FULL FRIENDLY INDEX SET-II *†

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Abstract

Let $G = (V,\ E)$ be a graph, a vertex labeling $f : V \to \mathbb{Z}_2$ induces an edge labeling $f^* : E \to \mathbb{Z}_2$ defined by $f^*(xy) = f(x) + f(y)$ for each $xy \in E$. For each, $i \in \mathbb{Z}_2$ define $v_f(i) = |f^{-1}(i)|$ and $e_f(i) = |f^{*-1}(i)|$. We call f friendly if $|v_f(1) - v_f(0)| \leq 1$. The full friendly index set of G is the set of all possible values of $e_f(1) - e_f(0)$, where f is a friendly labeling. In this paper, we study the full friendly index set of the wheel W_n, tensor product of paths P_2 and P_n *i.e* $(P_2 \otimes P_n)$ and double star $D(m,n)$.

Keywords: Labeling; Cordial labeling; Friendly labeling; Full friendly index set.
AMS Subject Classification 2000: 05C78, 05C25.

1 Introduction

We refer book [2, 7] for graph theory notations and terminology described in this paper and all graphs are finite, simple, and undirected. Let $G = (V, E)$ be a graph and $f : V(G) \to \mathbb{Z}_2$, a vertex labeling of G. For each $i \in \mathbb{Z}_2$, let $v_f(i) = \left|f^{-1}(i)\right|$. The labeling f is said to be *friendly* if $|v_f(1) - v_f(0)| \leq 1$.

Any vertex labeling $f : V(G) \to \mathbb{Z}_2$ induces an edge labeling $f^* : E(G) \to \mathbb{Z}_2$ defined by $f^*(xy) = f(x) + f(y)\ \forall\ xy \in E(G)$. For $i \in \mathbb{Z}_2$, let $e_f(i) = \left|f^{*-1}(i)\right|$. A vertex v is called a $k-$vertex if $f(v) = k$. Similarly, an edge e is called a $k-$edge if $f^*(e) = k$. The number $i_f(G) = e_f(1) - e_f(0)$ is called the

*Presented in International Conference of Mathematics and Computer Science held at Loyola college, Chennai on feb 5-6, 2010.
†Research is supported by the Department of Science and Technology (Govt. of India), New Delhi, India under the project SR/S4/MS:409/06.
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friendly index of f. The *friendly index set* of the graph G, denoted by $FI(G)$ is defined by

$$FI(G) = \{|i_f(G)| : f \text{ is friendly labeling of } G\}.$$

The *full friendly index set* of the graph G denoted by $FFI(G)$ is defined by

$$FFI(G) = \{i_f(G) : f \text{ is friendly labeling of } G\}.$$

Note that if $f : V(G) \to \mathbb{Z}_2$ is a friendly labeling, so is its inverse labeling $g : V(G) \to \mathbb{Z}_2$ defined by $g(v) = 1 - f(v)\ \forall\ v \in V(G)$. Moreover, $i_f(G) = i_g(G)$. Readers interested in friendly labeling and friendly index set of graph are referred to a number of relevant literature that are mentioned in the reference section, including [13, 16, 18, 19].

In 1987, Cahit [3, 4, 5] introduced the concept of cordial labeling as weakened version of the less tractable graceful and harmonious labeling. A graph G is said to be *cordial* if, it admits a friendly labeling with 0 or 1. Hovay [10], later generalized the concept of cordial graphs and introduced *A-cordial labeling*, where A is an abelian group. A graph G is said to be A-cordial if it admits a labeling $f : V(G) \to A$ such that for every $i,\ j \in A$,

$$|v_f(i) - v_f(j)| \leq 1 \text{ and } |e_f(i) - e_f(j)| \leq 1.$$

Cordial graphs have been studied extensively. Interested readers are referred to a few of relevant papers in the literature [1, 6, 9, 11, 12, 14, 17].

In this paper, we focus on the group $A = \mathbb{Z}_2$ and determine the full friendly index set of wheel W_n, tensor product of paths P_2 and P_n *i.e.* $(P_2 \otimes P_n)$ and double star $D(m, n)$.

2 Basic Properties

Lemma 1. *[18] Let G be a $(p,\ q)$-graph of order greater than 1. Then $FFI(G) \subseteq \{2i - q :\ 1 \leq i \leq q\}$.*

Lemma 2. *[18] Let f be a labeling of a graph G that contains a cycle C as its subgraph. If C contains an 1-edge, then the number of 1-edges in C is a positive even number.*

3 Full friendly index set of wheel (W_n)

A wheel W_n, $n \geq 4$ is a defined to be the graph $K_1 + C_{n-1}$. In this graph, one vertex lies at the center of wheel and $n-1$ vertices lie on the circumference.

In this section, by C_{n-1} we will always mean a circle formed by the vertices lying on the circumference of the wheel and by $K_{1,n-1}$ we will mean the edges due to central vertex of the wheel and the vertices lying on the circumference. Clearly, C_{n-1} and $K_{1,n-1}$ are edge disjoint.

Lemma 3. *Let f be a friendly labeling of the W_{2n}, $n \geq 2$ then the minimum and maximum value of $e_f(1)$ are $n+2$ and $3n-2$ respectively.*

Proof. W_{2n} can be labeled friendly with any one permutation of the either of following sequences:

A. $(n-1)$ $0's$ followed by n $1's$ on C_{2n-1} and central vertex labeled 0.

B. n $1's$ followed by $(n-1)$ $0's$ on C_{2n-1} and central vertex labeled 1.

Clearly, for all permutation of labelings in A and B value of $e_h(1)$ for $K_{1,2n-1}$ is n, where h is the labeling restricted to the subgraph $K_{1,2n-1}$ of W_{2n}.
Now, we will find minimum and maximum value of $e_g(1)$, where g is the labeling restricted to the cycle C_{2n-1} of the wheel W_{2n}.

Case 1: *Labeling of W_{2n} is one of permutation of sequence of $(n-1)$ $0's$ followed by n $1's$ on C_{2n-1} and central vertex labeled* 0.
We observe that amongst all the permutations of defined sequence of W_{2n}, in this case C_{2n-1} will have minimum value of $e_g(1)$ for the sequence $(n-1)0's$ followed by $n1's$ and the value is 2. Therefore, the minimum value of $e_f(1)$ on W_{2n} is $n+2$.

Case 2: *Labeling of W_{2n} is one of permutation of sequence of n $0's$ followed by $(n-1)$ $1's$ on C_{2n-1} and central vertex labeled* 1.
We observe that amongst all the permutations of defined sequence on W_{2n}, in this case C_{2n-1} will have minimum value of $e_g(1)$ for the sequence $n0's$ followed by $(n-1)1's$ and the value is 2. Therefore, the minimum value of $e_f(1)$ on W_{2n} is $n+2$.

Now, we will evaluate the corresponding maximum values.

Case 1: *Labeling of W_{2n} is one of permutation of sequence of $(n-1)$ $0's$ followed by n $1's$ on C_{2n-1} and central vertex labeled* 0.
We observe that amongst all the permutations of defined sequence on W_{2n}, in this

case C_{2n-1} will have maximum value of $e_g(1)$ for the sequence "1, 0, 1, 0, ... 1, 0, 1" only and the value is $2n-2$. Therefore, the maximum value of $e_f(1)$ on W_{2n} is $3n-2$.

Case 2: *Labeling of W_{2n} is one of permutation of sequence of n $0's$ followed by $(n-1)$ $1's$ on C_{2n-1} and central vertex labeled* 1.
We observe that amongst all the permutations of defined sequence on W_{2n}, in this case C_{2n-1} will have maximum value of $e_g(1)$ for the sequence "0, 1, 0, 1, ... 0, 1, 0" only and the value is $2n-2$. Therefore, the maximum value of $e_f(1)$ on W_{2n} is $3n-2$. □

Lemma 4. *Let f be the friendly labeling of W_{2n+1}, $n \geq 2$, the minimum and maximum value of $e_f(1)$ are $n+2$ and $3n$ respectively.*

Proof. For friendly labeling W_{2n+1} can be labeled with any permutation of either of the following sequences:

1. n $0's$ followed by n $1's$ on C_{2n} and the central vertex labeled 0 or 1.
2. $(n-1)$ $1's$ followed by $(n+1)$ $0's$ on $C_{2n}$and the central vertex labeled 1.
3. $(n-1)$ $0's$ followed by $(n+1)$ $1's$ on C_{2n} and the central vertex labeled 0.

Using similar arguments as in Lemma 3 on above defined sequences results can be easily obtained. □

Lemma 5. *For W_{2n}, $n \geq 2$, there exists no friendly labeling f such that $e_f(1) \in \{n+3, n+5, \ldots, 3n-3\}$.*

Proof. For friendly labeling of the wheel W_{2n}, $n \geq 2$ vertices can be labeled in any permutation of either of the following sequences:

1. $(n-1)$ $0's$ followed by n $1's$ on C_{2n-1} and the central vertex labeled 0.
2. n $0's$ followed by $(n-1)$ $1's$ on C_{2n-1} and the central vertex labeled 1.

Clearly, for both sequences and their permutations the value of $e_h(1)$ for $K_{1,2n-1}$ is n, where h is the labeling restricted to the subgraph $K_{1,2n-1}$ of W_{2n}.
By Lemma 2, $e_g(1)$ on C_{2n-1} can have only even values, where g is the labeling restricted to the cycle C_{2n-1} of the wheel W_{2n}. Which shows $e_g(1) \notin \{3, 5, 7, \ldots, 2n-3\}$. Thus, $e_f(1) \notin \{n+3, n+5, n+7, \ldots, 3n-3\}$. □

Lemma 6. *For W_{2n+1}, $n \geq 2$, there exists no friendly labeling f such that $e_f(1) \in \{n+3, n+5, \ldots, 3n-1\}$.*

Proof. For friendly labeling of the wheel W_{2n+1}, $n \geq 2$ vertices can be labeled in any permutation of either of the following sequences:

1. $(n-1)$ $0's$ followed by $(n+1)$ $1's$ on C_{2n} and the central vertex labeled 0.

2. $(n+1)$ $0's$ followed by $(n-1)$ $1's$ on C_{2n} and the central vertex labeled 1.

3. n $0's$ followed by n $1's$ on C_{2n} and the central vertex labeled 0 or 1.

results can be easily obtained by similar arguments as in Lemma 5 on above defined sequences. □

Theorem 1. *For $n \geq 4$ the full friendly index set of wheel W_n is*

$$FFI(W_n) = \begin{cases} \{2i - 2n + 2 : \ i = \frac{n}{2} + 2, \ \frac{n}{2} + 4, \ \ldots, \ \frac{3n}{2} - 2\} & n \text{ is even.} \\ \{2i - 2n + 2 : \ i = \frac{n+3}{2}, \ \frac{n+5}{2}, \ \ldots, \ \frac{3n-3}{2}\} & n \text{ is odd.} \end{cases}$$

Proof. Theorem can be proved by considering following two cases:

Case 1: *Let number of vertices are even (say, 2k).* By Lemma 1, we have

$$FFI(W_{2k}) \subseteq \{2i - 4k + 2 : \ 0 \leq i \leq 4k - 2\}.$$

By Lemma 3, we get

$$FFI(W_{2k}) \subseteq \{2i - 4k + 2 : \ k + 2 \leq i \leq 3k - 2\}.$$

By Lemma 5, we get

$$FFI(W_{2k}) \subseteq \{2i - 4k + 2 : \ i = k + 2, \ k + 4, \ \ldots, \ 3k - 2\}. \qquad (1)$$

For particular value of $i = e_f(1) \in \{k+2, \ k+4, \ \ldots, \ 3k-2\}$, define the labeling:

Label the central vertex by 0. Now, take the alternate sequence of $0's$ and $1's$ of length $i - k$ and add remaining $0's$ adjacent to any of the 0 of the sequence in all or in parts and add remaining $1's$ adjacent to any of the 1 of the sequence in all or in parts to make it friendly. The resultant sequence when assigned on W_{2k} gives required $e_f(1)$. Thus,

$$\{2i - 4k + 2 : \ i = k + 2, \ k + 4, \ \ldots, \ 3k - 2\} \subseteq FFI(W_{2k}). \qquad (2)$$

From (1) and (2), we get

$$FFI(W_{2k}) = \{2i - 4k + 2 : \ i = k + 2, \ k + 4, \ \ldots, \ 3k - 2\}.$$

Case 2: *Let number of vertices are odd (say, 2k+1).* By Lemma 1, we have

$$FFI(W_{2k+1}) \subseteq \{2i - 4k : \ 0 \leq i \leq 4k\}.$$

By Lemma 4, we get

$$FFI(W_{2k+1}) \subseteq \{2i-4k:\ k+2 \le i \le 3k\}.$$

By Lemma 6, we get

$$FFI(W_{2n+1}) \subseteq \{2i-4n:\ i=n+2,\ n+4,\ \ldots,\ 3n\}. \tag{3}$$

For particular value of $i = e_f(1) \in \{n+2,\ n+4,\ \ldots,\ 3n\}$. Define the labeling:

Label the central vertex by 0. Now, take the alternate sequence of $0's$ and $1's$ of length i-n and add remaining $0's$ adjacent to any of the 0 of the sequence in all or in parts and add remaining $1's$ adjacent to any of the 1 of the sequence in all or in parts to make it friendly. The resultant sequence when assigned on W_{2n+1} gives required $e_f(1)$. Thus,

$$\{2i-4n:\ i=n+2,\ n+4,\ \ldots,\ 3n\} \subseteq FFI(W_{2n+1}). \tag{4}$$

From (3) and (4), we get

$$FFI(W_{2n+1}) = \{2i-4n:\ i=n+2,\ n+4,\ \ldots,\ 3n\}.$$

□

4 Full friendly index set of tensor product of P_2 and P_n

Let $G_1 = (V_1,\ E_1)$ and $G_2 = (V_2,\ E_2)$ be two graphs. Then the *tensor product* $G = G_1 \otimes G_2$ of graphs G_1 and G_2 is the graph with vertex set $V_1 \times V_2$ such that $(u_1,\ u_2)$ is adjacent to (v_1, v_2) whenever $\{u_1, v_1\} \in E_1$ and $\{u_2, v_2\} \in E_2$. The construction was originally introduced by Weischel [20]. It is also called the Kronecker product, weak product, direct product, categorical product and conjuction in the literature. The path P_n is a tree with two vertices of degree 1 and other $n-2$ vertices of degree 2.

The addition of two full friendly index sets [18] A and B is defined as:
$A + B = \{a + b : a \in A,\ b \in B\}$.
We require following Lemma's for the proof of theorem.

Lemma 7. *Let H be a $(p_H,\ q_H)$-graph and K be a $(p_K,\ q_K)$-graph. Then,*

$$FFI(H) + FFI(K) \subseteq FFI(H \cup K).$$

Proof. Let H be a (p_H, q_H)-graph, K be a (p_K, q_K)-graph and $G = H \cup K$. There arises following cases:
Case 1: *Both p_H and p_K are even.* For every $a \in FFE(H)$, $b \in FFI(K)$ there exists friendly labelings f, g of H, K respectively. Thus, $|v_{Hf}(1) - v_{Hf}(0)| = 0$ and $|v_{Kg}(1) - v_{Kg}(0)| = 0$. Define labeling h of G as follows:

$$h(v) = \begin{cases} f(v) & \forall\, v \in V(H) \\ g(v) & \forall\, v \in V(K) \end{cases}$$

Clearly, $v_{Gh}(1) = v_{Hf}(1)+v_{Kg}(1)$ and $v_{Gh}(0) = v_{Hf}(0)+v_{Kg}(0) \Rightarrow v_{Gh}(1) - v_{Gh}(0) = 0$. Which shows that h is a friendly labeling of G. Also, $e_{Gh}(1) - e_{Gh}(0) = a + b$ or $i_h(G) = i_f(H) + i_g(K)$. So,

$$FFI(H) + FFI(K) \subseteq FFI(H \cup K).$$

Case 2: *p_H is even and p_K is odd.* For every $a \in FFE(H)$, $b \in FFI(K)$ there exists friendly labelings f, g of H, K respectively. Thus, $|v_{Hf}(1) - v_{Hf}(0)| = 0$ and $|v_{Kg}(1) - v_{Kg}(0)| = 1$. Define labeling h of G as follows:

$$h(v) = \begin{cases} f(v) & \forall\, v \in V(H) \\ g(v) & \forall\, v \in V(K) \end{cases}$$

Clearly, $v_{Gh}(1) = v_{Hf}(1)+v_{Kg}(1)$ and $v_{Gh}(0) = v_{Hf}(0)+v_{Kg}(0) \Rightarrow v_{Gh}(1) - v_{Gh}(0) = 1$. Which shows that h is a friendly labeling of G. Also, $i_h(G) = i_f(H) + i_g(K)$. So,

$$FFI(H) + FFI(K) \subseteq FFI(H \cup K).$$

Case 3: *p_H is odd and p_K is even.* This case is similar as in Case 2.

Case 4: *Both p_H and p_K* For every $a \in FFI(H)$ there exists friendly labeling f and its inverse labeling f' such that $v_f(1) - v_f(0) = 1$ and $v_{f'}(0) - v_{f'}(1) = 1$. Similarly, for every $b \in FFI(K)$ there exists friendly labelings g, g' s.t. $v_g(1) - v_g(0) = 1$ and $v_{g'}(0) - v_{g'}(1) = 1$. Define labeling h of G as follows:
$$h(v) = \begin{cases} f(v) & \forall\, v \in V(H) \\ g'(v) & \forall\, v \in V(K) \end{cases} \text{ or } h(v) = \begin{cases} f'(v) & \forall\, v \in V(H) \\ g(v) & \forall\, v \in V(K) \end{cases}$$
Thus, h is a friendly labeling of G. Also, $i_h(G) = i_f(H) + i_g(K)$. So,

$$FFI(H) + FFI(K) \subseteq FFI(H \cup K).$$

□

Lemma 8. *If G is a disconnected graph with two components $H = (p_H, q_H)-$graph and $K = (p_K, q_K)-$graph such that $p_H = p_K$ (even or odd) then $e_f(1) \neq 1$ for any friendly labeling f of G.*

Proof. Let there exists a friendly labeling f of G such that $e_f(1) = 1$ then two adjacent vertices in in one of the component H or K (without loss of generality say, H) must be labeled "0, 1". Clearly, all the vertices of second component K must be labeled 0(or 1). Now, by friendliness the remaining vertices of first component must be labeled 1(or 0). In this case, $|v_f(1) - v_f(0)| = 2$, a contradiction to f being friendly. □

Theorem 2. *The full friendly index set of tensor product of two paths P_2 and P_n, $P_2 \otimes P_n$ is*

$$FFI(P_2 \otimes P_n) = \{2i - (2n-2) : 2 \leq i \leq 2n-2\}$$

Proof. We can easily observe that $P_2 \otimes P_n$ will give two copies of P_n. By Lemma 1 and Lemma 8, we have

$$FFI(P_n \otimes P_2) \subseteq \{2i - (2n-2) : 2 \leq i \leq 2n-2\}. \quad (5)$$

We know that $FFI(P_n) = \{2i - (n-1) : 1 \leq i \leq n-1\}$ [16]. Using Lemma 7, we get $\{2i - (n-1) : 1 \leq i \leq n-1\} + \{2i - (n-1) : 1 \leq i \leq n-1\} \subseteq FFI(P_n \otimes P_2)$.

$$\{2i - (2n-2) : 2 \leq i \leq 2n-2\} \subseteq FFI(P_n \otimes P_2). \quad (6)$$

From (5) and (6), we get

$$FFI(P_n \otimes P_2) = \{2i - (2n-2) : 2 \leq i \leq 2n-2\}.$$

□

5 Full friendly index set of double star $D(m, n)$

The double star $D(m, n)$ is a tree of diameter three such that there are m appended edges on one end of P_2 and n appended edges on the other end of P_2 (see Figure 1). Without loss of generality, we may assume $m \leq n$.

Theorem 3. *The full friendly index set of $D(m, n)$ is given by*

$$FFI(D(m,n)) = \begin{cases} \{2m - 4j + 1 : \ 0 \leq j \leq m\} \cup \{1\} \\ \text{where } m + n = 2k \\ \{2m - 2j + 2 : \ 0 \leq j \leq 2m + 1\} \cup \{0, 2\} \\ \text{where } m + n = 2k + 1 \end{cases}$$

Proof. The theorem can be proved by considering following cases:

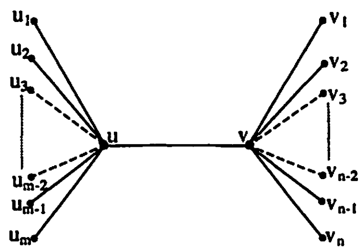

Figure 1: D(m,n)

Case 1: *Let* $m + n = 2k$. Then there are $2k + 2$ vertices. Without loss of generality, assume that the vertex u be labeled 0.

First label the vertex v by 0. Assume j of the vertices amongst $u_1, u_2, \ldots, u_m$ be labeled by 0 and the remaining $m - j$ vertices be labeled by 1. By friendliness, $(k - j - 1)$ of the vertices amongst $v_1, v_2, \ldots, v_n$ are labeled by 0 and the remaining $(n - k + j + 1)$ vertices are labeled by 1. Clearly, $e_f(0) = k$ and $e_f(1) = k + 1$. Thus, $e_f(1) - e_f(0) = 1$.

Next we label the vertex v by 1. Assume j of the vertices amongst $u_1, u_2, \ldots, u_m$ be labeled by 0 and the remaining $(m - j)$ vertices be labeled by 1. By friendliness, $(k - j)$ of the vertices amongst $v_1, v_2, \ldots, v_n$ are labeled by 0 and the remaining $(n - k + j)$ vertices are labeled by 1. Clearly, $e_f(0) = n - k + 2j$ and $e_f(1) = m + k - 2j + 1$. Thus, $e_f(1) - e_f(0) = 2m - 4j + 1$ where, $0 \leq j \leq m$.

Case 2: Let $m + n = 2k + 1$. Then there are $2k + 3$ vertices. Without loss of generality, assume that the vertex u be labeled 0.

First label the vertex v by 0. Assume j of the vertices amongst $u_1, u_2, \ldots, u_m$ be labeled by 0 and the remaining $m - j$ vertices be labeled by 1. By friendliness, either $(k - j - 1)$ (or $(k - j)$) of the vertices amongst $v_1, v_2, \ldots, v_n$ are labeled by 0 and the remaining $(n - k + j + 1)$ (or $(n - k + j)$) vertices are labeled by 1. Clearly, $e_f(0) = k$ (or $k + 1$) and $e_f(1) = k + 2$ (or $k + 1$). Thus, $e_f(1) - e_f(0) = 2$ (or 0).

Next we label the vertex v by 1. Assume j of the vertices amongst $u_1, u_2, \ldots, u_m$ be labeled by 0 and the remaining $(m - j)$ vertices be labeled by 1. By friendliness, either $(k - j)$ (or $(k - j + 1)$) of the vertices amongst $v_1, v_2, \ldots, v_n$ are labeled by 0 and the remaining $(n - k + j)$ (or $(n - k + j - 1)$) vertices are labeled by 1. Clearly, $e_f(0) = n - k + 2j$ (or $n - k + 2j - 1$) and $e_f(1) = m + k - 2j + 1$

(or $m + k - 2j + 2$). Thus, $e_f(1) - e_f(0) = 2m - 4j$ (or $2m - 4j + 2$) where, $0 \leq j \leq m$.

$$FFI(D(m,n)) = \begin{cases} \{2m - 4j + 1 : \ 0 \leq j \leq m\} \cup \{1\} \\ \qquad\qquad\qquad \text{When m + n = 2k} \\ \{2m - 2j + 2 : \ 0 \leq j \leq 2m + 1\} \cup \{0, 2\} \\ \qquad\qquad\qquad \text{When m + n = 2k + 1} \end{cases}$$

□

6 Conclusion

In this paper, the author has gone through the friendly labeling of some special graphs and hence calculated the full friendly index set of those graphs. Based on the arguments used in this paper edge friendly labeling can be very well defined and similar properties can be studied.

7 Acknowledgement

We express our gratitude to B.D. Acharya and M. Acharya for encouraging us to obtain more results on friendly labeling and also, for their constant readiness for discussions, which almost invariably yielded deeper insight. We are also thankful to S.M. Hegde who was visiting Centre for Mathematical Sciences, Bansthali University for intensive interaction in graph labelings. Their rigorous efforts in going through the paper have helped us to bring the paper in the present form.

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