Expansion techniques on the super edge antimagic total graphs

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Abstract. A (p,q)-graph G is called (a,d)-*edge antimagic total*, in short (a,d)-*EAMT*, if there exist integers $a > 0$, $d \geq 0$ and a bijection $\lambda : V \cup E \to \{1, 2, ..., p+q\}$ such that $W = \{w(xy) : xy \in E\} = \{a, a+d, ..., a+(q-1)d\}$, where $w(xy) = \lambda(x) + \lambda(y) + \lambda(xy)$ is the edge-weight of xy. An (a,d)-EAMT labeling λ of G is *super*, in short (a,d)-SEAMT, if $\lambda(V) = \{1, 2, \cdots, p\}$. In this paper, we propose some theorems how to construct the new (bigger) (a,d)-SEAMT graphs from old (smaller) ones.

Key words and phrases: *Labeling, (a,d)-EAMT, (a,d)-SEAMT, Dual Labeling*

1 Introduction

We consider finite undirected graphs without loops and multiple edges. The notation V and E stand for the vertex set and edge set of graph G, respectively. Let $e = \{u, v\}$ (in short, $e = uv$) denote an edge connecting vertices u and v in G. P_n denotes a path on n vertices. We denote by (p,q)-*graph* G a graph with p vertices and q edges. Other terminologies and notations for graph-theoretic ideas we follow the book of [6].

A (p,q)-graph G is called (a,d)-*edge antimagic total*, in short (a,d)-*EAMT*, if there exist integers $a > 0$, $d \geq 0$ and a bijection $\lambda : V \cup E \to$

$\{1, 2, ..., p+q\}$ such that the set of edge-weights is $W = \{w(xy) : xy \in E\} = \{a, a+b, ..., a+(q-1)d\}$, where $w(xy) = \lambda(x) + \lambda(y) + \lambda(xy)$. We shall follow [7] to call $w(xy) = \lambda(x) + \lambda(y) + \lambda(xy)$ the *edge-weight* of xy, and W the *set of edge-weights* of the graph G. In particular, an (a, d)-*EAMT* labeling λ of a (p, q)-graph G is *super* if $\lambda(V) = \{1, 2, \cdots, p\}$. For the rest of the paper, we will denote super (a, d)-EAMT of G by (a, d)-SEAMT.

For any (a, d)-SEAMT labeling on a (p, q)-graph G, the maximum edge-weight is no more than $p + (p-1) + (p+q)$. Thus

$$a + (q-1)d \leq 3p + q - 1 . \quad (1)$$

Similarly, the minimum possible edge-weight is at least $1 + 2 + p + 1$. This implies that

$$a \geq p + 4 . \quad (2)$$

So, from (1) and (2), we have

$$d \leq \frac{2p + q - 5}{q - 1} . \quad (3)$$

In general, for any (a, d)-EAMT labeling on a (p, q)-graph G, the maximum edge-weight is no more than $(p+q-2) + (p+q-1) + (p+q)$. Thus

$$a + (q-1)d \leq 3p + 3q - 3 . \quad (4)$$

Similarly, the minimum possible edge-weight is at least $1 + 2 + 3$. Consequently,

$$a \geq 6 . \quad (5)$$

So, from (4) and (5), we have

$$d \leq \frac{3p + 3q - 9}{q - 1} . \quad (6)$$

A number of classification studies on (a, d)-SEAMT (resp. (a, d)-EAMT) for connected graphs has been extensively investigated. For instances, in [3], Bača et al. showed that wheel W_n has a (a, d)-SEAMT labeling if and only if $d = 1$ and $n \equiv 1 (mod\ 4)$; Fan F_n has a (a, d)-SEAMT if $2 \leq n \leq 6$ and $d \in \{0, 1, 2\}$. Ngurah and Baskoro [7] proved that for every Petersen graph $P(n, m)$ has a $(4n+2, 1)$-SEAMT labeling, for $n \geq 3$ and $1 \leq m \leq \frac{n}{2}$. More results concerning antimagic total labeling, see for instances [9, 1] and a nice survey paper by Gallian [5].

People also consider how to construct a new (bigger) $(a, 0)$-SEAMT graphs from some known (smaller) $(a, 0)$-SEAMT graphs. These constructions are proposed by inserting some new pendant edges and points, see for instance [4, 10, 12, 8].

In this paper, we propose new constructions of (a,d)-SEAMT (bigger) graphs from old (smaller) ones. By using this construction, we can have more classes of (a,d)-SEAMT graphs. We also give (a,d)-SEAMT labeling of graph $nP_2 \cup F_{n+2}$, for even $n \geq 2$ and $d \in \{0,1,2\}$.

2 The Duality of EAMT and SEAMT Labeling

Given any (a,d)-EAMT labeling λ on a (p,q)-graph G. Then, its *dual* labeling λ' can be defined ([13]) by

$$\lambda'(x) = p + q + 1 - \lambda(x), \text{ for any vertex } x, \text{ and}$$

$$\lambda'(xy) = p + q + 1 - \lambda(xy), \text{ for any edge } xy.$$

By using this above duality property, we have the following theorem.

Theorem A. *(Wallis et al. [13]) If a (p,q)-graph G has an (a,d)-EAMT labeling then G has an $(3p+3q+3-a-(q-1)d, d)$-EAMT labeling as its dual.*

Theorem B. *(Sudarsana et al. [11]) Let λ_1 be a (a,d)-SEAMT labeling of a (p,q)-graph G. Then, the labeling λ_1' defined:*

$$\lambda_1'(x) = p + 1 - \lambda_1(x),\ \forall x \in V, \text{ and}$$
$$\lambda_1'(xy) = 2p + q + 1 - \lambda_1(xy),\ \forall xy \in E$$

is a $(4p+q+3-a-(q-1)d, d)$-SEAMT labeling of G.

The labeling λ_1' is called a *dual (a,d)-SEAMT labeling of* λ_1 on G.

3 Some Lemmas

The properties of (a,d)-SEAMT graph proposed in the next lemmas will be useful in the next section. Bača et al. [2] have proved the following lemma.

Lemma 1. *(Bača et al. [2]) If a (p,q)-graph G has a bijection $f: V(G) \to \{1,2,...,p\}$ such that the set $S_f = \{f(u)+f(v) : uv \in E(G)\}$ consists of q consecutive integers with difference d then G has a $(a_1, d-1)$-SEAMT and a $(a_2, d+1)$-SEAMT labeling with $a_1 = \min(S)+p+q$ and $a_2 = \min(S)+p+1$.*

Lemma 2. *Let G be a (p,q)-graph having a bijection $f': V(G) \to \{1,2,...,p\}$ such that the set $S_{f'} = \{f'(u)+f'(v) : uv \in E(G)\}$ consists of q consecutive integers. If q is odd then G has a $(a,1)$-SEAMT labeling with $a = \min(S_{f'}) + p + \frac{q+1}{2}$ or $a = \min(S_{f'}) + p + \frac{q-1}{2} + 1$.*

Proof. Suppose that a (p,q)-graph G with q odd, has a bijection $f' : V(G) \to \{1,2,...,p\}$ such that the set $S_{f'} = \{f'(u)+f'(v) : uv \in E(G)\}$ consists of q consecutive integers, namely, $S_{f'} = \{s, s+1, ..., s+(q-1)\}$. Let S' be a subset of $S_{f'}$ consisting of all even elements and S'' be a subset of $S_{f'}$ consisting of all odd elements.

Case 1. s even.
Since q odd then $|S'| = \frac{q+1}{2}$ and $|S''| = \frac{q-1}{2}$. Meanwhile, we can label the edges of G using labels $p+1, p+2, ..., p+q$. Let $e_j = uv$ be the edges of G with $f'(u)+f'(v) = s+2(j-1) \in S'$, for $j = 1,2,..., \frac{q+1}{2}$ and $e_i = xy$ be the edges of G with $f'(x)+f'(y) = s+2(i-1)+1 \in S''$, for $i = 1,2,..., \frac{q-1}{2}$. Define a bijection $f'' : E(G) \to \{p+1, p+2, ..., p+q\}$ as follows

$$f''(e_i) = p+q+1-i, \text{ for } i = 1,2,..., \tfrac{q-1}{2},$$
$$f''(e_j) = p+\tfrac{q+1}{2}+1-j, \text{ for } j = 1,2,..., \tfrac{q+1}{2}.$$

Combining the vertex bijection f' and the edge bijection f'' gives the set of edge-weights $W = \{s+p+\frac{q+1}{2}+j-1) : 1 \le j \le \frac{q+1}{2}\} \cup \{s+p+q+i : 1 \le i \le \frac{q-1}{2}\} = \{\min(S_{f'})+p+\frac{q+1}{2}, \min(S_{f'})+p+\frac{q+1}{2}+1, ..., \min(S_{f'})+p+\frac{q+1}{2}+\frac{q-1}{2}, \min(S_{f'})+p+\frac{q+1}{2}+\frac{q-1}{2}+1, ..., \min(S_{f'})+p+\frac{q+1}{2}+q-1\}$. This implies that combination of the bijection f' and f'' gives a $(a,1)$-SEAMT labeling of G with $a = \min(S_{f'})+p+\frac{q+1}{2}$.

Case 2. s odd.
Since q odd then $|S'| = \frac{q-1}{2}$ and $|S''| = \frac{q+1}{2}$. Let $e_j = uv$ be an edge of G with $f'(u)+f'(v) = s+2(j-1)+1 \in S'$, for $j = 1,2,..., \frac{q-1}{2}$ and $e_i = xy$ be an edge of G with $f'(x)+f'(y) = s+2(i-1) \in S''$, for $i = 1,2,..., \frac{q+1}{2}$. Define a bijection $f''' : E(G) \to \{p+1, p+2, ..., p+q\}$ as follows

$$f'''(e_i) = p+q+1-i, \text{ for } i = 1,2,..., \tfrac{q+1}{2},$$
$$f'''(e_j) = p+\tfrac{q-1}{2}+1-j, \text{ for } j = 1,2,..., \tfrac{q-1}{2}.$$

Combination of the bijection f' and f''' gives the set of edge-weights $W = \{s+1+p+\frac{q-1}{2}+j-1) : 1 \le j \le \frac{q-1}{2}\} \cup \{s+1+p+q+i-2 : 1 \le i \le \frac{q+1}{2}\} = \{\min(S_{f'})+p+\frac{q-1}{2}+1, \min(S_{f'})+p+\frac{q-1}{2}+2, ..., \min(S_{f'})+p+\frac{q-1}{2}+\frac{q-1}{2}, \min(S_{f'})+p+\frac{q-1}{2}+\frac{q-1}{2}+1, ..., \min(S_{f'})+p+\frac{q-1}{2}+1+q-1\}$. This concludes that G has a $(a,1)$-SEAMT labeling with $a = \min(S_{f'})+p+\frac{q-1}{2}+1$. This completes the proof of the lemma. □

4 The Expansion Techniques

The theorems proposed in this section are expansion techniques on the (a,d)-SEAMT graphs.

Theorem 1. *Let G be a (p,q)-graph having a bijection $f : V(G) \to \{1,2,...,p\}$ such that the set $S_f = \{f(u)+f(v) : uv \in E(G)\}$ consists of q consecutive integers with deference d. Let n be any positive integer with $n \leq \frac{2p+1-\max(S_f)}{d}$. If G_1 is a graph constructed from G by adding exactly one vertex adjacent to n distinct vertices $u_1, u_2, ..., u_n$ of G labeled by $f(u_i) = \max(S_f)-p-1+di$, for $1 \leq i \leq n$, then G_1 has a $(a_1, d-1)$-SEAMT and a $(a_2, d+1)$-SEAMT labeling with $a_1 = \min(S_f)+p+q+n+1$ and $a_2 = \min(S_f)+p+2$. Furthermore, if $q+n$ is odd then G_1 has a $(a_3, 1)$-SEAMT labeling with $a_3 = \min(S_f)+p+\frac{q+n+1}{2}+1$ or $a_3 = \min(S_f)+p+\frac{q+n-1}{2}+2$.*

Proof. Let x_0 be the new vertex. Define the new vertex bijection f_1 on G_1 as follows.

$$f_1(u) = f(u), \text{ for } u \in V(G),$$
$$f_1(x_0) = p+1.$$

To prove the first statement consider the set $S_{f_1} = \{f_1(u)+f_1(v) : uv \in E(G_1)\}$. Clearly, $S_{f_1} = S_f \cup \{f_1(x_0)+f(u_i) : 1 \leq i \leq n\}$, where x_0u_i are the new edges. Note that $S_f = \{f(u)+f(v) : uv \in E(G)\}$ consists of q consecutive integers with deference d, namely, $S_f = \{s, s+d, ..., s+(q-1)d\}$. So, we get $\{f_1(x_0)+f(u_i) : 1 \leq i \leq n\} = \{s+(q+i-1)d : 1 \leq i \leq n\} = \{s+qd, s+(q+1)d, ..., s+(q+n-1)d\}$. This implies that S_{f_1} consists of $q+n-1$ constitute an arithmetic progression with difference d and $\min(S_{f_1}) = \min(S_f) = s$. By Lemma 1, the graph G_1 has a $(a_1, d-1)$-SEAMT and a $(a_2, d+1)$-SEAMT labeling with $a_1 = \min(S_f)+p+q+n+1$ and $a_2 = \min(S_f)+p+2$. Furthermore, if $d=1$ this implies that the set S_{f_1} consists of $q+n$ consecutive integers with $\min(S_{f_1}) = \min(S_f) = s$. If $q+n$ is odd then Lemma 2 guarantees that G_1 has a $(a_3, 1)$-SEAMT labeling with $a_3 = \min(S_f)+p+\frac{q+n+1}{2}+1$ or $a_3 = \min(S_f)+p+\frac{q+n-1}{2}+2$. The theorem holds only if the highest label of the vertex adjacent to x_0 is less then or equal to p, namely $\max(S_f)-p-1+dn \leq p$. So, $n \leq \frac{2p+1-\max(S_f)}{d}$. □

Theorem 2. *Let G be a (p,q)-graph having a bijection $f : V(G) \to \{1,2,...,p\}$ such that the set $S_f = \{f(u)+f(v) : uv \in E(G)\}$ consists of q consecutive integers. Let G_3 be a graph constructed from G by joining $h \geq 1$ new vertices x_i to z_i and z_{i+1} $(1 \leq i \leq h)$, where $z_i \in V(G)$ with $f(z_i) = \min(S_f)+q-p-2+i$. If $h \leq 2p+1-\min(S_f)-q$ then G_3 has a $(a_1, 0)$-SEAMT and a $(a_2, 2)$-SEAMT labeling with $a_1 = \min(S_f)+p+q+3h$ and $a_2 = \min(S_f)+p+h+1$. Furthermore, if q is odd then G_3 has a $(a_3, 1)$-SEAMT labeling with $a_3 = \min(S_f)+p+\frac{q+1}{2}+2h$.*

Proof. Define the new bijection $f_3 : V(G_3) \to \{1,2,...,p+h\}$ as follows.

$$f_3(u) = f(u), \text{ for } u \in V(G),$$
$$f_3(x_i) = p+i, \text{ for } 1 \leq i \leq h.$$

Consider the set $S_{f_3} = \{f_3(u) + f_3(v) : uv \in E(G_3)\}$. It is not difficult to verify that $S_{f_3} = S_f \cup \{f_3(x_i) + f(z_i) : 1 \leq i \leq h\} \cup \{f_3(x_i) + f(z_{i+1}) : 1 \leq i \leq h\}$, where $x_i z_i$ and $x_i z_{i+1}$ are the new edges. Note that $S_f = \{f(u) + f(v) : uv \in E(G)\}$ consists of q consecutive integers, call $S_f = \{s, s+1, ..., s+(q-1)\}$. Consider the set $S_1 = \{f_3(x_i) + f(z_i) : 1 \leq i \leq h\}$ and $S_2 = \{f_3(x_i) + f(z_{i+1}) : 1 \leq i \leq h\}$. So, $S_1 = \{s + q + 2i - 2 : 1 \leq i \leq h\} = \{s+q, s+q+2, ..., s+q+2h-2\}$ and $S_2 = \{s+q+2i-1 : 1 \leq i \leq h\} = \{s+q+1, s+q+3, ..., s+q+2h-1\}$. Therefore, we have that the set $S_{f_3} = S_f \cup S_1 \cup S_2$ consists of $q + 2h$ consecutive integers and $\min(S_{f_3}) = \min(S_f) = s$. Again, Lemma 1 gives that G_3 has a $(a_1, 0)$-SEAMT and a $(a_2, 2)$-SEAMT labeling with $a_1 = \min(S_f) + p + q + 3h$ and $a_2 = \min(S_f) + p + h + 1$. Furthermore, if q is odd then $q + 2h$ is always odd. So, we can use Lemma 2 to show that G_3 has a $(a_3, 1)$-SEAMT labeling with $a_3 = \min(S_f) + p + \frac{q+1}{2} + 2h$. The theorem holds only if the highest label of z_i adjacent to x_h is less then or equal to p, namely $\min(S_f) + q - p - 2 + h + 1 \leq p$. So, $h \leq 2p + 1 - \min(S_f) - q$. □

Theorem 3. *Let G_1 be a (p_1, q_1)-graph having a bijection $f_1 : V(G_1) \to \{1, 2, ..., p_1\}$ such that the set $S_{f_1} = \{f_1(u) + f_1(v) : uv \in E(G_1)\}$ consists of q_1 consecutive integers. Let G_2 be a (p_2, q_2)-graph having a bijection $f_2 : V(G_2) \to \{1, 2, ..., p_2\}$ such that the set $S_{f_2} = \{f_2(u) + f_2(v) : uv \in E(G_2)\}$ consists of q_2 consecutive integers. Let G^* be a graph constructed by joining all vertices of G_2 to one vertex u_0 of G_1 with $f_1(u_0) = \min(S_{f_1}) + q_1 - p_1 - 1$. If $\min(S_{f_2}) = p_2 + \min(S_{f_1}) + q_1 - 2p_1$ then G^* has a $(a_1, 0)$-SEAMT and a $(a_2, 2)$-SEAMT labeling with $a_1 = \min(S_{f_1}) + p_1 + q_1 + 2p_2 + q_2$ and $a_2 = \min(S_{f_1}) + p_1 + p_2 + 1$. Furthermore, if $q_1 + p_2 + q_2$ is odd then G^* has a $(a_3, 1)$-SEAMT labeling with $a_3 = \min(S_{f_1}) + p_1 + p_2 + \frac{(q_1+p_2+q_2+1)}{2}$ or $a_3 = \min(S_{f_1}) + p_1 + p_2 + \frac{(q_1+p_2+q_2-1)}{2} + 1$.*

Proof. Call the new vertex bijection $f^* : V(G^*) \to \{1, 2, ..., p_1 + p_2\}$ as follows.

$$f^*(u) = f_1(u), \text{ for } u \in V(G_1),$$
$$f^*(v) = f_2(v) + p_1, \text{ for } v \in V(G_2).$$

Consider the set $S^* = \{f^*(x) + f^*(y) : xy \in E(G^*)\}$. Clearly, $S^* = S_{f_1} \cup \{f_1(u_0) + f^*(v_i) : 1 \leq i \leq p_2\} \cup S_2$, where $u_0 v_i$ are the new edges in G^*, $S_{f_1} = \{f_1(u) + f_1(x) : ux \in E(G_1)\}$ and $S_2 = \{f^*(v) + f^*(y) : vy \in E(G_2)\}$. Call $S_{f_1} = \{s_1 + j - 1 : 1 \leq j \leq q_1\}$. Since $f_1(u_0) = \min(S_{f_1}) + q_1 - p_1 - 1$ then the set $\{f_1(u_0) + f^*(v_i) : 1 \leq i \leq p_2\}$ is $\{s_1 + q_1 + i - 1 : 1 \leq i \leq p_2\}$. Let $S_{f_2} = \{s_2 + k - 1 : 1 \leq k \leq q_2\}$. Now, consider $S_2 = \{f^*(v) + f^*(y) : vy \in E(G_2)\}$. Since $f^*(v) = f_2(v) + p_1$, for $v \in V(G_2)$ then $S_2 = \{f_2(v) + f_2(y) + 2p_1 : vy \in E(G_2)\}$ and hence

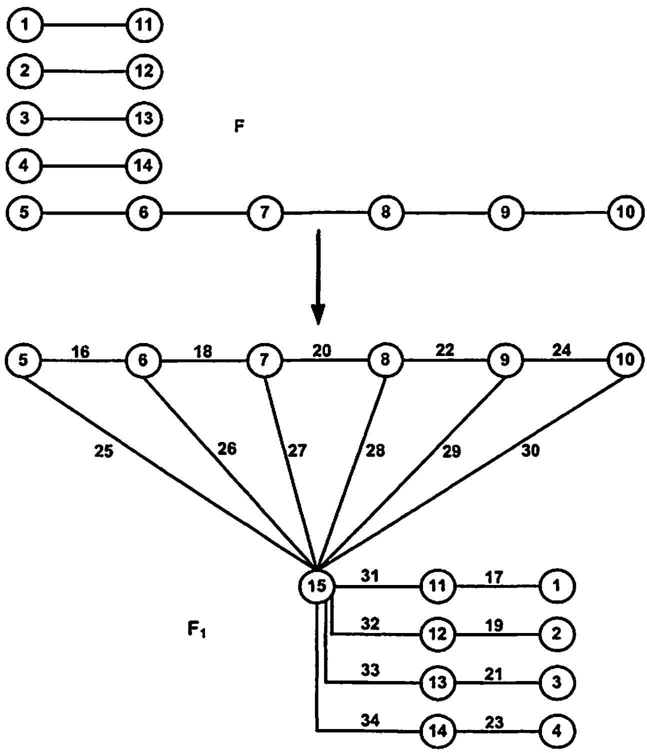

Fig. 1. The (27, 2)-SEAMT graph F_1 is formed from the graph F by applying Theorem 1.

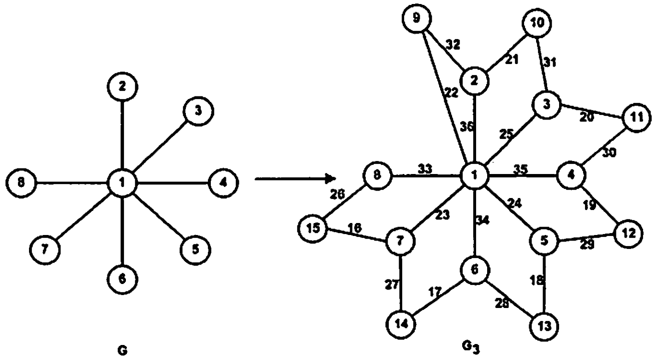

Fig. 2. The $(29,1)$-SEAMT graph G_3 is formed from the graph G by applying Theorem 2.

$S_2 = \{s_2+2p_1+k-1 : 1 \le k \le q_2\}$. Since $\min(S_{f_2}) = p_2+\min(S_{f_1})+q_1-2p_1$ then $S_2 = \{s_1+p_2+q_1+k-1 : 1 \le k \le q_2\}$. Finally, we have $S^* = \{s_1+j-1 : 1 \le j \le q_1\} \cup \{s_1+q_1+i-1 : 1 \le i \le p_2\} \cup \{s_1+p_2+q_1+k-1 : 1 \le k \le q_2\}$. It is not difficult to verify that the set S^* consists of $q_1+p_2+q_2$ consecutive integers. This implies that the bijection f^* extends to a $(a_1,0)$-SEAMT and a $(a_2,2)$-SEAMT labeling of G^* with $a_1 = \min(S_{f_1}) + p_1 + q_1 + 2p_2 + q_2$ and $a_2 = \min(S_{f_1}) + p_1 + p_2 + 1$. Furthermore, if $q_1 + p_2 + q_2$ is odd then Lemma 2 gives that G^* has a $(a_3,1)$-SEAMT labeling with $a_3 = \min(S_{f_1})+p_1+p_2+\frac{(q_1+p_2+q_2+1)}{2}$ or $a_3 = \min(S_{f_1})+p_1+p_2+\frac{(q_1+p_2+q_2-1)}{2}+1$. □

5 SEAMT Labeling of $nP_2 \cup F_{n+2}$

A Fan F_{n+2}, $n \ge 0$, is a graph obtained by joining all vertices of path P_{n+2} to a further vertex u_c called the center. The graph $nP_2 \cup F_{n+2}$ is a disjoint union of graph nP_2 and F_{n+2}. We denote that $V(nP_2 \cup F_{n+2}) = \{u_{1,i}, u_{2,i} | 1 \le i \le n\} \cup \{u_c, u_{3,j} | 1 \le j \le n+2\}$, and $E(nP_2 \cup F_{n+2}) = \{e_{1,i} | 1 \le i \le n\} \cup \{e_{2,j} | 1 \le j \le n+1\} \cup \{e_{3,j} | 1 \le j \le n+2\}$, where $e_{1,i} = u_{1,i}u_{2,i}$, for $1 \le i \le n$; $e_{2,j} = u_{3,j}u_{3,j+1}$, for $1 \le j \le n+1$; and $e_{3,j} = u_c u_{3,j}, u_{3,j} \in V(P_{n+2})$, for $1 \le j \le n+2$.

In this section, we will show that the graph $nP_2 \cup F_{n+2}$ has a (a,d)-SEAMT labeling, for $n \ge 1$ and $d \in \{0,1,2\}$. By (3) and (6), we have the following facts: for every $n \ge 1$, there is no a (a,d)-SEAMT labeling of $nP_2 \cup F_{n+2}$ with $d \ge 3$.

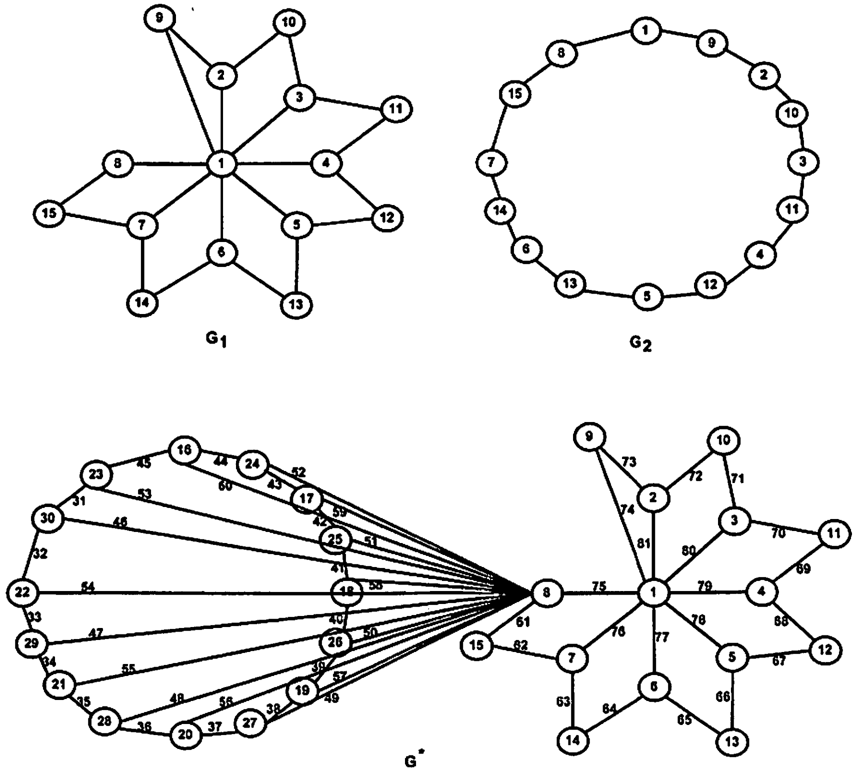

Fig. 3. The $(84, 0)$-SEAMT graph G^* is formed from the graphs G_1 and G_2 as required in Theorem 3.

Theorem 4. *For every $n \geq 1$, the graph $nP_2 \cup F_{n+2}$ has a $(8n+9, 0)$-SEAMT and a $(5n+7, 2)$-SEAMT labeling. Furthermore, for even $n \geq 2$ the graph $nP_2 \cup F_{n+2}$ has a $(\frac{13n+16}{2}, 1)$-SEAMT labeling.*

Proof. Define a vertex bijection $\lambda : V(nP_2 \cup P_{n+2}) \rightarrow \{1, 2, ..., 3n+2\}$, for every $n \geq 1$ in the following way.
$\lambda(u_{1,i}) = i$, for $1 \leq i \leq n$,
$\lambda(u_{2,i}) = 2n + 2 + i$, for $1 \leq i \leq n$,
$\lambda(u_{3,j}) = n + j$, for $1 \leq j \leq n + 2$.

By using Theorem 1, Lemma 1 and the definition of λ as above, it can be verified that the bijection λ extends to a $(8n+9, 0)$-SEAMT and a $(5n+7, 2)$-SEAMT labeling of $nP_2 \cup F_{n+2}$ for every $n \geq 1$. Furthermore, Lemma 2 ensures that the bijection λ can be extended to a $(\frac{13n+16}{2}, 1)$-SEAMT labeling of $nP_2 \cup F_{n+2}$ for even $n \geq 2$. □

By Theorems A and B, we have the following corollaries.

Corollary 1. *For every $n \geq 1$, the graph $nP_2 \cup F_{n+2}$ has a $(10n+12, 0)$-EAMT and a $(7n+10, 2)$-EAMT labeling. Furthermore, for odd $n \geq 1$ the graph $nP_2 \cup F_{n+2}$ has a $(\frac{17n+22}{2}, 1)$-EAMT labeling.*

Corollary 2. *For every $n \geq 1$, the graph $nP_2 \cup F_{n+2}$ has a $(7n+8, 0)$-SEAMT and a $(4n+7, 2)$-SEAMT labeling. Furthermore, for odd $n \geq 1$ the graph $nP_2 \cup F_{n+2}$ has a $(\frac{11n+4}{2}, 1)$-SEAMT labeling.*

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References

1. M. Bača, Y. Lin, M. Miller and R. Simanjuntak, New constructions of magic and antimagic graph labelings, *Utilitas Math.* 60 (2001), 229-239.
2. M. Bača, F. Bertault, J. A. MacDougall, M. Miller, R. Simanjuntak and Slamin, Vertex-antimagic total labeling of graphs, *Discuss. Math. Graph Theory* 23 (2003), 67-83.
3. M. Bača, Y. Lin, M. Miller and M. Z. Youssef, Edge-antimagic graphs, *Discrete Math.*, 307 (2007), 1232-1244.
4. E. T. Baskoro, I W. Sudarsana and Y. M. Cholily, How to construct new super edge-magic graphs from some old ones, *J. Indones. Math. Soc.*, 11, 2 (2005), 156-162.

5. J. A. Gallian, A dynamic survey of graph labellings, *Electron. J. Combin.*,#DS6, 2007.
6. N. Hartsfield and G. Ringel, *Pearls in Graph Theory*, Academic Press, San Diego (1994).
7. A. A. G. Ngurah and E. T. Baskoro, On magic and antimagic total labeling of generalized Petersen graph, *Utilitas Math.*, 63 (2003), 97-107.
8. Kiki A. Sugeng and Mirka Miller, Relationship between adjacency matrices and super (a,d)-edge-antimagic-total labeling of graphs, *J. Combin. Math. Combin. Comput.*, 55 (2005), 71-82.
9. R. Simanjuntak et al, Two new (a,d)-antimagic graph labelings, *Proc. AWOCA* (2000), 179-189.
10. I W. Sudarsana, E. T. Baskoro, D. Izmaimusa and H. Assiyatun, Creating new super edge-magic total labelings from old ones, *J. Combin. Math. Combin. Comput.*, 55 (2005), 83-90.
11. I W. Sudarsana, E. T. Baskoro, D. Izmaimusa and H. Assiyatun, On super (a,d)-edge antimagic total labeling of disconnected graphs, *J. Combin. Math. Combin. Comput.*, 55 (2005), 149-158.
12. I W. Sudarsana, E. T. Baskoro, S. Uttunggadewa and D. Izmaimusa, An expansion technique on super edge-magic total graphs, *Ars Combin.*, to appear.
13. W. D. Wallis, E. T. Baskoro, M. Miller and Slamin, Edge-magic total labellings, *Australasian J. Combin.* 22 (2000), 177-190.