \tag{2}$$

$$B_i \cong B_j \cong B_{j+1}^{\sigma}, B_{j+1} \cong B_{j+2}^{\sigma}, \ldots, B_k \cong B_1^{\sigma}. \tag{3}$$

Using lemma 3.1, α is an isomorphism between B_i^{σ} and B_j^{σ}. Since f_{i-1} and α are isomorphisms, $\alpha \circ f_{i-1}$ is an isomorphism between B_{i-1} and B_j^{σ}. Also $(\alpha \circ$

$f_{i-1})|\sigma = f_{i-1}|\sigma$. From (1) *and* (3), *we get* $C_1 = (B_1, B_2, \ldots, B_{i-1}, B_j, B_{j+1}, \ldots, B_k)$ *and from* (2), $C_2 = (B_i, B_{i+1}, \ldots, B_{j-1})$ *which are cycle switchings at* σ *in* G. *If* C_1 *and* C_2 *are proper, then the proof is over. Otherwise we repeat the above process to* C_1 *and* C_2, *separately and since the graph is finite, we obtain* C *as a product of proper cycle switchings. This completes the proof.* ■

Example 3.3. *Consider the graph* G_1 *given in figure 3.1. Let* $\sigma = \{1, 2, 3, 4, 5, 6\}$. *Then* D_1, D_2 *and* D_3 *are the three joints at* σ *as shown in the graph* G_1. *The graph* G_1^σ *is given in figure 3.2. Clearly* G_1 *and* G_1^σ *are isomorphic and a* σ*-preserving isomorphism is given by (1 2 3 4 5 6) (g k i) (h l j). Now* $D_1 \cong D_2^\sigma$, $D_2 \cong D_3^\sigma$ *and* $D_3 \cong D_1^\sigma$. *This implies,* (D_1, D_2, D_3) *is the cycle switching at* σ *in* G_1.

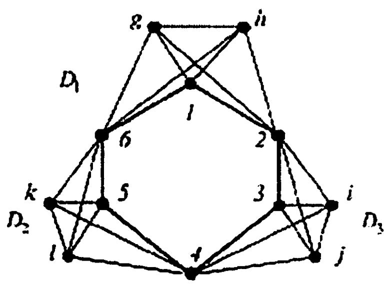

Fig.3.1: G_1

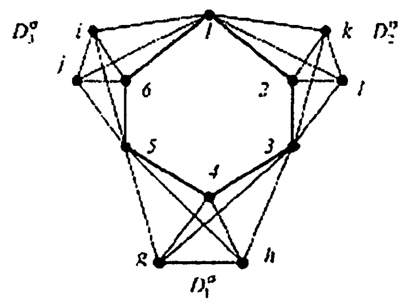

Fig.3.2: G_1^σ

Example 3.4. *Consider the graph* G_2 *given in figure 3.3. Let* $\sigma = \{1, 2, 3, 4, 5, 6\}$. *The six joints* B_1, B_2, B_3, B_4, B_5 *and* B_6 *at* σ *are shown in the graph* G_2. *The graph* G_2^σ *is given in figure 3.4.*

Clearly $G_2 \cong G_2^\sigma$ *and a* σ*-preserving isomorphism is given by (1 2 3 4 5 6) (a u c x f y) (b v d w e z). Also* $B_1 \cong B_2^\sigma$, $B_2 \cong B_3^\sigma$, $B_3 \cong B_4^\sigma$, $B_4 \cong B_5^\sigma$, $B_5 \cong B_6^\sigma$ *and* $B_6 \cong B_1^\sigma$. *This implies,* $(B_1, B_2, B_3, B_4, B_5, B_6)$ *is the only cycle switching at* σ *in* G_2.

Definition 3.5. ***The length of a cycle switching*** *is the number of joints in the cycle switching.*

In G_1 *cycle switching* (D_1, D_2, D_3) *is of length 3 and in* G_2 *cycle switching* $(B_1, B_2, B_3, B_4, B_5, B_6)$ *is of length 6.*

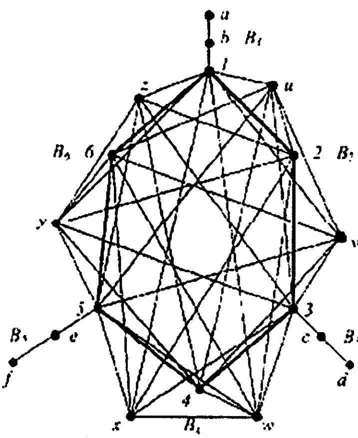

Fig.3.3: G_2

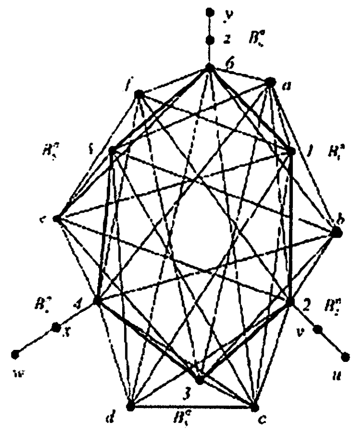

Fig.3.4: G_2^σ

Note 3.6. *In a graph two proper cycle switchings need not be of same length. As an example, consider the graph $G = G_1 \cup G_2$, where G_1 and G_2 are as given in figures 3.1 and 3.3, respectively. Then G has nine joints at $\sigma = \{1, 2, 3, 4, 5, 6\}$ namely D_1, D_2, D_3, B_1, B_2, B_3, B_4, B_5 and B_6. If we take f as a function from $V(G)$ to $V(G^\sigma)$ defined by $(1\ 2\ 3\ 4\ 5\ 6)\ (g\ k\ i)\ (h\ l\ j)\ (a\ u\ c\ x\ f\ y)\ (b\ v\ d\ w\ e\ z)$, then clearly f is an isomorphism and hence σ is a self switching of G. Now (D_1, D_2, D_3) and $(B_1, B_2, B_3, B_4, B_5, B_6)$ are two proper cycle switchings at σ but of different lengths.*

Lemma 3.7. *If the length of a proper cycle switching at σ is odd in a graph G, then each joint in it is a self switching joint at σ.* ■

Lemma 3.8. *If f is an isomorphism between the graphs G and G^σ and has a proper cycle switching at σ, then $f|\sigma$ is of order at least 3.*

Proof. *Let $(B_1, B_2, ..., B_k)$ be a proper cycle switching at σ in G. Now $f_i = f|_{B_i}$ is a σ-preserving isomorphism between B_i and B_{i+1}^σ under subscript modulo k, $1 \le i \le k$. Suppose $f|\sigma$ is of order at most 2. Then $f^2|\ \sigma = (f_2 \circ f_1)\ \sigma$ is the identity. Now $f_2 \circ f_1$ is an isomorphism between the joints B_1 and B_3 fixing all the elements of σ. This implies, B_1 and B_3 are the same joints at σ. This is a contradiction to proper cycle switching and hence $f|\sigma$ is of order at least 3.*
■

Theorem 3.9. *Let B_1, B_2, ..., B_k be the joints at σ in G such that $G = \bigcup_{i=1}^{k} B_i$, $k \geq 2$. If there exist isomorphisms f_1, f_2, ..., f_k such that each f_i is between the joint B_i in G and a unique joint B_j^σ in G^σ preserving σ and $f_1|\sigma = f_2|\sigma = \ldots = f_k|\sigma$, $1 \leq i \leq k$, then σ is a self switching of G.*

Proof. *Define $f: V(G) \rightarrow V(G^\sigma)$ such that $f(v) = f_i(v)$ if $v \in V(B_i)$. Clearly f is a bijection and it is easy to show that f is an isomorphism. Hence σ is a self switching of G.* ■

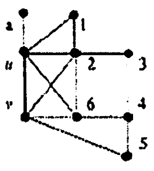

Fig.3.5: G

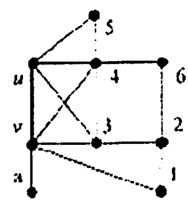

Fig.3.6: G^σ

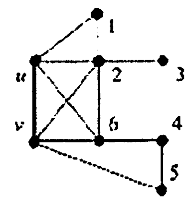

Fig.3.7: B_1

Fig.3.8: B_2

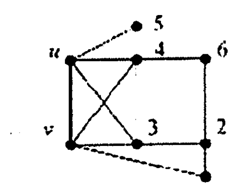

Fig.3.9: B_1^σ

Fig.3.10: B_2^σ

Note 3.10. *In the above theorem if we remove the condition that $f_1|\sigma = f_2|\sigma = \ldots = f_k|\sigma$, then σ need not be a self switching of G. As an example, consider the graphs G and switching G^σ by σ given in figures 3.5 and 3.6. The two joints B_1 and B_2 at $\sigma = \{u, v\}$ in G are given in figures 3.7 and 3.8 and the two joints B_1^σ and B_2^σ at σ in G^σ are given in figures 3.9 and 3.10. The functions*

f_1 *between* B_1 *and* B_1^σ *and* f_2 *between* B_2 *and* B_2^σ *are given by* (1 5)(2 4)(3 6)*and* (u v)*, respectively, are isomorphic. This implies,* $B_1 \cong B_1^\sigma$ *and* $B_2 \cong B_2^\sigma$ *and* $f_1|\sigma \neq f_2|\sigma$. *But* σ *is not a self switching of* G *since* G *is not isomorphic to* G^σ.

4 Main result

In this section, we assume that G is a connected graph such that any two self vertex switchings if exist, are interchange similar. In [9], we gave a characterization of a vertex in G to be a self vertex switching. Here the following theorem gives a characterization of a cut vertex in G to be a self vertex switching.

Theorem 4.1. *A cut vertex v in a connected graph G is a self vertex switching if and only if the branches at v in G are either self switching branches or complementary switching branches such that after pairing off the complementary switching branches the remaining branches are self switching branches and for each self switching branch B, there is an isomorphism between B and* B^v *fixing v.*

Proof. *Let v be a cut vertex and* $B_1, B_2, \ldots, B_k$ *be the branches at v in G such that* $G = \bigcup_{i=1}^k B_i$. *Using corollary 2.11,* $G^v = \bigcup_{j=1}^k B_j^v$. *Suppose v is a self vertex switching of G. Using theorem 1.2, let f be an isomorphism between G and* G^v *fixing v. This implies,* $B_i \cong B_j^v$ *for a unique j and an isomorphism is given by* $f_i = f|_{B_i}$. *Using lemma 3.8, f has no proper cycle switching at v. This implies that the branches are either self switching branches or complementary switching branches at v. Suppose* B_t *is a self switching branch at v. Then* $f_t = f|_{B_t}$ *is an isomorphism between* B_t *and* B_t^v *fixing v.*

Conversely, without loss of generality we assume that $(B_1, B_2), (B_3, B_4), \ldots, (B_{2r-1}, B_{2r})$ *are pairs of complementary switching branches and* $B_{2r+1}, B_{2r+2}, \ldots, B_k$ *are self switching branches at v in G. Let* $f_1, f_2, \ldots, f_k$ *be the isomorphisms such that for odd j,* $1 \leq j \leq 2r-1$, f_j *between* B_j *and* B_{j+1}^v *and for even j,* $2 \leq j \leq 2r$, f_j *between* B_j *and* B_{j-1}^v *and for* $2r+1 \leq j \leq k$, f_j *between* B_j *and* B_j^v *fixing v. Define* $f: V(G) \rightarrow V(G^v)$ *such that* $f(v) = f_j(v)$ *if* $v \in V(B_j)$. *Clearly f is an isomorphism. Hence v is a self vertex switching of G.* ■

Note 4.2. *In the above theorem if we remove the condition that for each self switching branch B at v in G, there is an isomorphism between B and* B^v *fixing v. then v need not be a self vertex switching. For example, consider the graph G given in figure 4.1. Then* B_1, B_2 *and* B_3 *are the branches at v shown in G. The branches* B_1 *and* B_2 *are complementary switching branches at v and* B_3 *is a self switching branch at v and no isomorphism between* B_3 *and* B_3^v *fixing v [9]. But v is not a self vertex switching of G since G is not isomorphic to* G^v.

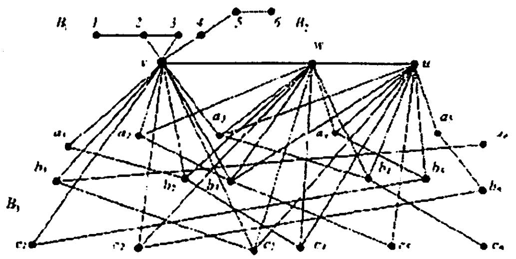

Fig.4.1: G

Notation 4.3. *Consider a cycle $C_r = (v_1, v_2, ..., v_r)$(clock-wise). For our convenience we denote it by $C_{r(v_1)}$. Identifying an end vertex of paths P_m at v_i and P_s at v_j in $C_{r(v_1)}$ is denoted by $C_{r(v_1)}(0, \ldots, P_m, 0, \ldots, P_s, 0, \ldots, 0)$. Identifying an end vertex of paths P_m and P_s to the vertex v_j in $C_{r(v_1)}$ is denoted by $C_{r(v_1)}(0, \ldots, P_m \cup P_s, 0, \ldots, 0)$. The graphs given in figures 4.2 and 4.3 are $C_{4(v)}(0, 2P_2 \cup P_3, P_2, P_3)$ and $C_{3(v)}(P_3, 0, P_3)$, respectively.*

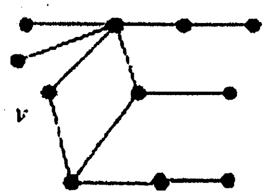

Fig.4.2: $C_{4(v)}(0, 2P_2 \cup P_3, P_2, P_3)$

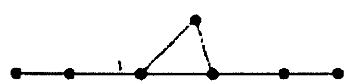

Fig.4.3: $C_{3(v)}(P_3, 0, P_3)$

Theorem 4.4. *Let G be a connected graph with a cut vertex v as a self vertex switching such that any two self vertex switchings, if exist, are interchange similar. Then $ss_1(G) > 1$ if and only if $G = C_{3(v)}(P_3, 0, P_3)$.*

Proof. *Let v be a self vertex switching of G. Using theorem 1.1. $d_G(v) = (p-1)/2 = n$, say. Let $ss_1(G) > 1$. Let u be another self vertex switching of G. Now consider the following two cases.*

Case 1. *G has only two branches, say B_1 and B_2, at v.*

Now u is in one component, say B_2. Since u and v are self vertex switchings, there exists an automorphism α of G interchanging u and v. This implies, there *are two branches* D_1 *and* D_2 *at* u *in* G *such that* $B_1 \cong D_1$ *and* $B_2 \cong D_2$. *Clearly* v *is in* D_2. *Since* B_2 *is a branch at* v *and* u *in* B_2 *is a self vertex switching,* n *vertices of* B_2 *are adjacent to* u. *Since* D_1 *is a sub graph of* B_2 *and a branch at* u*, at least one vertex* w *in* D_1 *is non adjacent to* u. *Otherwise* G^u *is disconnected as* D_1^u. *Now* $|V(B_2)| \geq n+1+1 = n+2$. *This implies,* $|V(B_2)| > |V(B_1)|$. *Then* B_1 *and* B_2 *are self switching branches at* v *and each has an odd number of vertices, using theorem 4.1. Similarly* D_1 *and* D_2 *are self switching branches at* u. *Let* $|(VB_1)| = 2r+1$ *and* $|(VB_2)| = 2s+1$, $r, s \in N$. *This implies,* $p = 2r+2s+1$. *In* G *the following two cases arise with respect* u *and* v.

Case 1.a. *u and v are adjacent in G.*

Here, $d_G(v) = d_G(u) = r+s$ *since* v *and* u *are self vertex switchings in* G *and using theorem 1.1. Also* $d_{B_2}(u) = r+s$ *and* $d_{B_2}(v) = s$. *Now* $|V(D_2)| \geq |V(B_1)|$*+number of vertices in* B_2 *which are adjacent to* v *in* G. *That is,* $2s+1 \geq 2r+1+s$ *which implies,* $s \geq 2r$. *Also* $|V(D_1)| \leq |V(B_2)|$*-number of vertices in* B_2 *which are adjacent to* v *in* G. *That is,* $2r+1 \leq 2s+1-s$ *which implies* $2r \leq s$. *Hence we get* $s = 2r$. *This implies,* $|(VB_1)| = 2r+1$, $|(VB_2)| = 4r+1$ *and* $|V(G)| = 6r+1$. *Hence* $d_G(u) = d_{B_2}(u) = 3r$.

Let B *be the subgraph obtained from* G *by deleting the branches* B_1 *at* v *and* D_1 *at* u. *Then* $B_2 = B \cup D_1$ *and* $D_2 = B \cup B_1$ *such that* $B \cap D_1 = \{u\}$ *and* $B \cap B_1 = \{v\}$. *Also* $|V(B)| = 6r+1-2r-2r = 2r+1$. *Since* B_2 *is a self switching branch at* v *and* $|V(B_2)| = 4r+1$, v *is adjacent only to* $2r$ *vertices in* B_2 *and hence* v *is adjacent to all the vertices of* B. *Similarly, we can show that* u *is adjacent to all the vertices of* B. *Figure 4.4 gives an idea about this structure. Now we prove that* $r = 1$.

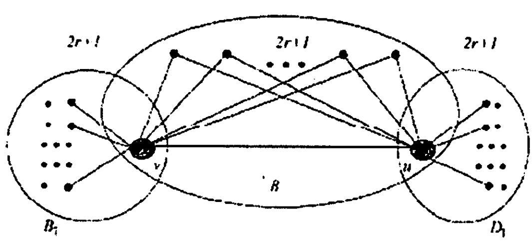

Fig.4.4:

Suppose $r \geq 2$. *Since* B_2 *is a self switching branch at* v *and* $G \cong G^v$, u *is mapped to a vertex, say* w*, in* B_2. *Now* $w \in V(D_1)$ *since* $B_2 = B \cup D_1$ *and all vertices in* B *are adjacent to* v *in* G. *Also* $d_G(w) \leq 2r$ *implies that* $d_{G^v}(w) \leq 2r+1$. *This is a contradiction to* $d_G(u) = 3r = d_{G^v}(w)$ *and hence* $r = 1$. *This implies,* $|V(G)| = 7$ *and* $G = C_{3(v)}(P_3, 0, P_3)$. *Clearly* v *and* u *are the only two self vertex switchings.*

Case 1.b. u and v are non adjacent in G.

In this case, we can prove that $|V(B_1)| = 2r+1$, $|V(B_2)| = 4r+3$ *and* $|V(G)| = 6r+3$ *and hence* $d_G(u) = d_{B_2}(u) = 3r+1$. *Now let us consider all possible values of r. If* $r = 1$, *then* $|V(G)| = 9$ *and G is the graph in figure 4.5. Clearly v is not a self vertex switching of G.*

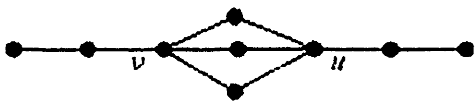

Fig.4.5:

When $r \geq 2$, *by a similar argument given in case-1.a, we can show that u is not a self vertex switching of G. Thus for all values of r, u is not a self vertex switching which is a contradiction and hence v and u are adjacent in G.*

Case 2. G has more than two branches at v.

Let $B_1, B_2, ..., B_k$ *be the branches at v in G,* $k \geq 3$. *Now u is in a branch, say* B_k. *As in case-1,* $|V(B_k)| > |(VB_1 \cup B_2 \ldots \cup B_{k-1})|$ *and hence* B_k *is a self switching branch at v. Since there is an automorphism interchanging u and v, there exist k branches* $D_1, D_2, ..., D_k$ *at u such that* $B_i \cong D_i$, $1 \leq i \leq k$.

Let B be the subgraph obtained from G by deleting the branches $B_1, B_2, ..., B_{k-1}$ *at v and* $D_1, D_2, ..., D_{k-1}$ *at u. Then* $B_k = B \cup D_1 \cup D_2 \ldots \cup D_{k-1}$ *and* $D_k = B \cup B_1 \cup B_2 \ldots \cup B_{k-1}$. *As in case-1, we can show that each vertex in B is adjacent to both u and v. Since* B_k *is a self switching branch at v and* $G \cong G^v$, *u is mapped to a vertex, say w, in* B_k. *In particular w is in some* D_i, $1 \leq i \leq k-1$. *This implies,* $d_{G^v}(w) < n$ *which is a contradiction to* $d_G(u) = d_{G^v}(w) = n$. *This implies that G cannot have a second self vertex switching.*

Thus the only possibility is $G = C_{3(v)}(P_3, 0, P_3)$.

Conversely, let $G = C_{3(v)}(P_3, 0, P_3)$. *Then v and a vertex* $u \neq v$ *of degree 3 in G are self vertex switchings and hence* $ss_1(G) > 1$. ■

5 Conclusion

We proved theorem 4.4 for graphs with any two self vertex switchings are interchange similar. One can try to prove this theorem by deleting the condition that any two self vertex switchings are interchange similar. Characterize trees and unicyclic graphs; each has a self vertex switching.

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