On k-star-forming sets in graphs

[1]Mustapha Chellali and [2]Odile Favaron

[1]Department of Mathematics, University of Blida.
B.P. 270, Blida, Algeria.
E-mail: m_chellali@yahoo.com

[2]L.R.I., URM 8623
Bât. 490, Université de Paris-Sud
91405-Orsay cedex, France
E-mail: of@lri.fr

Abstract

In [10], Fink and Jacobson gave a generalization of the concepts of domination and independence in graphs which extends only partially the well-known inequality chain $\gamma(G) \leq \mathrm{i}(G) \leq \beta(G) \leq \Gamma(G)$ between the usual parameters of domination and independence. If a k-independent set is defined as a subset of vertices inducing in G a subgraph of maximum degree less than k, we introduce the property which makes a k-independent set maximal. This leads us to the notion of k-star-forming set. The corresponding parameters $\mathrm{sf}_k(G)$ and $\mathrm{SF}_k(G)$ satisfy $\mathrm{sf}_k(G) \leq \mathrm{i}_k(G) \leq \beta_k(G) \leq \mathrm{SF}_k(G)$ where $\mathrm{i}_k(G)$ and $\beta_k(G)$ are respectively the minimum and the maximum cardinality of a maximal k-independent set. We initiate the study of $\mathrm{sf}_k(G)$ and $\mathrm{SF}_k(G)$ and give some results in particular classes of graphs as trees, chordal graphs and $K_{1,r}$-free graphs.

Keywords: Domination, independence, forming-set.
AMS subject classification: 05C69

1 Introduction

In a simple graph $G = (V, E)$ of order $n(G)$, the *neighborhood* of a vertex $v \in V$ is $N(v) = \{u \in V \mid uv \in E\}$ and the *degree* $d(v)$ of v is the order of its neighborhood. If S is a subset of vertices, its neighborhood is $N(S) = \cup_{v \in S} N(v)$, and $G[S]$ is the *subgraph* induced by the vertices of S. The closed neighborhoods of v and S are respectively $N[v] = N(v) \cup \{v\}$ and $N[S] = N(S) \cup S$. If T is another subset of vertices, $N_T(v)$ is the set of the neighbors of v in T, $d_T(v) = |N_T(v)|$, and $N_T(S) = \cup_{v \in S} N_T(v)$. The *corona* of two graphs G_1 and G_2 is the graph $G = G_1 \circ G_2$ formed from one copy of G_1 and $|V(G_1)|$ copies of G_2 where the i^{th} vertex of G_1 is adjacent to every vertex in the i^{th} copy of G_2. A graph is *chordal* if every induced cycle has length three.

In [10] Fink and Jacobson generalized the concepts of independent and dominating sets. We say that a subset S of V is *k-independent* if the maximum degree of the subgraph induced by the vertices of S is less or equal to $k - 1$. The subset S is *k-dominating* if every vertex of $V - S$ is dominated by at least k vertices of S. The property for a subset of V to be k-independent (k-dominating) is hereditary (superhereditary). A k-independent set S of G is maximal if for every vertex $v \in V \setminus S$, $S \cup \{v\}$ is not k-independent. A k-dominating set S is minimal if, for every vertex $v \in S$, $S \setminus \{v\}$ is not k-dominating in G. The *lower k-independence number* $i_k(G)$ is the minimum cardinality of a maximal k-independent set in G and the *k-independence number* $\beta_k(G)$ is the maximum cardinality of a k-independent set. Similarly, the *k-domination number* $\gamma_k(G)$ and the *upper k-domination number* $\Gamma_k(G)$ are respectively the minimum cardinality of a k-dominating set and the maximum cardinality of a minimal k-dominating set of G. A graph G is *well-covered* if $i(G) = \beta(G)$ and *well-k-covered* if $i_k(G) = \beta_k(G)$.

For $k = 1$, the 1-independent and 1-dominating sets are the classical independent and dominating sets. It is well kown that an independent set is maximal if and only if it is also dominating. So we can say that the domination, which is defined even for non-independent sets, is the property which makes an independent set maximal. Moreover every set which is both independent and dominating is a minimal dominating set of G. This observation leads to the well known inequality chain:

$$\gamma_1(G) \leq i_1(G) \leq \beta_1(G) \leq \Gamma_1(G) \quad \text{for all } G \qquad (1).$$

The k-independence and k-domination defined above generalize only partially the previous properties. If a k-independent set S of G is also k-dominating, then it is a maximal k-independent set and a minimal k-dominating set of G. But a maximal k-independent set of G is not necessarily k-dominating. For instance, if G is the 1-corona of a cycle C, $V(C)$ is a 3-independent set which is not 3-dominating. So all the inequalities of (1) do not necessarily extend when 1 is replaced by k. It has been proved that for every positive integer k, every graph G contains a set which is both k-independent and k-dominating [5]. For such sets S, $\mathrm{i}_k(G) \leq |S| \leq \beta_k(G)$ and $\gamma_k(G) \leq |S| \leq \Gamma_k(G)$. Therefore $\mathrm{i}_k(G) \leq \Gamma_k(G)$ and $\gamma_k(G) \leq \beta_k(G)$ for every G and every k. But there exist graphs satisfying $\mathrm{i}_k(G) \leq \gamma_k(G)$ or $\Gamma_k(G) \leq \beta_k(G)$ for some k (see for instance [6]).

Our purpose is to completely extend (1) and for that, to study which property makes a k-independent set maximal. Our result will be related to another generalization of domination and independence given by Haynes, Hedetniemi, Henning and Slater [12]. For graphs G and H, a set $S \subseteq V$ is an *H-forming set* of G if for every $v \in V \setminus S$, there exists a subset $R \subseteq S$, where $|R| = |V(H)| - 1$, such that the subgraph induced by $R \cup \{v\}$ contains H as a (not necessarily induced) subgraph. A set S of V is *H-independent set* if the subgraph induced by S does not contain any subgraph isomorphic to H. P_2-forming and P_2-independent sets are the classical dominating and independent sets.

For any parameter μ associated to a graph property $\mathcal{P}$, we refer to a set of vertices with Property $\mathcal{P}$ and cardinality $\mu(G)$ as a $\mu(G)$-set.

2 k-star-forming sets

Let S be a k-independent set of G. From the definition of the k-independence, S is maximal if and only if for each vertex $v \in V \setminus S$, $\Delta(G[S \cup \{v\}]) \geq k$, i.e., either v has at least k neighbors in S or v has a neighbor u in S such that $d_S(u) = k - 1$ (or both). If S is not required to be k-independent, we consider the following property $\mathcal{P}_k$.

Definition A subset S of vertices of G has Property $\mathcal{P}_k$ if for every $v \in V \setminus S$, either $d_S(v) \geq k$ or v has a neighbor u in S such that $d_S(u) \geq k - 1$.

In other words, S has Property $\mathcal{P}_k$ if for every $v \in V \setminus S$, there exist k vertices $u_1, \cdots, u_k$ in S such that $G[\{v, u_1, \cdots, u_k\}]$ contains a star $K_{1,k}$ as a subgraph. If it is the case, v is said to be k-star-dominated by S. With the terminology of Haynes et al., S has Property $\mathcal{P}_k$ if and only if it is a $K_{1,k}$-forming set. To be a $K_{1,k}$-forming set is a superhereditary property and a $K_{1,k}$-forming set is minimal if for every vertex $v \in S$, $S \setminus \{v\}$ is not $K_{1,k}$-forming. We call a $K_{1,k}$-forming set, a k-star-forming set and we denote by $\mathrm{sf}_k(G)$ (resp. $\mathrm{SF}_k(G)$) the minimum (resp. maximum) cardinality of a minimal k-star-forming set. In the particular case $k = 2$, and since $K_{1,2} = P_3$, 2-star-forming sets and $\mathrm{sf}_2(G)$ are respectively called in [12] P_3-forming-sets and $\gamma_{\{P_3\}}(G)$.

From what precedes, a k-independent set is maximal if and only if it is k-star forming. Moreover, if a k-star-forming set S is also k-independent, then for all $v \in S$ and all neighbors u of v in S, $d_{S\setminus\{v\}}(v) < k$ and $d_{S\setminus\{v\}}(u) < k-1$ so that $S\setminus\{v\}$ is not k-star-forming. Hence $\mathcal{P}_k$ is exactly the property to associate to the k-independence to generalize the relationship between independence and domination and the inequality chain (1). Therefore we can state

Theorem 2.1

1. A k-independent set is maximal if and only if it is k-star-forming.
2. A k-independent and k-star-forming set is a minimal k-star-forming set.
3. $\mathrm{sf}_k(G) \leq \mathrm{i}_k(G) \leq \beta_k(G) \leq \mathrm{SF}_k(G)$ for all G and all k (2).

It is interesting to compare the two new parameters sf_k and SF_k to the previous ones γ_k and Γ_k. Let the square G^2 of G be defined by $V(G^2) = V(G)$ and two vertices are adjacent in G^2 if they are at distance at most two in G.

Theorem 2.2 For every graph G and every k, $\gamma_k(G^2) \leq \mathrm{sf}_k(G) \leq \gamma_k(G)$.

Proof The second inequality comes from the fact that any k-dominating set is k-star-forming. To establish the first one, let S be a $\mathrm{sf}_k(G)$-set and v any vertex in $V \setminus S$. The star $K_{1,k}$ of G that contains v with k vertices of S becomes in G^2 a clique K_{k+1} on the same vertex set. Hence in G^2, v has k neighbors in S and S is a k-dominating set of G^2. □

For $k \geq 2$, the difference $\gamma_k(G) - \mathrm{sf}_k(G)$, and even the ratio $\gamma_k(G)/\mathrm{sf}_k(G)$, can be arbitrarily large. This can be seen on a star $K_{1,p}$ with $p \geq k \geq 2$, for which $\gamma_k(K_{1,p}) = p$ and $\mathrm{sf}_k(K_{1,p}) = k$. However $\mathrm{sf}_k(G)$ and $\gamma_k(G)$ may be equal and there exist upper bounds on $\gamma_k(G)$ which remain sharp for $\mathrm{sf}_k(G)$. This is checked in the next theorem with the bound of Cockayne, Gamble and Shepperd.

Theorem 2.3 Let G be a graph of order n and minimum degree δ, and k an integer with $2 \leq k \leq \delta$. Then $\mathrm{sf}_k(G) \leq kn/(k+1)$ with equality if and only if G is the disjoint union of cliques K_{k+1}.

Proof The inequality comes from $\gamma_k(G) \leq kn/(k+1)$ for $2 \leq k \leq \delta$ which was established in [3]. If this bound is sharp on $\mathrm{sf}_k(G)$, the extremal graphs belong to the family of graphs satisfying $\gamma_k(G) = kn/(k+1)$. These graphs have been determined in [8]. They are the K_k-coronas JoK_k where J is any graph. Let $G = JoK_k$ with J connected. Let S be the union of $V(J)$ and of $k-2$ vertices in each pendant clique K_k. If $|V(J)| > 1$, each vertex of $V(G) \setminus S$ is adjacent to a vertex of J of degree at least $k-1$ in S. Hence S is a k-star-forming set and $\mathrm{sf}_k(G) \leq (k-1)|V(J)| = (k-1)n/(k+1) < kn/(k+1)$. Therefore, for each connected component of G, $|V(J)| = 1$ and the proof is complete. □

On a similar way, the sharp upper bound on $i_k(G)$ established in [1], $i_k(G) \leq n - \Delta + k - 1$, remains sharp as a bound on $\mathrm{sf}_k(G)$ as shown for instance by the star $K_{1,k}$ for which $\mathrm{sf}_k(K_{1,k}) = i_k(K_{1,k}) = k$.

Contrarily to what happens with the small parameters, $\mathrm{SF}_k(G)$ may be smaller or larger than $\Gamma_k(G)$. That $\mathrm{SF}_k(G)$ may be larger than $\Gamma_k(G)$ is clear from (2) since there exist graphs with $\Gamma_k(G) < \beta_k(G)$ for some k. Let us consider the graph G obtained from $k \geq 2$ disjoint stars $C_i \simeq K_{1,k}$ with centers c_i and leaves $u_{i,1}, \cdots, u_{i,k}$ by adding a new vertex x and the k edges xc_i, $1 \leq i \leq k$. Then $n = k^2+k+1$ and since $k < \Delta$, $\Gamma_k(G) < n$. Moreover $\cup_{i=1}^{k} V(C_i)$ is a minimal k-dominating set of G. Therefore $\Gamma_k(G) = k^2 + k$. On the other hand, let S be a k-star-forming set of G. If $C_i \subseteq S$ for some i then $S \setminus \{u_{i,1}\}$ is a k-star-forming set too and S is not minimal. Hence every minimal k -star-forming set has at most k vertices in each star C_i and $\mathrm{SF}_k(G) \leq k^2 + 1 < \Gamma_k(G)$ (actually, $V(G) \setminus \{c_1, \cdots, c_k\}$ is a minimal k-star-forming set and $\mathrm{SF}_k(G) = k^2+1$). Note that the same graph G also provides an example for the opposite inequality since $\Gamma_{k+1}(G) = k^2 + 1 < k^2 + k = \mathrm{SF}_{k+1}(G)$.

Since every $(k+1)$-star-forming set is a k-star-forming set for every graph G and positive integer k, the sequence $\mathrm{sf}_k(G)$ is non-decreasing as was the sequence $\gamma_k(G)$. Moreover, since the vertex set V is the only $(\Delta+1)$-star forming set but is not a minimal Δ-star-forming set, every graph G satisfies

$$\gamma(G) = \mathrm{sf}_1(G) \leq \mathrm{sf}_2(G) \leq .... \leq \mathrm{sf}_\Delta(G) < \mathrm{sf}_{\Delta+1}(G) = |V|.$$

Note that the property shown in [10] that $\gamma_k(G) \geq \gamma(G) + k - 2$ for all G with $\Delta \geq k \geq 2$ has no counterpart with sf_k. If $G = K_poK_1$ is obtained by adding a pendant vertex at each vertex of a clique K_p, then $\Delta = p$ and $\mathrm{sf}_\Delta(G) = \gamma(G) = p$. In this example, all the large inequalities of the chain above are equalities.

3 k-star-forming sets in particular classes of graphs

It is known that the well-covered trees are P_1 and the coronas JoK_1 where J is any tree. They satisfy $\gamma(T) = i(T) = \beta(T) = \Gamma(T)$. Since $\mathrm{sf}_1(G) = \gamma(G)$ and $\mathrm{SF}_1(G) = \Gamma(G)$ for every graph, $\mathrm{sf}_1(T) = \beta(T)$ or $i(T) = \mathrm{SF}_1(T)$ or $\mathrm{sf}_1(T) = \mathrm{SF}_1(T)$ if and only if the tree T is well-covered. To generalize this result, we use the characterization of well-k-covered trees, given in [9].

Definition A tree belongs to the family $\mathcal{F}_k$, where k is an integer ≥ 1, if $\Delta(T) \leq k-1$ or if its vertex set is partitioned into an induced forest X and stars $K_{1,k}$ (called Stars) such that

(i) the centers $c_1, c_2, \cdots, c_p$ of the Stars have degree exactly k in T;

(ii) the vertices of X and their neighbors in the Stars have degree less than k in T;

(iii) for each i, at most one neighbor x of c_i has degree at least k in T or has a neighbor $y \neq c_i$ of degree at least k (or both).

The partition of T is necessarily unique. We can note that by (ii), if $k = 1$ and $n > 1$ or $k = 2$ and $n > 2$ then $X = \emptyset$. The trees of $\mathcal{F}_1$ are the well-covered ones. The trees of $\mathcal{F}_2$ are P_1, P_2 and the trees obtained by attaching a path of length 2 at each vertex of any tree J. For $k \geq 1$, the trees in $\mathcal{F}_k$ satisfy $i_k(T) = \beta_k(G) = n - p$ where p is the number of Stars $K_{1,k}$ in the partition of T. It is shown in [9] that a tree is well-k-covered if and only if it belongs to $\mathcal{F}_k$.

Theorem 3.1 A tree T satisfies $\mathrm{sf}_k(T) = \beta_k(T)$ or $i_k(T) = \mathrm{SF}_k(T)$ or $\mathrm{sf}_k(T) = \mathrm{SF}_k(T)$ for some integer $k \geq 2$ if and only if it is well-k-covered, that is belongs to $\mathcal{F}_k$.

Proof If $\mathrm{sf}_k(T) = \beta_k(T)$ or $i_k(T) = \mathrm{SF}_k(T)$ or $\mathrm{sf}_k(T) = \mathrm{SF}_k(T)$ then, by Item (3) of Theorem 2.1, T is well-k-covered. and thus belongs to $\mathcal{F}_k$ by [9].

Conversely let T be a tree of $\mathcal{F}_k$ and p the number of Stars of its unique partition. If $\Delta(T) < k$, that is if $p = 0$, then $\mathrm{sf}_k(T) = i_k(T) = \beta_k(T) = \mathrm{SF}_k(T) = n$. If $p > 0$, let S be a k-star-forming set of T. Then S contains all the vertices of the forest X by (ii). Let c be the center and $u_1, u_2, \cdots, u_k$ the leaves of a Star of T. Suppose $c \notin S$ and $u_1 \notin S$. Then c has a neighbor in S, say u_2, of degree at least $k-1$ in S. By (iii), and since $d_T(u_2) \geq k$,

u_1 and its neighbors different from c have degree at most $k-1$ in T, in contradiction to $u_1 \notin S$. Therefore if $u_1 \notin S$, then $c \in S$. Suppose now $u_1 \notin S$ and $u_2 \notin S$. Let u_1' be a neighbor of u_1 in S of degree at least $k-1$ in S. Since $u_2 \notin S$, $u_1' \neq c$. Then $d_T(u_1') \geq k$ and by (iii), the degrees of u_2 and of all its neighbors different from c are less than k. This contradicts S is a k-star-forming set not containing u_2. Therefore S contains at least k vertices in each Star. Moreover if S is a minimal k-star-forming set then it contains exactly k vertices in each Star for if $\{u_1, \cdots, u_k\} \subseteq S$, then $c \notin S$. Hence every minimal k-star-forming set has order $n-p$ and $\mathrm{sf}_k(T) = \mathrm{SF}_k(T)$, which completes the proof. □

Recall that the *total domination number* $\gamma_t(G)$ is the minimum cardinality of dominating set whose induced subgraph contains no isolated vertex. Every total dominating set is clearly a 2-star-forming set. Thus $\mathrm{sf}_2(G) \leq \gamma_t(G)$ for all G. In [12], Haynes, Hedetniemi, Henning and Slater gave an example of a family of graphs G for which $\gamma_t(G)$ is arbitrarily larger than $\mathrm{sf}_2(G)$, and proved that every tree T satisfies $\gamma_t(T) = \mathrm{sf}_2(T)$. As a consequence, we get the following

Corollary 3.2 Let T be a tree. Then $\gamma_t(T) = \beta_2(T)$ or $\gamma_t(T) = \mathrm{SF}_2(T)$ if and only if $T \in \mathcal{F}_2(T)$.

The following theorem extends to chordal graphs the property $\gamma_t(T) = \mathrm{sf}_2(T)$ for any tree T.

Theorem 3.3 Every connected chordal graph G satisfies $\mathrm{sf}_2(G) = \gamma_t(G)$.

Proof It is sufficient to prove that $\mathrm{sf}_2(G) \geq \gamma_t(G)$. Let S be a $\mathrm{sf}_2(G)$-set containing the minimum number of isolated vertices and I the set of isolated vertices of S. If $I \neq \emptyset$, let T be the set of vertices of $V \setminus S$ having at least two neighbors in I. Since S is 2-star-forming, every vertex in $V \setminus (S \cup T)$ has at least one neighbor in $S \setminus I$. Let $x \in I$. If $N_T(x) = \emptyset$, let y be a neighbor of x in $V \setminus (S \cup T)$. If $N_T(x)$ is a clique, let y be a neighbor of x in T. In both cases, $(S \setminus \{x\}) \cup \{y\}$ is a 2-star-forming set containing less isolated vertices than S, a contradiction. Hence every vertex x of I has at least two non-adjacent neighbors in T. This implies in particular $|T| \geq 2$ and $|I| \geq 2$. Let $y_1 \in T$. We construct a path alternating between T and I as follows. Let y_2 be a neighbor of y_1 in I, y_3 a neighbor of y_2 in $T \setminus N[y_1]$, y_4 a neighbour of y_3 in $I \setminus \{y_2\}$, y_5 (if y_4 is not adjacent to y_1) a neighbor of y_4 in $T \setminus N[y_3]$, a.s.o. while the current vertex y_i has no neighbor in $\{y_1, y_2, \cdots, y_{i-3}\}$. Since G is finite, the process stops at a vertex y_p adjacent to some vertex y_i with $i \leq p-3$. Then, $y_i y_{i+1} \cdots y_p y_i$ is an induced cycle of G longer than three, contradicting the hypothesis that G is chordal. Hence $I = \emptyset$, S is a total dominating set of G, and thus $\gamma_t(G) \leq \mathrm{sf}_2(G)$. □

To complete the comparison between $\gamma_t(G)$ and the k-star-forming numbers, we can wonder wether there exists an index k such that $\gamma_t(G) \leq \mathrm{sf}_k(G)$ for every graph G. The answer is negative as shown by the following example which generalizes the family satisfying $\gamma_t(G) > \mathrm{sf}_2(G)$ given in [12].

The integer $k \geq 2$ being given, let X be a set of $|X| > k^2$ vertices. For each subset $Y_{i_1 i_2 \cdots i_k} = \{y_{i_1}, y_{i_2}, \cdots, y_{i_k}\}$ of k vertices of X, we consider a set $Z_{i_1 i_2 \cdots i_k}$ of $q > 2|X|$ independent new vertices and join every vertex of $Z_{i_1 i_2 \cdots i_k}$ to every vertex of $Y_{i_1 i_2 \cdots i_k}$. The set X is a minimum k-star-forming set of the resulting graph G and $\mathrm{sf}_k(G) = |X|$. To dominate G without taking the q vertices of a set $Z_{i_1 i_2 \cdots i_k}$, we need to take a set A of at least $|X| - k + 1$ vertices of X. To make the dominating set total, we must add at least $\lceil |A|/k \rceil$ vertices from some sets Z. Hence $\gamma_t(G) \geq |X| - k + 1 + \lceil \frac{|X|-k+1}{k} \rceil > |X| = \mathrm{sf}_k(G)$. Moreover, the difference $\gamma_t(G) - \mathrm{sf}_k(G)$ can be made arbitrarily large. However, Theorem 3.4 gives a class of graphs in which $\gamma_t(G) \leq \mathrm{sf}_k(G)$.

Theorem 3.4 If G is a connected $K_{1,k}$-free graph with $k \geq 2$, then $\gamma_t(G) \leq \mathrm{sf}_k(G)$.

Proof If $k = 2$, then G is complete and $\mathrm{sf}_2(G) = \gamma_t(G) = 2$. For $k \geq 3$, let S be a $\mathrm{sf}_k(G)$-set and I the set of isolated vertices of S. If $I = \emptyset$, then S is a total dominating set and we are done. So we can assume that $I \neq \emptyset$. Since G is $K_{1,k}$-free, every vertex u in $V \setminus S$ has less than k neighbors in I, and since S is k-star-forming, u has at least one neighbor in $S \setminus I$. By the connectedness of G, every vertex x of I has at least one neighbor in $V \setminus S$. Hence there exists a set $Y \subseteq V \setminus S$ such that every vertex of I has at least one neighbor in Y and $|Y| \leq |I|$. The set $(S \setminus I) \cup Y$ is a total dominating set of G of order at most $|S|$, which completes the proof. □

4 Star-irredundance

Irredundance has been defined as the property which makes a dominating set minimal [4]. Two possible definitions of the k-irredundance have already been given [13, 7]. In the second one, the property characterizing a k-irredundant set was choosen as that which makes a k-dominating set minimal. To completely generalize the classical scheme maximal independent set - minimal dominating set - maximal irredundant set from the initial definition of the k-independence, we should consider the concept of k-irredundance by the property which makes a k-star forming set minimal. This gives

Definition A subset S of vertices of a graph G is k-star-irredundant if for every vertex $y \in S$

(i) $d_S(t) \leq k-1 \ \ \forall t \in N_S[y]$

or

(ii) $\exists x \in N_{V\setminus S}(y)$ such that $d_{S\setminus\{y\}}(x) = k-1$ and $d_{S\setminus\{y\}}(z) \leq k-2 \ \forall z \in N_{S\setminus\{y\}}(x)$

or

(iii) $\exists t \in N_S(y)$ and $x \in N_{V\setminus S}(t)$ such that $d_{S\setminus\{y\}}(t) = k-2$, $d_{S\setminus\{y\}}(x) \leq k-1$ and $d_{S\setminus\{y\}}(w) \leq k-2 \ \forall w \in N_{S\setminus\{y,t\}}(x)$.

This notion seems to be too complicated to lead to interesting results

References

[1] M. Blidia, M. Chellali, O. Favaron and N. Meddah, On k-independence in graphs with emphasis on trees. *Discrete Mathematics* 307 (2007) 2209–2216.

[2] G. Chartrand and L. Lesniak, *Graphs & Digraphs: Third Edition*, Chapman & Hall, London, 1996.

[3] E. J. Cockayne, B. Gamble and B. Shepherd, An upper bound for the k-domination number of a graph. *J. Graph Theory* 9 (1985) 533-534.

[4] E. J. Cockayne, S. T. Hedetniemi and D. J. Miller, Properties of hereditary hypergraphs and middle graphs. *Canad. Math. Bull.* 21 (1978) 461-468.

[5] O. Favaron, On a conjecture of Fink and Jacobson concerning k-domination and k-dependence. *J. Combin. Theory Series B* 39 (1985) 101-102.

[6] O. Favaron, k-domination and k-independence in graphs. Ars Combin. 25C (1988) 159-167.

[7] O. Favaron, An alternative definition of the k-irredundance. *AKCE International Journal of Graphs and Combinatorics* 2(1) (2005) 33-38.

[8] O. Favaron, A. Hansberg and L. Volkmann, k-domination and minimum degree. *Journal of Graph Theory* 57 (2008) 33–40.

[9] O. Favaron and B. L. Hartnell On well-k-covered graphs. *JCMCC* 6 (1989) 199-205.

[10] J.F. Fink and M.S. Jacobson, n-domination in graphs. *Graph Theory with Applications to Algorithms and Computer.* John Wiley and sons, New York (1985) 283-300.

[11] J.F. Fink and M.S. Jacobson, n-domination, n-dependence and forbidden subgraphs. *Graph Theory with Applications to Algorithms and Computer.* John Wiley and sons, New York (1985) 301-311.

[12] T. W. Haynes, S. T. Hedetniemi, M.A. Henning, and P. J. Slater, H-forming sets in graphs. *Discrete Math.,* 262 (2003) 159-169.

[13] M. S. Jacobson, K. Peters and D. F. Rall, On n-irredundance and n-domination. *Ars Combin.* 29 B (1990) 151-160.