A New Bridge Tournament Schedule

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Abstract. Consider n bridge teams each consisting of two pairs (the two pairs are called *teammates*). A match is a triple (i, j, b) where pair i opposes pair j on a board b; here i and j are not teammates and "oppose" is an ordered relation. The problem is to schedule a tournament for the n teams satisfying the following conditions with a minimum number of boards:

(i) Every pair must play against every other pair not on its team exactly once.
(ii) Every pair must play one match at every round.
(iii) Every pair must play every board exactly once except for odd n each pair can skip a board.
(iv) If pair i opposes pair j on a board, then the teammate of j must oppose the teammate or i on the same board.
(v) Every board is played in at most one match at a round.

We call a schedule satisfying the above five conditions a *complete coupling round robin schedule* (CCRRS) and one satisfying the first four conditions a *coupling round robin schedule (CRRS)*. While the construction of CCRRS is a difficult combinatorial problem, we construct CRRS for every $n \geq 2$.

1. Introduction

In a *bridge* tournament the basic unit of competition is a *pair* of players who play together in all matches like a pair of tennis players in a doubles tournament. A match consists of two opposing pairs playing a board which has four hands, usually designated by N (north), S (south) E (east) and W (west), dealt to the four players, with one pair sitting the $N-S$ direction and the other the $E-W$ direction (we will use the convention that pair p *opposes* pair q means $p(q)$ holds the $N-S$ $(E-W)$ hands). What is peculiar to bridge is that the relative strength of the $N-S$ hands versus the $E-W$ hands greatly determines the outcome of the match. This is like playing tennis on a court with one half court doubling the size of the other half court. Another peculiarity of bridge is that a player can only play any given board once since estimating the three unseen hands is a crucial part of a player's skill, and once a board is played, then all hands are known to all four players. Therefore, while we can eliminate the disparity of the half-court size in tennis by switching the half courts between the opposing pairs, we cannot switch the directions in bridge. The standing of a pair on a given board is actually obtained by comparing all pairs playing the same direction on that board and rank order their scores (we say these pairs *compete* with each other on that board).

Ideally, a bridge tournament schedule should balance all factors which affect the tournament outcome, except those pertaining to the pair's own merit, and in the meantime minimize the tournament length. Parker and Mood [8] defined a *balanced Howell rotation* (BHR) which satisfies conditions (i) (no exception for team-mate), (ii) and (iii), listed in the abstract as well as the following condition:

(iv′) Every pair competes with every other pair on the same number of boards.

Note that condition (ii) restricts the number of pairs to be even. So let us assume that there are $2n$ pairs. The construction of a BHR for $2n$ pairs turns out to be a difficult mathematical problem. Parker and Mood [8] proved that a necessary condition for BHR to exist is that n is even. A BHR is *complete*, denoted by CBHR, if in addition condition (v) is also satisfied. However, although many constructions of BHR's have been given [1,2,3,5,8,9], the general problem of existence remains open.

A defect of BHR is its use of the rank-order transformation of the original scores with the resulting loss of the magnitude of the scores. For example, a pair obtains the same top rank on a board regardless whether it scores ten or a thousand points more than other competing pairs. A second defect is that the standing of a pair on a given board depends not only on the outcome of its own match, but also on many other matches over which the pair has no control. To remedy this, an alternative tournament format called *team-of-four* is often used and actually has become the dominant format in important tournaments. In the team-of-four format a team formed by two pairs is the basic unit of competition. A match consists of two opposing teams playing a set of boards twice, the first time pair 1 of team 1 opposes pair 1 of team 2, the second time pair 2 of team 2 opposes pair 2 of team 1. For each board, the actual scores from the two submatches are compared to determine not only which team wins that board, but by how much. A team wins if it achieves a greater sum over the set of boards. Note that a match between two teams is decided only by the play of the two teams but not by the play of any other team. Therefore, condition (iv′) of BHR is no longer a relevant concept. A CBHR without condition (iv′) is known as a Room design.

A social bridge club with $2n$ pairs desires to construct a tournament schedule which satisfies the five conditions listed in the abstract (except that condition (iv) can now be made more precise by saying if p opposes q on board b, then the teammate of q must oppose the teammate of p also on board b). The rationale for condition (i) is to enhance the social aspect of the game, that for condition (ii) is to maximize the use of time, that for condition (iv) is to use the team-of-four format for greater fun, that for condition (v) is to have designated boards for every match at a round. The rationale for condition (iii) is multifold: to minimize the equipment (boards) required as well as the number of hands to be analyzed by experts at post mortem, to create a community of interests among the teams by playing essentially the same set of boards. The exception for odd n is necessitated by the fact that a match involves two teams and hence a board can be played in at most $(n-1)/2$ matches. Table 1 gives an example of a CCRRS.

TABLE 1. *A CCRRS WITH 8 PAIRS*

round				
1	8 6 (1)	3 7 (3)	2, 4(2)	5, 1(4)
2	8 5 (2)	4 7 (5)	1, 2(1)	6, 3(6)
3	8 2 (3)	5 7 (1)	4, 6(4)	1, 3(2)
4	8 3 (4)	1 7 (6)	6, 2(5)	4, 5(3)
5	8 1 (5)	6 7 (2)	2, 5(6)	3, 4(1)
6	8 4 (6)	2 7 (4)	6, 1(3)	3, 5(5)

Teams: 1-4, 2-3, 5-6, 7-8
number in the parenthesis after a pair is the board assigned

This example is taken from [6] in a different context. The construction of CCRRS is a difficult problem in general.

2. The Construction of CRRS

In practice condition (v) is often ignored for two reasons:

(i) A board in a CCRRS typically represents a set of two to eight hands since it is too unreliable to let a single hand determine a match and also too much time would be consumed in moving pairs versus playtime. But a round playing a set of s hands implies the existence of s subrounds; hence up to s matches can share the set of boards without having two matches playing the same hand at any subround.

(ii) Computers are often employed to deal hands in tournaments. Hence a board (or hands) can be duplicated in advance.

Thus CRRS is of practical interests provided a board is not shared by too many matches at a round. We first show that a CRRS with n even can be constructed with no more than two matches sharing a board.

Let R_n denote a Room design on the set $P = \{1, \ldots, n\}$ of pairs and the set $B = \{1, \ldots, n-1\}$ of boards. It is well known [7] that R_n exists for all even n except 4 and 6. Let $P' = \{1', \ldots, n'\}$ and $B' = \{1', \ldots, (n-1)'\}$. Then for even $n \geq 8$ a CRRS of n teams can be constructed from R_n by

(i) Delete the first round of R_n and define pair i and j to be teammate if i opposes j at that round.

(ii) Expand each match at the remaining rounds of R_n into four matches. Suppose that pair i opposes pair j on board b at round r in R_n. Then we add three other matches:

i' opposes j' on board b at round r,
i opposes j' on board b' at round r',
i' opposes j on board b' at round r'.

It is easily verified the expanded R_n is a CRRS with n teams where each board is shared by two matches at a round.

For n odd let T_{2n} denote a round robin schedule on $2n$ pairs. Delete the first round of T_{2n} and define (i, j) to be teammates if i opposes j at that round. Assign the numbers $1, \ldots, n$ to the n teams and for each team arbitrarily designate one teammate as *male* and the other as *female*. Let t_i denote the team number of pair i. Assign to the match i opposing j in T_{2n} the board

$$t_i + t_j \pmod{n} \quad \text{if } i \text{ and } j \text{ are of the same sex}$$

and

$$t_i + t_j \pmod{n} + n \quad \text{if } i \text{ and } j \text{ are of different sex.}$$

It is easily verified the T_{2n} with such a board assignment is a CRRS (if the order of the two pairs in a match is wrong, we simply switch the order). Unfortunately, we don't have a good bound on the maximum number of matches sharing a board at a round. An example of CRRS constructed from T_{10} is shown below:

TABLE 2. *A CRRS WITH 5 TEAMS*

1	2, 10(8)	1, 3(4)	9, 4(6)	8, 5(8)	7, 6(4)
2	10, 3(9)	2, 4(1)	1, 5(1)	9, 6(2)	8, 7(2)
3	4, 10(10)	3, 5(3)	2, 6(7)	1, 7(10)	9, 8(5)
4	10, 5(6)	4, 6(9)	3, 7(7)	2, 8(10)	1, 9(8)
5	6, 10(1)	5, 7(9)	4, 8(7)	3, 9(10)	2, 1(3)
6	10, 7(5)	6, 8(3)	5, 9(7)	4, 1(5)	3, 2(5)
7	8, 10(4)	7, 9(1)	6, 1(6)	5, 2(2)	4, 3(2)
8	10, 9(3)	8, 1(9)	7, 2(6)	6, 3(8)	5, 4(4)

teams: 10-1, 9-2, 8-3, 7-4, 6-5.
male: pairs 1,2,3,4,5; female: pairs 6,7,8,9,10.
number in the parenthesis after a pair is the board assigned.

The above construction actually works for all n odd or even. Since T_{2n} exists for all n [4], we have

Theorem. *A CRRS with $n \geq 2$ teams always exists.*

A cyclic schedule has the advantage that pair i, except pair $2n$, only needs to follow pair $i - 1$ in its movement from round to round. This is a very nice property to have in practice since one wrong move of a pair at a round can mess up latter rounds. Fortunately, the CRRS constructed in this session retain most of the cyclic property of T_{2n} and of R_n if the latter is cyclic and the primed rounds are played after the unprimed ones.

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