The intersection problem for disjoint 2-flowers in Steiner triple systems

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Abstract

We give a solution for the intersection problem for disjoint 2-flowers in Steiner triple systems.

1 Preliminaries

A *partial triple system*, $(V, \mathcal{V})$, of order v is a set V of v elements and a collection $\mathcal{V}$ of triples of the elements of V, so that each unordered pair of distinct elements occurs in at most one triple of $\mathcal{V}$. If each element occurs in precisely one triple of $\mathcal{V}$ we say $(V, \mathcal{V})$ is a *Steiner triple system* (or STS).

It is a well known result due to Kirkman [11] that there exists a Steiner triple system of order v if and only if $v \equiv 1, 3 \pmod 6$.

An *m-flower*, $(F, \mathcal{F})$, is a partial triple system where:

$$\mathcal{F} = \{\{x, y_i, z_i\} \mid \{y_i, z_i\} \cap \{y_j, z_j\} = \emptyset, \text{ for } 0 \leq i, j \leq m-1, i \neq j\}; \text{ and}$$

$$F = \bigcup_{X \in \mathcal{F}} X.$$

Let $V^j = \{x_j \mid 0 \leq x \leq l-1\}$, where $j \in \{r, c, s\}$. Then, let $(L, \mathcal{L}) = (V^r \cup V^c \cup V^s, \mathcal{L})$ be a partial triple system of order $3l$, where $|\mathcal{L}| = l^2$ and i, j and k are distinct when $\{x_i, y_j, z_k\} \in \mathcal{L}$. Then we say $(L, \mathcal{L})$ is a *latin square* (or LS) of order l.

Intersection problems between pairs of Steiner triple systems were first considered by Lindner and Rosa [13]. Subsequently, the intersection problem between pairs of Steiner triple systems, $(V, \mathcal{V}_1)$ and $(V, \mathcal{V}_2)$, in which the intersection of $\mathcal{V}_1$ and $\mathcal{V}_2$ is composed of a number of isomorphic copies

JCMCC 75 (2010), pp. 41-63

of some specified partial triple system has also been considered. Mullin, Poplove and Zhu [16] considered the case where the partial triple system in question was a *triangle*. Whereas, Lindner and Hoffman [10] considered pairs of Steiner triple systems of order v intersecting in a $(\frac{v-1}{2})$-flower and some other (possibly empty) set of triples; Chang and Lo Faro [3] considered the same problem for Kirkman triple systems.

The intersection problem for disjoint triples in Steiner triple systems was considered in papers by Chee [4] and Srinivasan [17].

A natural progression is to consider the intersection problem for pairs of Steiner triple systems in which the intersection is composed of a number of disjoint isomorphic copies of some specified partial triple system. This paper solves one such problem, in which the partial triple system in question is a 2-flower.

Intersection problems have also been considered with respect to latin squares, initialy by Fu [8]. In 1990 C.M. Fu and H.L. Fu [5] gave a survey paper on the topic. Subsequently, further papers have been written on the subject in [9], [6], [1], [2] and [7].

We note the following results, in regards to intersection problems for pairs of latin squares, from [14] and [15].

Lemma 1.1. *(Donovan, Lefevre and McCourt [14]) There exists a pair of latin squares of order $v \geq 6$ whose intersection is composed precisely of i disjoint 2-flowers where $0 \leq i \leq \lfloor \frac{3v}{5} \rfloor$. Furthermore there does not exist a pair of latin squares of order v that intersect precisely in $i > \lfloor \frac{3v}{5} \rfloor$ disjoint m-flowers.*

Lemma 1.2. *(Donovan, Lefevre and McCourt [14]) Assume $l \equiv 1 \pmod 5$ and $16 \leq l$. There exists a pair of latin squares of order l that intersect precisely in $\lfloor \frac{3l}{5} \rfloor$ disjoint 2-flowers and one additional triple.*

Lemma 1.3. *(McCourt [15]) Assume $l \equiv 3 \pmod 5$ and $l \neq 13$. There exists a pair of latin squares of order l that intersect precisely in $\lfloor \frac{3l}{5} \rfloor$ disjoint 2-flowers and one additional disjoint triple.*

Let $(L, \mathcal{L}_a)$ and $(L, \mathcal{L}_b)$ be a pair of latin squares that satisfy Lemma 1.3. There is one element in L that is not contained in any of the triples in the intersection. Without loss of generality this element is $(l-2)_s$.

Lemma 1.4. *(Donovan, Lefevre and McCourt [14]) Assume $l \equiv 4 \pmod 5$ and $l \geq 19$ then there exists a pair of latin squares of order l that intersect precisely in $\lfloor \frac{3l}{5} \rfloor$ disjoint 2-flowers. Furthermore, none of these 2-flowers contain $(l-2)_s$ or $(l-1)_s$.*

Now, we will consider intersection problems for pairs of Steiner triple systems. The following theorem was proved by Lindner [12].

Theorem 1.5. *(Lindner [12]) Let* $(V, \mathcal{V}_1)$ *and* $(V, \mathcal{V}_2)$ *be partial Steiner triple systems of order* v*. Then for every* $u \equiv 1,3 \pmod 6$ *and* $u \geq 6v + 1$ *there exists a pair of Steiner triple systems* $(U, \mathcal{U}_1)$ *and* $(U, \mathcal{U}_2)$ *of order* u *such that* $\mathcal{U}_1 \cap \mathcal{U}_2 = \mathcal{V}_1 \cap \mathcal{V}_2$.

Combining Theorem 1.5 with pairs of Steiner triple systems from the Appendix we achieve the following corollary.

Corollary 1.6. *Let* $(V, \mathcal{V})$ *be a Steiner triple system of order seven. For every* $u \equiv 3 \pmod 6$ *and* $u \geq 15$ *there exists a pair of Steiner triple systems* $(U, \mathcal{U}_1)$ *and* $(U, \mathcal{U}_2)$ *of order* u *such that* $\mathcal{U}_1 \cap \mathcal{U}_2 = \mathcal{V}$.

Let $u, v \equiv 1,3 \pmod 6$ and $v < u$. Assume there exists a Steiner triple system $(U, \mathcal{U})$ of order u and a Steiner triple system $(V, \mathcal{V})$ of order v such that $V \subset U$ and $\mathcal{V} \subset \mathcal{U}$. Then we call the partial Steiner triple system $(U, \mathcal{U} \setminus \mathcal{V})$ a Steiner triple system of order u with a *hole* V of size v and denote it by $(U_V, \mathcal{U})$.

We now rewrite Corollary 1.6 in terms of holes to arrive at the following lemma.

Lemma 1.7. *For every* $u \equiv 3 \pmod 6$ *and* $u \geq 15$ *there exists a pair of Steiner triple systems, namely* $(U_V, \mathcal{U}_1)$ *and* $(U_V, \mathcal{U}_2)$ *of order* u*, both with a hole* V *of size seven such that* $\mathcal{U}_1 \cap \mathcal{U}_2 = \emptyset$.

We also note that Lindner and Rosa [13] proved (in the context of a much stronger result) the following lemma.

Lemma 1.8. *(Lindner and Rosa [13]) Let* $v \equiv 1,3 \pmod 6$ *and* $|V| = v$*. There exists a pair of Steiner triple systems* $(V, \mathcal{V}_1)$ *and* $(V, \mathcal{V}_2)$*, where* $|V| \neq 3$*, such that* $\mathcal{V}_1 \cap \mathcal{V}_2 = \emptyset$*. Furthermore, there exists a pair of Steiner triple systems* $(V, \mathcal{V}_3)$ *and* $(V, \mathcal{V}_4)$*, where* $|V| \neq 1$*, such that* $|\mathcal{V}_3 \cap \mathcal{V}_4| = 1$.

Using the above, we can rewrite the second statement in Lemma 1.8 in terms of holes to arrive at the following lemma.

Lemma 1.9. *For every* $u \equiv 1,3 \pmod 6$ *and* $u \geq 3$ *there exists a pair of Steiner triple systems, namely* $(U_V, \mathcal{U}_1)$ *and* $(U_V, \mathcal{U}_2)$ *of order* u*, both with a hole* V *of size three such that* $\mathcal{U}_1 \cap \mathcal{U}_2 = \emptyset$.

At this point we note another result due to Lindner and Rosa [13].

Lemma 1.10. *(Lindner and Rosa [13]) There does not exist a pair of Steiner triple systems of order seven that intersect precisely in two triples.*

Hence, there does not exist a pair of Steiner triple systems of order seven that intersect precisely in one 2-flower.

Note 1.11. *Let $u \equiv 1, 3 \,(mod\, 6)$. Assume there are k disjoint 2-flowers in a Steiner triple system $(U, \mathcal{U})$ of order u. The triples of each 2-flower contain five distinct elements of U, consequently, $\mathcal{U}$ contains a maximum of $\lfloor \frac{u}{5} \rfloor$ disjoint 2-flowers.*

Consequently, the possible values of i such that there exists a pair of Steiner triple systems of order v that intersect precisely in i disjoint 2-flowers is the set $J(v) = \{i \mid 0 \leq i \leq \lfloor \frac{v}{5} \rfloor\}$ where $v \neq 7$ and $J(7) = 0$.

In this paper we will prove the following result.

Theorem 1. *Let $u \equiv 1, 3 \,(mod\, 6)$, then there exists a pair of Steiner triple systems which intersect precisely in i disjoint 2-flowers where $0 \leq i \leq \lfloor \frac{u}{5} \rfloor$ when $u \neq 7$ and $i = 0$ when $u = 7$. Furthermore, there does not exist a pair of Steiner triple systems of order u that intersect precisely in $x > \lfloor \frac{u}{5} \rfloor$ disjoint 2-flowers.*

The sufficient conditions for the proof of Theorem 1 are proved in the remainder of this paper. To aid the reader we now provide the following table (Table 1) indicating the lemmas used to establish our sufficient conditions.

Table 1: Sufficient Conditions

Case	**Lemma**
Pairs of STS of small order	Appendix
Pairs of STS intersecting in zero 2-flowers	Lemma 1.12
Pairs of STS of order $u \equiv 3, 9 \,(\text{mod } 18)$	Lemma 3.1
Pairs of STS of order $u \equiv 1, 7 \,(\text{mod } 18)$	Lemmas 3.3 and 3.4
Pairs of STS of order $u \equiv 15 \,(\text{mod } 18)$	Lemmas 3.5, 3.6 and 3.7
Pairs of STS of order $u \equiv 13 \,(\text{mod } 18)$	Lemmas 3.9 and 3.10

By Lemma 1.8 we have the following result.

Lemma 1.12. *Let $u \equiv 1, 3 \,(mod\, 6)$ and $u \neq 3$. There exists a pair of Steiner triple systems intersecting precisely in zero disjoint 2-flowers.*

At this point, we establish the notation used in the following sections. Let $V^j = \{x_j \mid 0 \leq x \leq v - 1\}$, where $j \in \{r, c, s\}$. We establish the notation for pairs of Steiner triples systems, Steiner triples systems with holes and latin squares in Table 2, 3 and 4 respectively.

Table 2: Ingredient pairs of Steiner triple systems

Pair of STS	Existence ($\|V\| = v$)	Intersection	Proof
$(V^j, \mathcal{V}_1^j)$, $(V^j, \mathcal{V}_2^j)$	$v \equiv 1,3 \pmod 6$ and $v \neq 3$	$\mathcal{V}_1^j \cap \mathcal{V}_2^j = \emptyset$	Lemma 1.8
$(V^j, \mathcal{V}_3^j)$, $(V^j, \mathcal{V}_4^j)$	$v \equiv 1,3 \pmod 6$ and $v \neq 1$	$\mathcal{V}_3^j \cap \mathcal{V}_4^j = \{\{(v-3)_j, (v-2)_j, (v-1)_j\}\}$	Lemma 1.8
$(V^j, \mathcal{V}_5^j)$, $(V^j, \mathcal{V}_6^j)$	$v \equiv 1,3 \pmod 6$ and $v \neq 1$	$\mathcal{V}_5^j \cap \mathcal{V}_6^j = \{\{(v-4)_j, (v-3)_j, (v-2)_j\}\}$	Lemma 1.8
$(V^j, \mathcal{V}_7^j)$, $(V^j, \mathcal{V}_8^j)$	$v \equiv 1 \pmod 6$ and $v \geq 13$	$\mathcal{V}_7^j \cap \mathcal{V}_8^j = \{\{(v-5)_j, (v-4)_j, (v-3)_j\}, \{(v-3)_j, (v-2)_j, (v-1)_j\}\}$	Lemmas 3.3, 3.9 and Appendix
$(V^j, \mathcal{V}_9^j)$, $(V^j, \mathcal{V}_{10}^j)$	$v \equiv 3 \pmod 6$ and $v \neq 3$	$\mathcal{V}_9^j \cap \mathcal{V}_{10}^j = \{\{(v-5)_j, (v-4)_j, (v-3)_j\}, \{(v-3)_j, (v-2)_j, (v-1)_j\}\}$	Lemmas 3.1, 3.5 and Appendix
$(V^j, \mathcal{V}_{11}^j)$, $(V^j, \mathcal{V}_{12}^j)$	$v \equiv 3 \pmod 6$ and $v > 15$	$\mathcal{V}_{11}^j \cap \mathcal{V}_{12}^j = \{\{(v-8)_j, (v-7)_j, (v-6)_j\}, \{(v-5)_j, (v-4)_j, (v-3)_j\}, \{(v-3)_j, (v-2)_j, (v-1)_j\}\}$	Lemmas 3.2 and 3.8

Table 3: Ingredient pairs of Steiner triple systems with holes

Pair of STS	Existence ($\|V\| = v$)	Intersection	Proof
$(V^j_{W^j}, \mathcal{V}_{13})$, $(V^j_{W^j}, \mathcal{V}_{14})$	$v \equiv 1,3 \pmod 6$, $v \geq 3$ and $W^j = \{(v-3)_j, (v-2)_j, (v-1)_j\}$	$\mathcal{V}_{13}^j \cap \mathcal{V}_{14}^j = \emptyset$	Lemma 1.9
$(V^j_{W^j}, \mathcal{V}_{15}^j)$, $(V^j_{W^j}, \mathcal{V}_{16}^j)$	$v \equiv 3 \pmod 6$, $v \geq 15$ and $W^j = \{(v-7)_j, (v-6)_j, (v-5)_j, (v-4)_j, (v-3)_j, (v-2)_j, (v-1)_j\}$	$\mathcal{V}_{15}^j \cap \mathcal{V}_{16}^j = \emptyset$	Lemma 1.7

Table 4: Ingredient pairs of Latin Squares

Pair of LS	Existence ($\|L\| = 3l$)	Intersection	Proof
$(L, \mathcal{L}_1)$, $(L, \mathcal{L}_2)$	$l \geq 6$	$\mathcal{L}_1 \cap \mathcal{L}_2$ in $0 \leq g \leq \lfloor \frac{3l}{5} \rfloor$ disjoint 2-flowers	Lemma 1.1
$(L, \mathcal{L}_3)$, $(L, \mathcal{L}_4)$	$l \geq 6$	$\mathcal{L}_3 \cap \mathcal{L}_4 = \{\{0_r, 0_c, 0_s\}, \{0_r, 1_c, 1_s\}\}$	Lemma 1.1
$(L, \mathcal{L}_5)$, $(L, \mathcal{L}_6)$	$l \equiv 1, 3 (\text{mod } 5)$ and $l \geq 16$ or $l = 8$	$\mathcal{L}_5 \cap \mathcal{L}_6$ in $\lfloor \frac{3l}{5} \rfloor$ disjoint 2-flowers and the disjoint triple $\{(l-1)_r, (l-1)_c, (l-1)_s\}$ and if $l \equiv 3 \,(\text{mod } 5)$ none of the 2-flowers contain $(l-2)_s$	Lemmas 1.2 and 1.3
$(L, \mathcal{L}_7)$, $(L, \mathcal{L}_8)$	$l \equiv 4 \,(\text{mod } 5)$ and $l \geq 19$	$\mathcal{L}_7 \cap \mathcal{L}_8$ in $\lfloor \frac{3l}{5} \rfloor$ disjoint 2-flowers none of which contain $(l-1)_s$ or $(l-2)_s$	Lemma 1.4

2 Steiner triple system constructions

There are four constructions we will make extensive use of to prove Theorem 1.

Construction 2.1. *For $v \equiv 1$ or $3 \,(mod\, 6)$. Let $(V^r \cup V^c \cup V^s, \mathcal{L})$ be a latin square of order v and let $(V^r, \mathcal{A})$, $(V^c, \mathcal{B})$ and $(V^s, \mathcal{C})$ be Steiner triple systems. Then, $(V_r \cup V^c \cup V^s, \mathcal{L} \cup \mathcal{A} \cup \mathcal{B} \cup \mathcal{C})$ is a Steiner triple system of order $3v \equiv 3, 9 \,(mod\, 18)$.*

Construction 2.2. *For $v \equiv 1$ or $3 \,(mod\, 6)$, let $V^{j\prime} = V^j \setminus \{v - 1_j\}$, where $j \in \{r, c, s\}$ and $(v-1)_r = (v-1)_c = (v-1)_s$. Let $(V^{r\prime} \cup V^{c\prime} \cup V^{s\prime}, \mathcal{L})$ be a latin square of order $v - 1$ and let $(V^r, \mathcal{A})$, $(V^c, \mathcal{B})$ and $(V^s, \mathcal{C})$ be Steiner triple systems. Then, $(V^r \cup V^c \cup V^s, \mathcal{L} \cup \mathcal{A} \cup \mathcal{B} \cup \mathcal{C})$ is a Steiner triple system of order $3v - 2 \equiv 1, 7 \,(mod\, 18)$.*

Construction 2.3. *For $v \equiv 1 \,(mod\, 6)$, let $W = \{v-3, v-2, v-1\}$, $W^j = \{x_j \mid x \in W\}$ and $V^{j\prime} = V^j \setminus W^j$, where $j \in \{r, c, s\}$, $x_r = x_c = x_s$ for all $x \in W$ and the common entry is denoted by x. Let $(V^{r\prime} \cup V^{c\prime} \cup V^{s\prime}, \mathcal{L})$ be a latin square of order $v - 3$ and let $(V^r_{W^r}, \mathcal{A})$ and $(V^c_{W^c}, \mathcal{B})$ be Steiner triple systems with holes of size three and $(V^s, \mathcal{C})$ be a Steiner triple system. Then, $(V^r \cup V^c \cup V^s, \mathcal{L} \cup \mathcal{A} \cup \mathcal{B} \cup \mathcal{C})$ is a Steiner triple system of order $3v - 6 \equiv 15 \,(mod\, 18)$.*

Construction 2.4. *For $v \equiv 3\,(mod\ 6)$, let $W = \{v-7, v-6, v-5, v-4, v-3, v-2, v-1\}$, $W^j = \{x_j \mid x \in W\}$ and $V^{j'} = V^j \setminus W^j$, where $j \in \{r, c, s\}$, $x_r = x_c = x_s$ for all $x \in W$ and the common entry is denoted by x. Let $(V^{r'} \cup V^{c'} \cup V^{s'}, \mathcal{L})$ be a latin square of order $v-7$ and let $(V^r_{W^r}, \mathcal{A})$ and $(V^c_{W^c}, \mathcal{B})$ be Steiner triple systems with holes of size seven and $(V^s, \mathcal{C})$ be a Steiner triple system. Then, $(V^r \cup V^c \cup V^s, \mathcal{L} \cup \mathcal{A} \cup \mathcal{B} \cup \mathcal{C})$ is a Steiner triple system of order $3v - 14 \equiv 13\,(mod\ 18)$.*

Note 2.5. *When $(V^r \cup V^c \cup V^s, \mathcal{L}_1 \cup \mathcal{A}_1 \cup \mathcal{B}_1 \cup \mathcal{C}_1)$ and $(V^r \cup V^c \cup V^s, \mathcal{L}_2 \cup \mathcal{A}_2 \cup \mathcal{B}_2 \cup \mathcal{C}_2)$ are two Steiner triple systemsconstructed as above, the intersection of the triples is clearly the union of the following sets $\mathcal{L}_1 \cap \mathcal{L}_2$, $\mathcal{A}_1 \cap \mathcal{A}_2$, $\mathcal{B}_1 \cap \mathcal{B}_2$ and $\mathcal{C}_1 \cap \mathcal{C}_2$.*

3 Proof of Theorem 1

3.1 $u \equiv 3, 9\,(\text{mod } 18)$

Lemma 3.1. *Let $u \equiv 3, 9\,(mod\ 18)$ and $u \geq 21$. There exists a pair of Steiner triple systems of order u which intersect precisely in g disjoint 2-flowers, where $0 \leq g \leq \lfloor \frac{u}{5} \rfloor$.*

Proof. Let $i \in \{1, 2\}$, $j \in \{r, c, s\}$, $(V^j, \mathcal{V}^j_i)$ as given in Table 2 and $(L, \mathcal{L}_i)$ as given in Table 4. We use Construction 2.1 where $\mathcal{A} = \mathcal{V}^r_i$, $\mathcal{B} = \mathcal{V}^c_i$, $\mathcal{C} = \mathcal{V}^s_i$ and $\mathcal{L} = \mathcal{L}_i$ to construct the Steiner triple system $(U, \mathcal{U}_i)$. By Note 2.5, $(U, \mathcal{U}_1)$ and $(U, \mathcal{U}_2)$ intersect precisely in g disjoint 2-flowers where $0 \leq g \leq \lfloor \frac{3v}{5} \rfloor$. □

The following lemma will be used in Subsection 3.4.

Lemma 3.2. *Let $u \equiv 3, 9\,(mod\ 18)$ and $u \geq 21$. There exists a pair of Steiner triple systems of order u which intersect precisely in one 2-flower and one additional disjoint triple.*

Proof. Let $i \in \{3, 4\}$, $j \in \{r, c\}$, $(V^j, \mathcal{V}^j_{i-2})$, $(V^s, \mathcal{V}^s_i)$ as in Table 2 and $(L, \mathcal{L}_i)$ as in Table 4. We use Construction 2.1 where $\mathcal{A} = \mathcal{V}^r_{i-2}$, $\mathcal{B} = \mathcal{V}^c_{i-2}$, $\mathcal{C} = \mathcal{V}^s_i$ and $\mathcal{L} = \mathcal{L}_i$ to construct the Steiner triple system $(U, \mathcal{U}_i)$. By Note 2.5, $\mathcal{U}_3 \cap \mathcal{U}_4 = \{\{0_r, 0_c, 0_s\}, \{0_r, 1_c, 1_s\}, \{(v-3)_s, (v-2)_s, (v-1)_s\}\}$ one 2-flower and one additional disjoint triple. □

3.2 $u \equiv 1, 7\,(\text{mod } 18)$

Lemma 3.3. *Let $u \equiv 1, 7\,(mod\ 18)$ and $u \geq 19$. There exists a pair of Steiner triple systems of order u which intersect precisely in g disjoint 2-flowers, where $0 \leq g \leq \lfloor \frac{u-1}{5} \rfloor$.*

Proof. Let $i \in \{5,6\}$, $j \in \{r,c,s\}$, $(V^j, \mathcal{V}^j_{i-4})$ as in Table 2 and $(L, \mathcal{L}_{i-4})$ as in Table 4. We use Construction 2.2 where $\mathcal{A} = \mathcal{V}^r_{i-4}$, $\mathcal{B} = \mathcal{V}^c_{i-4}$, $\mathcal{C} = \mathcal{V}^s_{i-4}$ and $\mathcal{L} = \mathcal{L}_{i-4}$ to construct the Steiner triple system $(U, \mathcal{U}_i)$. By Note 2.5, $(U, \mathcal{U}_5)$ and $(U, \mathcal{U}_6)$ intersect precisely in g disjoint 2-flowers where $0 \le g \le \lfloor \frac{3(v-1)}{5} \rfloor$. □

Note that if $v \equiv 0,1,2,3 \pmod 5$ then $\lfloor \frac{3(v-1)}{5} \rfloor = \lfloor \frac{3(v-1)+1}{5} \rfloor$. However if $v \equiv 4 \pmod 5$ then $\lfloor \frac{3(v-1)}{5} \rfloor + 1 = \lfloor \frac{3(v-1)+1}{5} \rfloor$.

Let $v \equiv 4 \pmod 5$ and $v \equiv 1,3 \pmod 6$. Hence, $v \equiv 9, 19 \pmod{30}$ and therefore $u = 3v - 2 \equiv 25, 55 \pmod{90}$. With this in mind, we prove the following result.

Lemma 3.4. *Let $u \equiv 25, 55 \pmod{90}$. There exists a pair of Steiner triple systems of order u which intersect precisely in $\lfloor \frac{u-1}{5} \rfloor + 1$ disjoint 2-flowers.*

Proof. Let $i \in \{7,8\}$, $j \in \{r,c\}$, $(V^j, \mathcal{V}^j_{i-6})$, $(V^s, \mathcal{V}^s_{i-4})$ as in Table 2 and $(L, \mathcal{L}_{i-2})$ as in Table 4. We use Construction 2.2 where $\mathcal{A} = \mathcal{V}^r_{i-6}$, $\mathcal{B} = \mathcal{V}^c_{i-6}$, $\mathcal{C} = \mathcal{V}^s_{i-4}$ and $\mathcal{L} = \mathcal{L}_{i-2}$ to construct the Steiner triple system $(U, \mathcal{U}_i)$. By Note 2.5, $\mathcal{U}_7$ intersects $\mathcal{U}_8$ precisely in $\lfloor \frac{3(v-1)}{5} \rfloor$ disjoint 2-flowers and the additional disjoint 2-flower $\{\{(v-2)_r, (v-2)_c, (v-2)_s\}, \{(v-3)_s, (v-2)_s, v-1\}\}$. □

3.3 $u \equiv 15 \pmod{18}$

Lemma 3.5. *Let $u \equiv 15 \pmod{18}$ and $u \neq 15$. There exists a pair of Steiner triple systems of order u which intersect precisely in g disjoint 2-flowers where $0 \le g \le \lfloor \frac{u-3}{5} \rfloor\}$.*

Proof. Let $i \in \{9,10\}$, $j \in \{r,c\}$, $(V^s, \mathcal{V}^s_{i-8})$ as in Table 2, $(V^j, \mathcal{V}^j_{i+4})$ as in Table 3 and $(L, \mathcal{L}_{i-8})$ as in Table 4. We use Construction 2.3 where $\mathcal{A} = \mathcal{V}^r_{i+4}$, $\mathcal{B} = \mathcal{V}^c_{i+4}$, $\mathcal{C} = \mathcal{V}^s_{i-8}$ and $\mathcal{L} = \mathcal{L}_{i-8}$ to construct the Steiner triple systems $(U, \mathcal{U}_i)$. By Note 2.5, $\mathcal{U}_9$ intersects $\mathcal{U}_{10}$ precisely in g disjoint 2-flowers where $0 \le g \le \lfloor \frac{3(v-3)}{5} \rfloor$. □

Note that if $v \equiv 0,3 \pmod 5$ then $\lfloor \frac{3(v-3)}{5} \rfloor = \lfloor \frac{3(v-3)+3}{5} \rfloor$. However if $v \equiv$ 1, 2, 4 $\pmod 5$ then $\lfloor \frac{3(v-3)}{5} \rfloor + 1 = \lfloor \frac{3(v-3)+3}{5} \rfloor$.

Let $v \equiv 1,2,4 \pmod 5$ and $v \equiv 1 \pmod 6$. Hence, $v \equiv 7, 19, 31 \pmod{30}$ and therefore $u = 3v - 6 \equiv 15, 51, 87 \pmod{90}$. With this in mind, we prove the following results.

Lemma 3.6. *Let $u \equiv 51, 87 \pmod{90}$. There exists a pair of Steiner triple systems of order u which intersect precisely in $\lfloor \frac{u-3}{5} \rfloor + 1$ disjoint 2-flowers.*

Proof. Let $i \in \{11, 12\}$, $j \in \{r, c\}$, $(V^s, \mathcal{V}^s_{i-6})$ as in Table 2, $(V^j, \mathcal{V}^j_{i+2})$ as in Table 3 and $(L, \mathcal{L}_{i-6})$ as in Table 4. We use Construction 2.3 where $\mathcal{A} = \mathcal{V}^r_{i+2}$, $\mathcal{B} = \mathcal{V}^c_{i+2}$, $\mathcal{C} = \mathcal{V}^s_{i-6}$ and $\mathcal{L} = \mathcal{L}_{i-6}$ to construct the Steiner triple system $(U, \mathcal{U}_i)$. By Note 2.5, $\mathcal{U}_{11}$ intersects $\mathcal{U}_{12}$ precisely in $\lfloor \frac{3(v-3)}{5} \rfloor$ disjoint 2-flowers and the additional disjoint 2-flower $\{\{(v-4)_r, (v-4)_c, (v-4)_s\}, \{(v-4)_s, (v-3)_s, v-2\}\}$. □

Lemma 3.7. *Let $u \equiv 15 \,(mod\, 90)$ where $u > 15$. There exists a pair of Steiner triple systems of order u which intersect precisely in $\lfloor \frac{u-3}{5} \rfloor + 1$ disjoint 2-flowers.*

Proof. Let $i \in \{13, 14\}$, $j \in \{r, c\}$, $(V^s, \mathcal{V}^s_{i-6})$ as in Table 2, $(V^j, \mathcal{V}^j_i)$ as in Table 3 and $(L, \mathcal{L}_{i-6})$ as in Table 4. We use Construction 2.3 where $\mathcal{A} = \mathcal{V}^r_i$, $\mathcal{B} = \mathcal{V}^c_i$, $\mathcal{C} = \mathcal{V}^s_{i-6}$ and $\mathcal{L} = \mathcal{L}_{i-6}$ to construct the Steiner triple system $(U, \mathcal{U}_i)$. By Note 2.5, $\mathcal{U}_{13}$ intersects $\mathcal{U}_{14}$ precisely in $\lfloor \frac{3(v-3)}{5} \rfloor$ disjoint 2-flowers and the additional disjoint 2-flower $\{\{(v-5)_s, (v-4)_s, (v-3)_s\}, \{(v-3)_s, (v-2)_s, v-1\}\}$. □

We now extend the result in Lemma 3.2 to include the congruency class $u \equiv 15 \,(\text{mod } 18)$.

Lemma 3.8. *Let $u \equiv 15 \,(mod\, 18)$ and $u > 15$. There exists a pair of Steiner triple systems of order u which intersect precisely in one 2-flower and one additional disjoint triple.*

Proof. Let $i \in \{15, 16\}$, $j \in \{r, c\}$, $(V^s, \mathcal{V}^s_{i-12})$ as in Table 2, $(V^j, \mathcal{V}^j_{i-2})$ as in Table 3 and $(L, \mathcal{L}_{i-12})$ as in Table 4. We use Construction 2.3 where $\mathcal{A} = \mathcal{V}^r_{i-2}$, $\mathcal{B} = \mathcal{V}^c_{i-2}$, $\mathcal{C} = \mathcal{V}^s_{i-12}$ and $\mathcal{L} = \mathcal{L}_{i-12}$ to construct the Steiner triple systems $(U, \mathcal{U}_i)$. By Note 2.5, $\mathcal{U}_{15} \cap \mathcal{U}_{16} = \{\{0_r, 0_c, 0_s\}, \{0_r, 1_c, 1_s\}, \{(v-3)_s, (v-2)_s, (v-1)_s\}\}$ one 2-flower and one additional disjoint triple. □

3.4 $u \equiv 13 \,(\text{mod } 18)$

Lemma 3.9. *Let $u \equiv 13 \,(mod\, 18)$ and $13 < u$. There exists a pair of Steiner triple systems of order u which intersect precisely in $g + 1$ disjoint 2-flowers where $0 \le g \le \lfloor \frac{u-7}{5} \rfloor$.*

Proof. Let $i \in \{17, 18\}$, $j \in \{r, c\}$, $(V^s, \mathcal{V}^s_{i-8})$ as in Table 2, $(V^j, \mathcal{V}^j_{i-2})$ as in Table 3 and $(L, \mathcal{L}_{i-16})$ as in Table 4. We use Construction 2.4 where $\mathcal{A} = \mathcal{V}^r_{i-2}$, $\mathcal{B} = \mathcal{V}^c_{i-2}$, $\mathcal{C} = \mathcal{V}^s_{i-8}$ and $\mathcal{L} = \mathcal{L}_{i-16}$ to construct the Steiner triple systems $(U, \mathcal{U}_i)$. By Note 2.5, $\mathcal{U}_{17}$ intersects $\mathcal{U}_{18}$ precisely in $0 \le g \le \lfloor \frac{3(v-7)}{5} \rfloor$ disjoint 2-flowers and the additional disjoint 2-flower $\{\{(v-5)_s, (v-4)_s, (v-3)_s\}, \{(v-3)_s, (v-2)_s, v-1\}\}$. □

Note that if $v \equiv 1,2,4 \pmod 5$ then $\lfloor \frac{3(v-7)}{5} \rfloor + 1 = \lfloor \frac{3(v-7)+7}{5} \rfloor$. However if $v \equiv 0,3 \pmod 5$ then $\lfloor \frac{3(v-7)}{5} \rfloor + 2 = \lfloor \frac{3(v-7)}{5} \rfloor$.

Let $v \equiv 0,3 \pmod 5$ and $v \equiv 3 \pmod 6$. Hence, $v \equiv 3,15 \pmod{30}$ and therefore $u = 3v - 14 \equiv 31,85 \pmod{90}$. With this in mind we prove the following result.

Lemma 3.10. *Let $u \equiv 31,85\ (mod\ 90)$. There exists a pair of Steiner triple systems of order u which intersect precisely in $\lfloor \frac{u}{5} \rfloor$ disjoint 2-flowers.*

Proof. Let $i \in \{19,20\}$, $j \in \{r,c\}$, $(V^s, \mathcal{V}^s_{i-8})$ as in Table 2, $(V^j, \mathcal{V}^j_{i-4})$ as in Table 3 and $(L, \mathcal{L}_{i-14})$ as in Table 4. We use Construction 2.4 where $\mathcal{A} = \mathcal{V}^r_{i-4}$, $\mathcal{B} = \mathcal{V}^c_{i-4}$, $\mathcal{C} = \mathcal{V}^s_{i-8}$ and $\mathcal{L} = L_{i-14}$ to construct the Steiner triple system $(U, \mathcal{U}_i)$. By Note 2.5, $\mathcal{U}_{19}$ intersects $\mathcal{U}_{20}$ precisely in $\lfloor \frac{3(v-7)}{5} \rfloor$ disjoint 2-flowers and the additional two disjoint 2-flowers $\{\{(v-8)_r, (v-8)_c, (v-8)_s\}, \{(v-8)_s, (v-7)_s, (v-6)_s\}\}$ and $\{\{(v-5)_s, (v-4)_s, (v-3)_s\}, \{(v-3)_s, (v-2)_s, v-1\}\}$. □

All the minor results required for the proof of Theorem 1 have now been established.

Theorem 1. *Let $u \equiv 1,3\ (mod\ 6)$, then there exists a pair of Steiner triple systems which intersect precisely in i disjoint 2-flowers where $0 \le i \le \lfloor \frac{u}{5} \rfloor$ when $u \ne 7$ and $i = 0$ when $u = 7$. Furthermore, there does not exist a pair of Steiner triple systems of order u that intersect precisely in $i > \lfloor \frac{u}{5} \rfloor$ disjoint 2-flowers.*

Proof. This result follows directly from Lemmas 1.12, 3.1, 3.3, 3.4, 3.9, 3.6, 3.7 and 3.10 and the pairs of Steiner triple systems provided in the Appendix. □

4 Acknowledgements

The author would like to thank Diane Donovan and Chris Rodger for their help and insight. The author would also like to thank Şule Yazıcı for the use of her code.

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Appendix

A pair of Steiner triple systems of order 9 that intersect precisely in one 2-flower.

$$\left\{\begin{array}{llll}
\mathbf{\{0,1,2\}}, & \{0,5,7\}, & \{1,5,6\}, & \{2,5,8\}, \\
\mathbf{\{0,3,6\}}, & \{1,3,8\}, & \{2,3,7\}, & \{3,4,5\}, \\
\{0,4,8\}, & \{1,4,7\}, & \{2,4,6\}, & \{6,7,8\}
\end{array}\right\}$$

$$\left\{\begin{array}{llll}
\mathbf{\{0,1,2\}}, & \{0,5,8\}, & \{1,6,7\}, & \{2,6,8\}, \\
\mathbf{\{0,3,6\}}, & \{1,3,5\}, & \{2,3,4\}, & \{3,7,8\}, \\
\{0,4,7\}, & \{1,4,8\}, & \{2,5,7\}, & \{4,5,6\}
\end{array}\right\}$$

A pair of Steiner triple systems of order 13 that intersect precisely in one 2-flower.

$$\left\{\begin{array}{llll}
\mathbf{\{0,1,2\}}, & \{1,4,7\}, & \{2,6,12\}, & \{4,8,12\}, \\
\mathbf{\{0,3,4\}}, & \{2,3,8\}, & \{2,9,10\}, & \{5,8,9\}, \\
\{0,5,6\}, & \{1,5,12\}, & \{3,5,11\}, & \{6,7,8\}, \\
\{0,9,12\}, & \{1,8,10\}, & \{3,7,9\}, & \{6,10,11\}, \\
\{0,7,10\}, & \{1,9,11\}, & \{3,10,12\}, & \{7,11,12\} \\
\{0,8,11\}, & \{2,4,11\}, & \{4,5,10\}, & \\
\{1,3,6\}, & \{2,5,7\}, & \{4,6,9\}, &
\end{array}\right\}$$

$$\left\{\begin{array}{llll}
\mathbf{\{0,1,2\}}, & \{1,4,8\}, & \{2,7,8\}, & \{4,11,12\}, \\
\mathbf{\{0,3,4\}}, & \{2,4,9\}, & \{2,10,12\}, & \{5,8,12\}, \\
\{0,5,9\}, & \{1,6,11\}, & \{3,6,8\}, & \{5,10,11\}, \\
\{0,6,12\}, & \{1,7,12\}, & \{3,7,10\}, & \{6,7,9\}, \\
\{0,7,11\}, & \{1,9,10\}, & \{3,9,12\}, & \{8,9,11\} \\
\{0,8,10\}, & \{2,3,11\}, & \{4,5,7\}, & \\
\{1,3,5\}, & \{2,5,6\}, & \{4,6,10\}, &
\end{array}\right\}$$

A pair of Steiner triple systems of order 13 that intersect precisely in two disjoint 2-flowers.

$$\left\{\begin{array}{llll}
\mathbf{\{0,1,2\}}, & \{0,11,12\}, & \{2,4,8\}, & \{4,6,11\}, \\
\mathbf{\{0,3,4\}}, & \{1,3,7\}, & \{2,5,12\}, & \{4,7,12\}, \\
\mathbf{\{5,6,7\}}, & \{1,4,5\}, & \{2,7,10\}, & \{4,9,10\}, \\
\mathbf{\{5,8,9\}}, & \{1,6,9\}, & \{2,9,11\}, & \{6,10,12\}, \\
\{0,5,10\}, & \{1,8,12\}, & \{3,5,11\}, & \{7,8,11\} \\
\{0,6,8\}, & \{1,10,11\}, & \{3,8,10\}, & \\
\{0,7,9\}, & \{2,3,6\}, & \{3,9,12\}, &
\end{array}\right\}$$

$$\left\{\begin{array}{llll}
\mathbf{\{0,1,2\}}, & \{0,11,9\}, & \{2,3,5\}, & \{4,5,10\}, \\
\mathbf{\{0,3,4\}}, & \{2,11,12\}, & \{2,4,7\}, & \{4,6,11\}, \\
\mathbf{\{5,6,7\}}, & \{1,3,10\}, & \{2,6,8\}, & \{4,9,12\}, \\
\mathbf{\{5,8,9\}}, & \{1,4,8\}, & \{2,9,10\}, & \{7,10,11\}, \\
\{0,5,12\}, & \{1,5,11\}, & \{3,6,9\}, & \{8,10,12\} \\
\{0,6,10\}, & \{1,6,12\}, & \{3,7,12\}, & \\
\{0,7,8\}, & \{1,7,9\}, & \{3,8,11\}, &
\end{array}\right\}$$

A pair of Steiner triple systems of order 15 that intersect precisely in one 2-flower.

$$\left\{\begin{array}{llll}
\mathbf{\{0,1,2\}}, & \{5,4,10\}, & \{2,14,10\}, & \{5,6,13\}, \\
\mathbf{\{2,3,4\}}, & \{5,7,2\}, & \{14,9,11\}, & \{6,12,0\}, \\
\{0,4,9\}, & \{5,8,9\}, & \{9,10,12\}, & \{13,1,4\}, \\
\{0,7,10\}, & \{6,1,10\}, & \{10,11,13\}, & \{1,14,7\}, \\
\{0,8,11\}, & \{6,4,11\}, & \{11,12,2\}, & \{14,4,8\}, \\
\{3,1,9\}, & \{6,7,9\}, & \{13,0,3\}, & \{4,7,12\}, \\
\{3,7,11\}, & \{6,8,2\}, & \{0,14,5\}, & \{7,8,13\}, \\
\{3,8,10\}, & \{12,13,14\}, & \{14,3,6\}, & \{8,12,1\} \\
\{5,1,11\}, & \{13,2,9\}, & \{3,5,12\}, &
\end{array}\right\}$$

$$\left\{\begin{array}{llll}
\mathbf{\{0,1,2\}}, & \{5,4,9\}, & \{13,2,11\}, & \{5,6,14\}, \\
\mathbf{\{2,3,4\}}, & \{5,7,11\}, & \{2,14,12\}, & \{6,12,3\}, \\
\{0,4,11\}, & \{5,8,2\}, & \{14,9,13\}, & \{14,1,4\}, \\
\{0,7,9\}, & \{6,1,9\}, & \{9,10,2\}, & \{13,4,7\}, \\
\{0,8,10\}, & \{6,4,10\}, & \{11,10,14\}, & \{13,1,8\}, \\
\{3,1,11\}, & \{6,7,2\}, & \{14,0,3\}, & \{1,7,12\}, \\
\{3,7,10\}, & \{6,8,11\}, & \{13,3,5\}, & \{7,8,14\}, \\
\{3,8,9\}, & \{12,9,11\}, & \{13,0,6\}, & \{8,12,4\} \\
\{5,1,10\}, & \{12,13,10\}, & \{0,5,12\}, &
\end{array}\right\}$$

A pair of Steiner triple systems of order 15 that intersect precsiely in 2 disjoint 2-flowers.

$$\left\{\begin{array}{llll}
\mathbf{\{0,1,2\}}, & \{1,3,14\}, & \{2,6,14\}, & \{4,8,12\}, \\
\mathbf{\{0,3,4\}}, & \{1,4,5\}, & \{2,7,12\}, & \{4,9,14\}, \\
\mathbf{\{5,6,7\}}, & \{1,6,9\}, & \{2,8,13\}, & \{5,12,14\}, \\
\mathbf{\{5,8,9\}}, & \{1,7,8\}, & \{3,5,13\}, & \{6,10,13\}, \\
\{0,5,10\}, & \{1,10,11\}, & \{3,6,12\}, & \{7,10,14\}, \\
\{0,6,8\}, & \{1,12,13\}, & \{3,7,11\}, & \{8,11,14\}, \\
\{0,7,9\}, & \{2,3,9\}, & \{3,8,10\}, & \{9,10,12\}, \\
\{0,11,12\}, & \{2,4,10\}, & \{4,6,11\}, & \{9,11,13\} \\
\{0,13,14\}, & \{2,5,11\}, & \{4,7,13\}, &
\end{array}\right\}$$

{0,1,2},	{0,9,13},	{2,5,13},	{4,9,10},
{0,3,4},	{1,3,5},	{2,6,9},	{4,11,13},
{5,6,7},	{2,11,12},	{2,7,10},	{5,10,14},
{5,8,9},	{1,4,6},	{3,6,11},	{6,8,12},
{0,6,10},	{1,7,11},	{3,7,9},	{6,13,14},
{1,9,12},	{1,8,14},	{3,8,13},	{7,12,13},
{0,12,14},	{1,10,13},	{3,10,12},	{8,10,11},
{0,5,11},	{2,3,14},	{4,5,12},	{9,11,14}
{0,7,8},	{2,4,8},	{4,7,14},	

A pair of Steiner triple systems of order 15 that intersect precisely in 3 disjoint 2-flowers.

{0,1,2},	{0,7,9},	{2,3,14},	{4,6,11},
{0,3,4},	{0,11,13},	{2,4,5},	{4,7,10},
{5,6,7},	{0,12,14},	{2,6,12},	{4,8,12},
{5,8,9},	{1,3,5},	{2,8,13},	{4,9,14},
{10,11,12},	{1,4,13},	{2,9,10},	{5,11,14},
{10,13,14},	{1,6,14},	{3,6,10},	{5,12,13},
{2,7,11},	{1,7,12},	{3,7,13},	{6,9,13},
{0,5,10},	{1,8,10},	{3,8,11},	{7,8,14}
{0,6,8},	{1,9,11},	{3,9,12},	

{0,1,2},	{0,7,8},	{2,3,12},	{4,5,10},
{0,3,4},	{0,11,14},	{2,5,11},	{4,6,12},
{5,6,7},	{0,9,12},	{2,6,14},	{4,8,13},
{5,8,9},	{1,3,13},	{2,8,10},	{4,9,11},
{10,11,12},	{1,4,14},	{2,9,13},	{6,11,13},
{10,13,14},	{1,5,12},	{3,5,14},	{7,9,14},
{2,4,7},	{1,6,9},	{3,6,8},	{7,12,13},
{0,5,13},	{1,7,10},	{3,7,11},	{8,12,14}
{0,6,10},	{1,8,11},	{3,9,10},	

A pair of Steiner triple systems of order 15 intersecting in a subsystem of order 7.

{0,7,8},	{2,8,10},	{4,9,12},	{6,10,14},
{0,9,10},	{2,11,13},	{4,11,14},	**{0,1,3},**
{0,11,12},	{2,12,14},	{5,7,12},	**{1,2,4},**
{0,13,14},	{3,7,13},	{5,8,11},	**{2,3,5},**
{1,8,9},	{3,8,14},	{5,9,14},	**{3,4,6},**
{1,10,11},	{3,9,11},	{5,10,13},	**{4,5,0},**
{1,12,13},	{3,10,12},	{6,7,11},	**{5,6,1},**
{1,7,14},	{4,7,10},	{6,8,12},	**{6,0,2}**
{2,7,9},	{4,8,13},	{6,9,13},	

$$\left\{\begin{array}{cccc}
\{0,8,9\}, & \{2,8,14\}, & \{4,9,14\}, & \{6,13,14\}, \\
\{0,10,11\}, & \{2,11,9\}, & \{4,10,13\}, & \mathbf{\{0,1,3\}}, \\
\{0,12,13\}, & \{2,12,10\}, & \{5,7,11\}, & \mathbf{\{1,2,4\}}, \\
\{0,7,14\}, & \{3,7,10\}, & \{5,8,12\}, & \mathbf{\{2,3,5\}}, \\
\{1,7,9\}, & \{3,8,13\}, & \{5,9,13\}, & \mathbf{\{3,4,6\}}, \\
\{1,10,8\}, & \{3,9,12\}, & \{5,10,14\}, & \mathbf{\{4,5,0\}}, \\
\{1,11,13\}, & \{3,11,14\}, & \{6,7,8\}, & \mathbf{\{5,6,1\}}, \\
\{1,12,14\}, & \{4,7,12\}, & \{6,9,10\}, & \mathbf{\{6,0,2\}} \\
\{2,7,13\}, & \{4,8,11\}, & \{6,11,12\}, &
\end{array}\right\}$$

A pair of Steiner triple systems of order 21 intersecting in a subsystem of order 7.

$$\left\{\begin{array}{cccc}
\{0,7,8\}, & \{1,7,20\}, & \{2,7,10\}, & \{3,7,18\}, \\
\{0,9,10\}, & \{1,8,9\}, & \{2,8,19\}, & \{3,8,11\}, \\
\{0,11,12\}, & \{1,10,11\}, & \{2,9,12\}, & \{3,9,20\}, \\
\{0,13,14\}, & \{1,12,13\}, & \{2,11,14\}, & \{3,10,13\}, \\
\{0,15,16\}, & \{1,14,15\}, & \{2,13,16\}, & \{3,12,15\}, \\
\{0,17,18\}, & \{1,16,17\}, & \{2,15,18\}, & \{3,14,17\}, \\
\{0,19,20\}, & \{1,18,19\}, & \{2,17,20\}, & \{3,16,19\}, \\
\{4,7,12\}, & \{5,7,16\}, & \{6,7,14\}, & \mathbf{\{0,1,3\}}, \\
\{4,8,17\}, & \{5,8,13\}, & \{6,8,15\}, & \mathbf{\{1,2,4\}}, \\
\{4,9,14\}, & \{5,9,18\}, & \{6,9,16\}, & \mathbf{\{2,3,5\}}, \\
\{4,10,19\}, & \{5,10,15\}, & \{6,10,17\}, & \mathbf{\{3,4,6\}}, \\
\{4,11,16\}, & \{5,11,20\}, & \{6,11,18\}, & \mathbf{\{4,5,0\}}, \\
\{4,13,18\}, & \{5,12,17\}, & \{6,12,19\}, & \mathbf{\{5,6,1\}}, \\
\{4,15,20\}, & \{5,14,19\}, & \{6,13,20\}, & \mathbf{\{6,0,2\}}
\end{array}\right\}$$

$$\cup \{\{(7+i \pmod{14})+7,\ (9+i \pmod{14})+7,\ (13+i \pmod{14})+7\} \mid 0 \le i \le 13\}$$

$$\left\{\begin{array}{cccc}
\{0,7,17\}, & \{1,7,11\}, & \{2,7,19\}, & \{3,7,14\}, \\
\{0,8,18\}, & \{1,8,12\}, & \{2,8,10\}, & \{3,8,15\}, \\
\{0,9,13\}, & \{1,9,19\}, & \{2,9,16\}, & \{3,9,11\}, \\
\{0,10,20\}, & \{1,10,14\}, & \{2,11,18\}, & \{3,10,17\}, \\
\{0,11,14\}, & \{1,13,15\}, & \{2,12,14\}, & \{3,12,19\}, \\
\{0,12,15\}, & \{1,16,18\}, & \{2,13,20\}, & \{3,13,16\}, \\
\{0,16,19\}, & \{1,17,20\}, & \{2,15,17\}, & \{3,18,20\}, \\
\{4,7,9\}, & \{5,7,10\}, & \{6,7,18\}, & \mathbf{\{0,1,3\}}, \\
\{4,8,20\}, & \{5,8,19\}, & \{6,8,11\}, & \mathbf{\{1,2,4\}}, \\
\{4,10,12\}, & \{5,9,20\}, & \{6,9,12\}, & \mathbf{\{2,3,5\}}, \\
\{4,11,13\}, & \{5,11,15\}, & \{6,10,13\}, & \mathbf{\{3,4,6\}}, \\
\{4,14,16\}, & \{5,12,16\}, & \{6,14,17\}, & \mathbf{\{4,5,0\}}, \\
\{4,15,18\}, & \{5,13,17\}, & \{6,15,19\}, & \mathbf{\{5,6,1\}}, \\
\{4,17,19\}, & \{5,14,18\}, & \{6,16,20\}, & \mathbf{\{6,0,2\}}
\end{array}\right\}$$

$\cup\ \{\{(7+i\,(\mathrm{mod}\ 14))+7,\ (8+i\,(\mathrm{mod}\ 14))+7,\ (13+i\,(\mathrm{mod}\ 14))+7\}\mid 0\le i\le 13\}$

A pair of Steiner triple systems of order 27 intersecting in a subsystem of order 7.

$$\left\{\begin{array}{cccc}
\{0,7,8\}, & \{1,7,26\}, & \{2,7,10\}, & \{3,7,24\}, \\
\{0,9,10\}, & \{1,8,9\}, & \{2,8,25\}, & \{3,8,11\}, \\
\{0,11,12\}, & \{1,10,11\}, & \{2,9,12\}, & \{3,9,26\}, \\
\{0,13,14\}, & \{1,12,13\}, & \{2,11,14\}, & \{3,10,13\}, \\
\{0,15,16\}, & \{1,14,15\}, & \{2,13,16\}, & \{3,12,15\}, \\
\{0,17,18\}, & \{1,16,17\}, & \{2,15,18\}, & \{3,14,17\}, \\
\{0,19,20\}, & \{1,18,19\}, & \{2,17,20\}, & \{3,16,19\}, \\
\{0,21,22\}, & \{1,20,21\}, & \{2,19,22\}, & \{3,18,21\}, \\
\{0,23,24\}, & \{1,22,23\}, & \{2,21,24\}, & \{3,20,23\}, \\
\{0,25,26\}, & \{1,24,25\}, & \{2,23,26\}, & \{3,22,25\}, \\
\{4,7,16\}, & \{5,7,18\}, & \{6,7,17\}, & \mathbf{\{0,1,3\}}, \\
\{4,8,19\}, & \{5,8,17\}, & \{6,8,18\}, & \mathbf{\{1,2,4\}}, \\
\{4,9,18\}, & \{5,9,20\}, & \{6,9,19\}, & \mathbf{\{2,3,5\}}, \\
\{4,10,21\}, & \{5,10,19\}, & \{6,10,20\}, & \mathbf{\{3,4,6\}}, \\
\{4,11,20\}, & \{5,11,22\}, & \{6,11,21\}, & \mathbf{\{4,5,0\}}, \\
\{4,12,23\}, & \{5,12,21\}, & \{6,12,22\}, & \mathbf{\{5,6,1\}}, \\
\{4,13,22\}, & \{5,13,24\}, & \{6,13,23\}, & \mathbf{\{6,0,2\}} \\
\{4,14,25\}, & \{5,14,23\}, & \{6,14,24\}, & \\
\{4,15,24\}, & \{5,15,26\}, & \{6,15,25\}, & \\
\{4,17,26\}, & \{5,16,25\}, & \{6,16,26\}, &
\end{array}\right\}$$

$\cup\ \{\{(7+i\,(\mathrm{mod}\ 20))+7,\ (12+i\,(\mathrm{mod}\ 20))+7,\ (19+i\,(\mathrm{mod}\ 20))+7\}\mid 0\le i\le 19\}$

$\cup\ \{\{(7+i\,(\mathrm{mod}\ 20))+7,\ (9+i\,(\mathrm{mod}\ 20))+7,\ (13+i\,(\mathrm{mod}\ 20))+7\}\mid 0\le i\le 19\}$

$$
\left\{
\begin{array}{cccc}
\{0,7,17\}, & \{1,7,16\}, & \{2,7,18\}, & \{3,7,8\}, \\
\{0,8,18\}, & \{1,8,19\}, & \{2,8,17\}, & \{3,9,10\}, \\
\{0,9,19\}, & \{1,9,18\}, & \{2,9,20\}, & \{3,11,12\}, \\
\{0,10,20\}, & \{1,10,21\}, & \{2,10,19\}, & \{3,13,14\}, \\
\{0,11,21\}, & \{1,11,20\}, & \{2,11,22\}, & \{3,15,16\}, \\
\{0,12,22\}, & \{1,12,23\}, & \{2,12,21\}, & \{3,17,18\}, \\
\{0,13,23\}, & \{1,13,22\}, & \{2,13,24\}, & \{3,19,20\}, \\
\{0,14,24\}, & \{1,14,25\}, & \{2,14,23\}, & \{3,21,22\}, \\
\{0,15,25\}, & \{1,15,24\}, & \{2,15,26\}, & \{3,23,24\}, \\
\{0,16,26\}, & \{1,17,26\}, & \{2,16,25\}, & \{3,25,26\}, \\
\{4,7,26\}, & \{5,7,12\}, & \{6,7,22\}, & \mathbf{\{0,1,3\}}, \\
\{4,8,9\}, & \{5,8,13\}, & \{6,8,23\}, & \mathbf{\{1,2,4\}}, \\
\{4,10,11\}, & \{5,9,14\}, & \{6,9,24\}, & \mathbf{\{2,3,5\}}, \\
\{4,12,13\}, & \{5,10,15\}, & \{6,10,25\}, & \mathbf{\{3,4,6\}}, \\
\{4,14,15\}, & \{5,11,16\}, & \{6,11,26\}, & \mathbf{\{4,5,0\}}, \\
\{4,16,17\}, & \{5,17,22\}, & \{6,12,17\}, & \mathbf{\{5,6,1\}}, \\
\{4,18,19\}, & \{5,18,23\}, & \{6,13,18\}, & \mathbf{\{6,0,2\}} \\
\{4,20,21\}, & \{5,19,24\}, & \{6,14,19\}, & \\
\{4,22,23\}, & \{5,20,25\}, & \{6,15,20\}, & \\
\{4,24,25\}, & \{5,21,26\}, & \{6,16,21\}, &
\end{array}
\right\}
$$

$$\cup \{\{(7+i \pmod{20})+7,\ (9+i \pmod{20})+7,\ (15+i \pmod{20})+7\} \mid 0 \le i \le 19\}$$

$$\cup \{\{(7+i \pmod{20})+7,\ (10+i \pmod{20})+7,\ (14+i \pmod{20})+7\} \mid 0 \le i \le 19\}$$

A pair of Steiner triple systems of order 33 intersecting in a subsystem of order 7.

$$\left\{\begin{array}{llll}
\{0,7,11\}, & \{1,7,29\}, & \{2,7,8\}, & \{3,7,32\}, \\
\{0,8,12\}, & \{1,8,30\}, & \{2,9,10\}, & \{3,8,9\}, \\
\{0,9,13\}, & \{1,9,31\}, & \{2,11,12\}, & \{3,10,11\}, \\
\{0,10,14\}, & \{1,10,32\}, & \{2,13,14\}, & \{3,12,13\}, \\
\{0,15,19\}, & \{1,11,15\}, & \{2,15,16\}, & \{3,14,15\}, \\
\{0,16,20\}, & \{1,12,16\}, & \{2,17,18\}, & \{3,16,17\}, \\
\{0,17,21\}, & \{1,13,17\}, & \{2,19,20\}, & \{3,18,19\}, \\
\{0,18,22\}, & \{1,14,18\}, & \{2,21,22\}, & \{3,20,21\}, \\
\{0,23,27\}, & \{1,19,23\}, & \{2,23,24\}, & \{3,22,23\}, \\
\{0,24,28\}, & \{1,20,24\}, & \{2,25,26\}, & \{3,24,25\}, \\
\{0,25,29\}, & \{1,21,25\}, & \{2,27,31\}, & \{3,26,27\}, \\
\{0,26,30\}, & \{1,22,26\}, & \{2,28,32\}, & \{3,28,29\}, \\
\{0,31,32\}, & \{1,27,28\}, & \{2,29,30\}, & \{3,30,31\}, \\
\{4,7,14\}, & \{5,7,26\}, & \{6,7,20\}, & \mathbf{\{0,1,3\}}, \\
\{4,8,27\}, & \{5,8,15\}, & \{6,8,21\}, & \mathbf{\{1,2,4\}}, \\
\{4,9,16\}, & \{5,9,28\}, & \{6,9,22\}, & \mathbf{\{2,3,5\}}, \\
\{4,10,29\}, & \{5,10,17\}, & \{6,10,23\}, & \mathbf{\{3,4,6\}}, \\
\{4,11,18\}, & \{5,11,30\}, & \{6,11,24\}, & \mathbf{\{4,5,0\}}, \\
\{4,12,31\}, & \{5,12,19\}, & \{6,12,25\}, & \mathbf{\{5,6,1\}}, \\
\{4,13,20\}, & \{5,13,32\}, & \{6,13,26\}, & \mathbf{\{6,0,2\}} \\
\{4,15,22\}, & \{5,14,21\}, & \{6,14,27\}, & \\
\{4,17,24\}, & \{5,16,23\}, & \{6,15,28\}, & \\
\{4,19,26\}, & \{5,18,25\}, & \{6,16,29\}, & \\
\{4,21,28\}, & \{5,20,27\}, & \{6,17,30\}, & \\
\{4,23,30\}, & \{5,22,29\}, & \{6,18,31\}, & \\
\{4,25,32\}, & \{5,24,31\}, & \{6,19,32\}, &
\end{array}\right\}$$

$$\cup \{\{(7+i \pmod{26})+7,\ (12+i \pmod{26})+7,\ (22+i \pmod{26})+7\} \mid 0 \le i \le 25\}$$

$$\cup \{\{(7+i \pmod{26})+7,\ (10+i \pmod{26})+7,\ (19+i \pmod{26})+7\} \mid 0 \le i \le 25\}$$

$$\cup \{\{(7+i \pmod{26})+7,\ (9+i \pmod{26})+7,\ (15+i \pmod{26})+7\} \mid 0 \le i \le 25\}$$

$$
\left\{
\begin{array}{llll}
\{0,7,14\}, & \{1,7,26\}, & \{2,7,32\}, & \{3,7,8\}, \\
\{0,8,27\}, & \{1,8,15\}, & \{2,8,9\}, & \{3,9,10\}, \\
\{0,9,16\}, & \{1,9,28\}, & \{2,10,11\}, & \{3,11,12\}, \\
\{0,10,29\}, & \{1,10,17\}, & \{2,12,13\}, & \{3,13,14\}, \\
\{0,11,18\}, & \{1,11,30\}, & \{2,14,15\}, & \{3,15,16\}, \\
\{0,12,31\}, & \{1,12,19\}, & \{2,16,17\}, & \{3,17,18\}, \\
\{0,13,20\}, & \{1,13,32\}, & \{2,18,19\}, & \{3,19,20\}, \\
\{0,15,22\}, & \{1,14,21\}, & \{2,20,21\}, & \{3,21,22\}, \\
\{0,17,24\}, & \{1,16,23\}, & \{2,22,23\}, & \{3,23,24\}, \\
\{0,19,26\}, & \{1,18,25\}, & \{2,24,25\}, & \{3,25,26\}, \\
\{0,21,28\}, & \{1,20,27\}, & \{2,26,27\}, & \{3,27,31\}, \\
\{0,23,30\}, & \{1,22,29\}, & \{2,28,29\}, & \{3,28,32\}, \\
\{0,25,32\}, & \{1,24,31\}, & \{2,30,31\}, & \{3,29,30\}, \\
\{4,7,29\}, & \{5,7,20\}, & \{6,7,11\}, & \boldsymbol{\{0,1,3\}}, \\
\{4,8,30\}, & \{5,8,21\}, & \{6,8,12\}, & \boldsymbol{\{1,2,4\}}, \\
\{4,9,31\}, & \{5,9,22\}, & \{6,9,13\}, & \boldsymbol{\{2,3,5\}}, \\
\{4,10,32\}, & \{5,10,23\}, & \{6,10,14\}, & \boldsymbol{\{3,4,6\}}, \\
\{4,11,15\}, & \{5,11,24\}, & \{6,15,19\}, & \boldsymbol{\{4,5,0\}}, \\
\{4,12,16\}, & \{5,12,25\}, & \{6,16,20\}, & \boldsymbol{\{5,6,1\}}, \\
\{4,13,17\}, & \{5,13,26\}, & \{6,17,21\}, & \boldsymbol{\{6,0,2\}} \\
\{4,14,18\}, & \{5,14,27\}, & \{6,18,22\}, & \\
\{4,19,23\}, & \{5,15,28\}, & \{6,23,27\}, & \\
\{4,20,24\}, & \{5,16,29\}, & \{6,24,28\}, & \\
\{4,21,25\}, & \{5,17,30\}, & \{6,25,29\}, & \\
\{4,22,26\}, & \{5,18,31\}, & \{6,26,30\}, & \\
\{4,27,28\}, & \{5,19,32\}, & \{6,31,32\}, &
\end{array}
\right\}
$$

$$\cup \{\{(7+i \pmod{26})+7,\ (12+i \pmod{26})+7,\ (22+i \pmod{26})+7\} \mid 0 \le i \le 25\}$$

$$\cup \{\{(7+i \pmod{26})+7,\ (10+i \pmod{26})+7,\ (19+i \pmod{26})+7\} \mid 0 \le i \le 25\}$$

$$\cup \{\{(7+i \pmod{26})+7,\ (9+i \pmod{26})+7,\ (15+i \pmod{26})+7\} \mid 0 \le i \le 25\}$$

A pair of Steiner triple systems of order 39 intersecting in a subsystem of order 7.

{0,7,12},	{1,7,34},	{2,7,15},	{3,7,31},
{0,8,35},	{1,8,13},	{2,8,16},	{3,8,32},
{0,9,14},	{1,9,36},	{2,9,17},	{3,9,33},
{0,10,37},	{1,10,15},	{2,10,18},	{3,10,34},
{0,11,16},	{1,11,38},	{2,11,19},	{3,11,35},
{0,13,18},	{1,12,17},	{2,12,20},	{3,12,36},
{0,15,20},	{1,14,19},	{2,13,21},	{3,13,37},
{0,17,22},	{1,16,21},	{2,14,22},	{3,14,38},
{0,19,24},	{1,18,23},	{2,23,31},	{3,23,15},
{0,21,26},	{1,20,25},	{2,24,32},	{3,24,16},
{0,23,28},	{1,22,27},	{2,25,33},	{3,25,17},
{0,25,30},	{1,24,29},	{2,26,34},	{3,26,18},
{0,27,32},	{1,26,31},	{2,27,35},	{3,27,19},
{0,29,34},	{1,28,33},	{2,28,36},	{3,28,20},
{0,31,36},	{1,30,35},	{2,29,37},	{3,29,21},
{0,33,38},	{1,32,37},	{2,30,38},	{3,30,22},
{4,7,22},	{5,7,24},	{6,7,23},	**{0,1,3},**
{4,8,25},	{5,8,23},	{6,8,24},	**{1,2,4},**
{4,9,24},	{5,9,26},	{6,9,25},	**{2,3,5},**
{4,10,27},	{5,10,25},	{6,10,26},	**{3,4,6},**
{4,11,26},	{5,11,28},	{6,11,27},	**{4,5,0},**
{4,12,29},	{5,12,27},	{6,12,28},	**{5,6,1},**
{4,13,28},	{5,13,30},	{6,13,29},	**{6,0,2}**
{4,14,31},	{5,14,29},	{6,14,30},	
{4,15,30},	{5,15,32},	{6,15,31},	
{4,16,33},	{5,16,31},	{6,16,32},	
{4,17,32},	{5,17,34},	{6,17,33},	
{4,18,35},	{5,18,33},	{6,18,34},	
{4,19,34},	{5,19,36},	{6,19,35},	
{4,20,37},	{5,20,35},	{6,20,36},	
{4,21,36},	{5,21,38},	{6,21,37},	
{4,23,38},	{5,22,37},	{6,22,38},	

$$\cup \{\{(7+i \pmod{32})+7,\ (8+i \pmod{32})+7,\ (20+i \pmod{32})+7\} \mid 0 \le i \le 31\}$$

$$\cup \{\{(7+i \pmod{32})+7,\ (9+i \pmod{32})+7,\ (16+i \pmod{32})+7\} \mid 0 \le i \le 31\}$$

$$\cup \{\{(7+i \pmod{32})+7,\ (10+i \pmod{32})+7,\ (21+i \pmod{32})+7\} \mid 0 \le i \le 31\}$$

$$\cup \{\{(7+i \pmod{32})+7,\ (11+i \pmod{32})+7,\ (17+i \pmod{32})+7\} \mid 0 \le i \le 31\}$$

$$
\left\{
\begin{array}{cccc}
\{0,7,23\}, & \{1,7,20\}, & \{2,7,26\}, & \{3,7,17\}, \\
\{0,8,24\}, & \{1,8,27\}, & \{2,8,21\}, & \{3,8,18\}, \\
\{0,9,25\}, & \{1,9,22\}, & \{2,9,28\}, & \{3,9,31\}, \\
\{0,10,26\}, & \{1,10,29\}, & \{2,10,23\}, & \{3,10,32\}, \\
\{0,11,27\}, & \{1,11,24\}, & \{2,11,30\}, & \{3,11,21\}, \\
\{0,12,28\}, & \{1,12,31\}, & \{2,12,25\}, & \{3,12,22\}, \\
\{0,13,29\}, & \{1,13,26\}, & \{2,13,32\}, & \{3,13,35\}, \\
\{0,14,30\}, & \{1,14,33\}, & \{2,14,27\}, & \{3,14,36\}, \\
\{0,15,31\}, & \{1,15,28\}, & \{2,15,34\}, & \{3,15,25\}, \\
\{0,16,32\}, & \{1,16,35\}, & \{2,16,29\}, & \{3,16,26\}, \\
\{0,17,33\}, & \{1,17,30\}, & \{2,17,36\}, & \{3,19,29\}, \\
\{0,18,34\}, & \{1,18,37\}, & \{2,18,31\}, & \{3,20,30\}, \\
\{0,19,35\}, & \{1,19,32\}, & \{2,19,38\}, & \{3,23,33\}, \\
\{0,20,36\}, & \{1,21,34\}, & \{2,20,33\}, & \{3,24,34\}, \\
\{0,21,37\}, & \{1,23,36\}, & \{2,22,35\}, & \{3,27,37\}, \\
\{0,22,38\}, & \{1,25,38\}, & \{2,24,37\}, & \{3,28,38\}, \\
\{4,7,29\}, & \{5,7,34\}, & \{6,7,12\}, & \boldsymbol{\{0,1,3\}}, \\
\{4,8,30\}, & \{5,8,13\}, & \{6,8,35\}, & \boldsymbol{\{1,2,4\}}, \\
\{4,9,19\}, & \{5,9,36\}, & \{6,9,14\}, & \boldsymbol{\{2,3,5\}}, \\
\{4,10,20\}, & \{5,10,15\}, & \{6,10,37\}, & \boldsymbol{\{3,4,6\}}, \\
\{4,11,33\}, & \{5,11,38\}, & \{6,11,16\}, & \boldsymbol{\{4,5,0\}}, \\
\{4,12,34\}, & \{5,12,17\}, & \{6,13,18\}, & \boldsymbol{\{5,6,1\}}, \\
\{4,13,23\}, & \{5,14,19\}, & \{6,15,20\}, & \boldsymbol{\{6,0,2\}} \\
\{4,14,24\}, & \{5,16,21\}, & \{6,17,22\}, & \\
\{4,15,37\}, & \{5,18,23\}, & \{6,19,24\}, & \\
\{4,16,38\}, & \{5,20,25\}, & \{6,21,26\}, & \\
\{4,17,27\}, & \{5,22,27\}, & \{6,23,28\}, & \\
\{4,18,28\}, & \{5,24,29\}, & \{6,25,30\}, & \\
\{4,21,31\}, & \{5,26,31\}, & \{6,27,32\}, & \\
\{4,22,32\}, & \{5,28,33\}, & \{6,29,34\}, & \\
\{4,25,35\}, & \{5,30,35\}, & \{6,31,36\}, & \\
\{4,26,36\}, & \{5,32,37\}, & \{6,33,38\}, &
\end{array}
\right\}
$$

$$\cup\ \{\{(7+i \pmod{32})+7,\ (8+i \pmod{32})+7,\ (15+i \pmod{32})+7\} \mid 0 \le i \le 31\}$$

$$\cup\ \{\{(7+i \pmod{32})+7,\ (9+i \pmod{32})+7,\ (21+i \pmod{32})+7\} \mid 0 \le i \le 31\}$$

$$\cup\ \{\{(7+i \pmod{32})+7,\ (10+i \pmod{32})+7,\ (16+i \pmod{32})+7\} \mid 0 \le i \le 31\}$$

$$\cup\ \{\{(7+i \pmod{32})+7,\ (11+i \pmod{32})+7,\ (22+i \pmod{32})+7\} \mid 0 \le i \le 31\}$$

A pair of Steiner triple systems of order 31 intersecting in 6 disjoint 2-flowers.

{6,13,11},	{12,10,14},	{6,12,3},	{6,0,10},
{6,2,5},	{6,4,1},	{12,0,5},	{12,2,1},
{12,4,13},	{0,2,13},	{0,4,3},	{2,4,10},
{10,13,8},	{10,1,11},	{10,3,9},	{10,5,7},
{13,1,9},	{13,3,7},	{13,5,14},	{1,3,14},
{1,5,8},	{3,5,11},	{11,14,2},	{11,7,12},
{11,8,4},	{11,9,0},	{14,7,4},	{14,8,0},
{14,9,6},	{7,8,6},	{7,9,2},	{8,9,12},
{15,24,7},	{15,25,9},	{15,26,8},	{15,27,14},
{15,28,11},	{15,29,12},	{15,30,13},	{16,23,7},
{16,25,8},	{16,26,9},	{16,27,13},	{16,28,12},
{16,29,11},	{16,30,14},	{17,23,8},	{17,24,9},
{17,25,10},	{17,26,7},	{17,28,14},	{17,29,13},
{17,30,12},	{18,23,9},	{18,24,8},	{18,25,7},
{18,26,10},	{18,28,13},	{18,29,14},	{18,30,11},
{19,23,11},	{19,24,12},	{19,27,9},	{19,28,8},
{19,29,10},	{19,30,7},	{20,23,13},	{20,24,14},
{20,25,11},	{20,26,12},	{20,27,10},	{20,29,9},
{20,30,8},	{21,23,12},	{21,24,11},	{21,25,14},
{21,26,13},	{21,27,7},	{21,28,9},	{21,30,10},
{22,23,14},	{22,24,13},	{22,25,12},	{22,26,11},
{22,27,8},	{22,28,10},	{22,29,7},	{0,15,16},
{2,16,18},	{4,17,20},	{6,18,22},	{0,17,18},
{2,19,21},	{4,19,22},	{0,19,20},	{2,20,22},
{5,15,20},	{0,21,22},	{3,15,21},	{5,16,19},
{1,16,17},	{3,16,22},	{5,17,22},	{1,18,19},
{3,17,19},	{5,18,21},	{1,20,21},	**{15,23,10},**
{3,18,20},	{6,15,19},	{1,15,22},	**{16,24,10},**
{4,15,18},	{6,16,20},	{2,15,17},	**{17,27,11},**
{4,16,21},	{6,17,21},	{0,23,24},	**{18,27,12},**
{2,24,26},	{4,25,28},	{6,26,30},	**{19,25,13},**
{0,25,26},	{2,27,29},	{4,27,30},	**{19,26,14},**
{0,27,28},	{2,28,30},	{5,23,28},	**{20,28,7},**
{0,29,30},	{3,23,29},	{5,24,27},	**{0,1,7},**
{1,24,25},	{3,24,30},	{5,25,30},	**{21,29,8},**
{1,26,27},	{3,25,27},	{5,26,29},	**{2,3,8},**
{1,28,29},	{3,26,28},	{6,23,27},	**{22,30,9},**
{1,23,30},	{4,23,26},	{6,24,28},	**{4,5,9}**
{2,23,25},	{4,24,29},	{6,25,29},	

{6,10,11},	{12,13,14},	{6,12,8},	{6,0,14},
{6,2,9},	{6,4,7},	{12,0,9},	{12,2,7},
{12,4,11},	{0,2,11},	{0,4,8},	{2,4,14},
{10,13,2},	{10,1,12},	{10,3,4},	{10,5,0},
{13,1,4},	{13,3,0},	{13,5,6},	{1,3,6},
{1,5,2},	{3,5,12},	{11,14,3},	{11,7,13},
{11,8,5},	{11,9,1},	{14,7,5},	{14,8,1},
{14,9,10},	{7,8,10},	{7,9,3},	{8,9,13},
{15,24,9},	{15,25,8},	{15,26,7},	{15,27,13},
{15,28,12},	{15,29,14},	{15,30,11},	{16,23,9},
{16,25,7},	{16,26,8},	{16,27,14},	{16,28,11},
{16,29,13},	{16,30,12},	{17,23,7},	{17,24,8},
{17,25,12},	{17,26,10},	{17,28,13},	{17,29,9},
{17,30,14},	{18,23,8},	{18,24,7},	{18,25,10},
{18,26,9},	{18,28,14},	{18,29,11},	{18,30,13},
{19,23,12},	{19,24,11},	{19,27,10},	{19,28,9},
{19,29,7},	{19,30,8},	{20,23,14},	{20,24,13},
{20,25,9},	{20,26,11},	{20,27,8},	{20,29,12},
{20,30,10},	{21,23,13},	{21,24,14},	{21,25,11},
{21,26,12},	{21,27,9},	{21,28,10},	{21,30,7},
{22,23,11},	{14,22,25},	{13,22,26},	{12,22,24},
{22,27,7},	{22,28,8},	{22,29,10},	{0,16,17},
{2,16,22},	{4,17,22},	{6,15,16},	{0,18,19},
{2,19,17},	{4,18,21},	{0,20,21},	{2,20,18},
{5,15,19},	{0,15,22},	{3,15,18},	{5,16,20},
{1,15,17},	{3,16,21},	{5,17,21},	{1,18,16},
{3,17,20},	{5,18,22},	{1,19,21},	**{15,23,10},**
{3,19,22},	{6,17,18},	{1,20,22},	**{16,24,10},**
{4,15,20},	{6,19,20},	{2,15,21},	**{17,27,11},**
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