The Domination Number of Fibonacci Cubes

David A. Pike[1,*], Yubo Zou[2,†]

[1]*Department of Mathematics and Statistics*
Memorial University
St. John's, NL, Canada A1C 5S7

[2]*Department of Mathematics*
University of South Carolina
Columbia, SC 29208, *USA*

Abstract

A dominating set is a vertex subset D of a graph G such that each vertex of G is either in D or adjacent to a vertex in D. The domination number, $\gamma(G)$, is the minimum cardinality of a dominating set of a graph G. In this paper, we will investigate the domination number of Fibonacci cubes. We firstly study the degree sequence of the Fibonacci cubes. Then, a lower bound for the domination number of Fibonacci cube of order n is obtained, and the exact value of the domination number of Fibonacci cubes of order at most 8 is determined.

Key words: Domination number, Fibonacci cube, Degree sequence, Hypercube.

AMS Classification: 05C69, 05C07.

1 Introduction

A vertex subset D is a *dominating* set of a graph $G(V, E)$ if each vertex in V is either in D or is adjacent to a vertex in D. A vertex in D is said to dominate itself and all its neighbours. The *domination number* $\gamma(G)$ is the minimum cardinality of a dominating set of G. The idea of domination has

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†Corresponding author (*Email:* zou@mailbox.sc.edu).

various applications in design and analysis of communication networks, social sciences, optimization, bioinformatics, computational complexity, and algorithm design [6, 7].

Several variations of domination exist. For instance, an *independent dominating* set is a dominating set which is also an independent set. A dominating set D is called a *perfect dominating* set if every vertex in $V - D$ is adjacent to exactly one vertex in D. In this paper, we will focus our discussion on the domination number.

Several papers, e.g. [3, 4, 5], briefly mention the influence of a vertex subset to define and subsequently study the redundance of a graph, where the *influence* of a vertex subset D is $I(D) = \sum_{v \in D} \big(\deg(v) + 1\big)$, and the *redundance* of a graph G is the minimum, over all dominating sets D, of $I(D)$. In many cases, the property of redundance is the primary interest.

In [1], the terminology of excess was introduced to study the domination number of hypercubes. We now extend the idea of excess to general graphs, and introduce the *over-domination* of a graph G with respect to a dominating set D of G as: $OD_G(D) = \Big(\sum_{v \in D} \big(\deg_G(v) + 1\big)\Big) - |V(G)|$, using $OD(D)$ for short if there is no confusion. For example, if a vertex not in D is dominated by two vertices in D, then it contributes 1 to the over-domination. We observe that over-domination differs from influence by a constant, $I(D) = OD(D) + |V(G)|$. But instead of using $\min\{OD(D)\}$ over all dominating sets, which corresponds to redundance, we concentrate our attention on over-domination itself.

In this paper, we will investigate the domination number of Fibonacci cubes. We firstly study the degree sequence of the Fibonacci cubes. Then, a lower bound for the domination number of Fibonacci cube of order n is obtained by applying over-domination. Furthermore, the exact value of the domination number of Fibonacci cubes of order at most 8 is determined.

For additional graph theory terminology and notational conventions we follow [11].

2 Degrees

A Fibonacci code of length n is a binary code $b_{n-1} \ldots b_1 b_0$ with $b_{i-1} \cdot b_i = 0$ for $1 \leqslant i \leqslant n-1$. So, a Fibonacci code is a binary code without consecutive ones. Recall that the Fibonacci numbers form a sequence of positive integers $\{f_n\}_{n=0}^{\infty}$ where $f_n = f_{n-1} + f_{n-2}$, $f_0 = 1$, and $f_1 = 2$. By *Zeckendorf's Theorem* [13], any non-negative integer $i \leqslant f_n - 1$ can be uniquely represented in the form $i = \sum_{j=0}^{n-1} b_j f_j$ where b_j is either 0 or 1, for $0 \leqslant j \leqslant n-1$ with the condition $b_{i-1} \cdot b_i = 0$ for $1 \leqslant i \leqslant n-1$.

Hence, i uniquely determines a Fibonacci code of length n. For example, $i = 11 = 8 + 3 = f_4 + f_2$ has Fibonacci code 10100.

In [8], Hsu introduced a new interconnection topology — Fibonacci cubes. The Fibonacci cube Γ_n of order n is the graph (V_n, E_n) where $V_n = \{0, 1, \cdots, f_n - 1\}$ and two vertices i and j are adjacent if and only if their Fibonacci codes differ in exactly one bit. The Fibonacci cubes for the first few values of n are depicted in Figure 1. We want to point out that the Fibonacci cube Γ_n is an induced subgraph of the n-cube Q_n. More properties of Fibonacci cubes are described in [2, 8, 9, 10, 12].

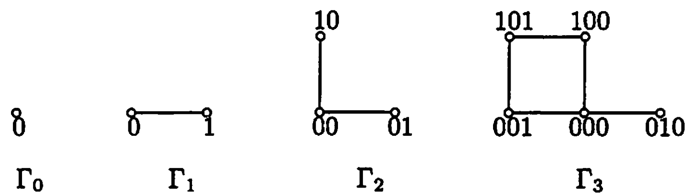


Figure 1. Fibonacci cubes Γ_n for $n = 0, 1, 2, 3$.

To investigate the domination number of Fibonacci cubes, we might look at the degree sequence. Firstly, we review the result regarding the degrees in [2].

Lemma 2.1 *For $n \geqslant 2$,*

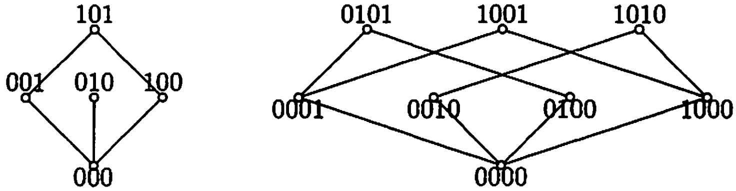
$$\deg_{\Gamma_n}(i) = \begin{cases} \deg_{\Gamma_{n-1}}(i) + 1, & 0 \leqslant i < f_{n-2}, \\ \deg_{\Gamma_{n-1}}(i), & f_{n-2} \leqslant i < f_{n-1}, \\ \deg_{\Gamma_{n-2}}(i - f_{n-1}) + 1, & f_{n-1} \leqslant i < f_n, \end{cases}$$

where $\deg_{\Gamma_0}(0) = 0$, $\deg_{\Gamma_1}(0) = 1$, and $\deg_{\Gamma_1}(1) = 1$. The maximum degree $\Delta(\Gamma_n) = n$, and vertex 0 is the only vertex of degree n. For $n \geqslant 4$, vertices 1 and f_{n-1} are the only vertices of degree $n - 1$.

In [10], Munarini and Zagaglia Salvi mentioned the Fibonacci semilattices. Let C_n be the set of Fibonacci codes of length n. An order relation on two codes $\alpha = a_{n-1} \ldots a_1 a_0$ and $\beta = b_{n-1} \ldots b_1 b_0$ in C_n is defined by

$$\alpha \leqslant \beta \Longleftrightarrow a_i \leqslant b_i, \quad i = n-1, \ldots, 1, 0.$$

In the Hasse diagram of the poset $\mathcal{F}_n := \langle C_n, \leqslant \rangle$, two codes are connected by an edge if and only if their Hamming distance is one. So the graph given by the Hasse diagram of $\mathcal{F}_n$ is isomorphic to Γ_n. Figure 2 gives the diagrams for $n = 3, 4$.



$\mathcal{F}_3$ $\mathcal{F}_4$

Figure 2. Hasse diagrams of $\mathcal{F}_3$ and $\mathcal{F}_4$

In the Hasse diagram of $\mathcal{F}_n$, vertices with the same number of ones are placed in the same level. Let $L_{n,k}$ be the set of length n codes having k ones. We can easily find that $|L_{n,0}| = 1, |L_{n,1}| = n$ and $|L_{n,2}| = \binom{n}{2} - (n-1)$ for $n \geqslant 3$. Since there are no consecutive ones in the Fibonacci codes, k is at most $\left\lceil \frac{n}{2} \right\rceil$. Furthermore, each vertex in $L_{n,k}$ has precisely k neighbours downwards (replacing a one by a zero to get its neighbour in $L_{n,k-1}$).

Now, we investigate the degrees of vertices in $L_{n,1}$.

Lemma 2.2 *In $L_{n,1}$, all vertices except vertices f_0 and f_{n-1} have degree $n-2$ for $n \geqslant 2$.*

Proof. The n vertices in $L_{n,1}$ are $f_0, f_1, \ldots, f_{n-1}$. Vertex f_i, $i = 1, \ldots, n-2$, has a one in the $i+1^{\text{th}}$ position, counting from right to left, of its Fibonacci code. The two positions on both sides of that one have to be zeros. Consequently, there are $n-3$ choices to replace a zero by one to construct the Fibonacci codes of vertex f_i's neighbours in $L_{n,2}$. Together with the neighbour downwards in $L_{n,0}$, vertex f_i has $n-2$ neighbours and the degree is $n-2$. □

Now, consider the degrees of the vertices in $L_{n,2}$.

Lemma 2.3 *In $L_{n,2}$, vertices $x = f_{n-1} + f_{n-3}$, $y = f_{n-1} + f_0$ and $z = f_2 + f_0$ are the only three having degree $n-2$ for $n \geqslant 4$.*

Proof. There are $\binom{n}{2} - (n-1)$ vertices in $L_{n,2}$ and each of them has two ones in its Fibonacci code. For the vertex z, the two ones locate on the first and third positions of its Fibonacci code, counting from right to left. The fourth position has to be a zero. Hence, there are $n-4$ possible choices to replace a zero by a one to construct the Fibonacci codes of vertex z's neighbours in $L_{n,3}$. Together with the two neighbours downwards, vertices f_0 and f_2, in $L_{n,1}$, vertex z has $n-2$ neighbours and the degree is $n-2$.

A similar argument applies to the vertices x and y. Since the three vertices x, y and z play important role on studying the domination number of Γ_n, we keep using these notations in the rest of this paper.

For any other vertex in $L_{n,2}$, at least one side or both sides of each ones in its Fibonacci codes have to be zeros, therefore, the degree is at most $n-3$. □

Applying a similar argument, we have the following result.

Lemma 2.4 *The maximum degree in* $L_{n,k}$ *is* $\max\{k, n-k\}$, *while the minimum degree in* $L_{n,k}$ *is* $n-2k$ *if* $k \leqslant \frac{n}{3}$, *or* k *otherwise. Furthermore, there are* $k+1$ *vertices in* $L_{n,k}$ *having degree* $n-k$

Proof. A vertex v in $L_{n,k}$ has k ones in its Fibonacci code, and k neighbours downwards. To reach the maximum degree, the vertex v should have as many as possible neighbours upwards (in $L_{n,k+1}$). Since there are no consecutive ones in Fibonacci codes, the ones should occur tightly, i.e. 10101010... and/or ...01010101 has to occur at the beginning and/or the end of the Fibonacci code. So there are $n-2k$ choices to replace a zero by a one in v's Fibonacci code to get neighbours of v in $L_{n,k+1}$, and the maximum degree of v is $n-2k+k=n-k$. For the k ones in v's Fibonacci code, there are $k+1$ ways to partition these k ones into two parts (empty part is allowed). Hence, there are $k+1$ vertices in $L_{n,k}$ having degree $n-k$.

For the minimum degree of a vertex in $L_{n,k}$, v should have as few as possible neighbours upwards. So, there are two zeros on both sides of each one in v's Fibonacci code, and the pattern 010 will occur k times if $k \leqslant \frac{n}{3}$. There are $n-3k$ possible choices to replace a zero by a one in v's Fibonacci code to get neighbours of v in $L_{n,k+1}$. Since there are k neighbours downwards, the minimum degree is $n-3k+k=n-2k$, if $k \leqslant \frac{n}{3}$. For the case $k > \frac{n}{3}$, the minimum degree is k. □

Corollary 2.5 *The minimum degree of* Γ_n *is at least* $\frac{n}{3}$.

In conclusion, we have the following result:

Theorem 2.6 *In* Γ_n $(n \geqslant 4)$, *vertex* 0 *is the only vertex having degree* n; *vertices* f_0 *and* f_{n-1} *are the only vertices having degree* $n-1$; *vertices* f_i $(i=1,\ldots,n-2)$, x, y, *and* z *are the only vertices having degree* $n-2$; *all other vertices in* Γ_n *have degree at most* $n-3$.

3 Lower Bound

Recalling Lemma 2 in [8], the Fibonacci cube Γ_n can be decomposed into two vertex disjoint subgraphs, LOW(n) and HIGH(n), isomorphic to Γ_{n-1} and Γ_{n-2}, respectively. Furthermore, LOW(n) and HIGH(n) are connected exactly by the edge set LINK(n) $= \{\{i,j\} : |i-j| = f_{n-1}, \{i,j\} \in E_n\}$. Dominating both of the two subgraphs is quite enough to dominate Γ_n. Furthermore, for any given minimum dominating set D_n in Γ_n, let $D_{n-1} = (D_n \cap \text{LOW}(n)) \cup \{u : u = v - f_{n-1} \text{ where } v \in D_n \cap \text{HIGH}(n)\}$. It is easy to verify that D_{n-1} is a dominating set of LOW(n). Then, we obtain the following bounds for the domination number of Γ_n.

Lemma 3.1 (a) $\gamma(\Gamma_n) \leqslant \gamma(\Gamma_{n-1}) + \gamma(\Gamma_{n-2})$.
(b) $\gamma(\Gamma_{n-1}) \leqslant \gamma(\Gamma_n)$.

Notice that the maximum degree of Γ_n is n and each vertex could dominate at most $n+1$ vertices, we have a trivial lower bound $\gamma(\Gamma_n) \geqslant \left\lceil \frac{f_n}{n+1} \right\rceil$. By applying the concept of over-domination, we obtain the following improved lower bound for the domination number of Γ_n.

Theorem 3.2 $\gamma(\Gamma_n) \geqslant \left\lceil \frac{f_n - 3}{n-2} \right\rceil$ *for* $4 \leqslant n < 9$, *and* $\left\lceil \frac{f_n - 2}{n-2} \right\rceil$ *for* $n \geqslant 9$

Proof. Suppose that D is a minimum dominating set in Γ_n, and D contains k ($0 \leqslant k \leqslant n-2$) vertices of degree $n-2$ from $L_{n,1}$ and l ($0 \leqslant l \leqslant 3$) vertices of degree $n-2$ from $L_{n,2}$. Let $n_2 = \left\lceil \frac{n}{2} \right\rceil$. Notice that the vertex/vertices in L_{n,n_2} must be dominated. A vertex in L_{n,n_2-1} or L_{n,n_2} must be included in D, and that vertex has maximum degree at most $n-4$ for $n \geqslant 9$. We consider the following cases:

1) Vertices $0, 1$ and f_{n-1} are in D. The number of dominated vertices is at most

$$(n+1) + 2n + (k+l)(n-1) + (\gamma(\Gamma_n) - 3 - k - l)(n-2) - OD(D) \geqslant f_n.$$

Simplification gives us

$$\gamma(\Gamma_n)(n-2) \geqslant f_n - k - l - 7 + OD(D).$$

Since vertices f_0 and f_{n-1} are neighbours of vertex 0; the three vertices of degree $n-2$ in $L_{n,2}$ are vertices x, y and z, and each of them has two common neighbours with vertex 0 in which either or both are vertices f_0 and f_{n-1}; each vertex in $L_{n,1}$ is a neighbour of vertex 0, we have

$$OD(D) \geqslant 5 + 3l + 2k.$$

Hence, we have

$$\gamma(\Gamma_n)(n-2) \geqslant f_n + k + 2l - 2 \geqslant f_n - 2.$$

Therefore, $\gamma(\Gamma_n) \geqslant \left\lceil \frac{f_n - 2}{n-2} \right\rceil$.

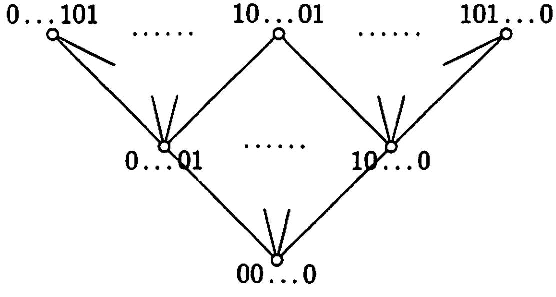


Figure 3. Configuration of $L_{n,0}, L_{n,1}$, and $L_{n,2}$ in Γ_n

2) Vertices 0 and exactly one of the vertices 1 and f_{n-1} are in D. Without lose of generality, suppose that vertices 0 and 1 are in D. The number of dominated vertices is

$$\begin{aligned} &n+1+n+(k+l)(n-1)+(\gamma(\Gamma_n)-3-k-l)(n-2) \\ &-OD \geqslant f_n \qquad \text{for } 4 \leqslant n < 9, \\ &n+1+n+n-3+(k+l)(n-1)+(\gamma(\Gamma_n)-3-k-l)(n-2) \\ &-OD \geqslant f_n \qquad \text{for } n \geqslant 9. \end{aligned}$$

That is

$$\gamma(\Gamma_n)(n-2) \geqslant \begin{cases} f_n - k - l - 5 + OD & \text{for } 4 \leqslant n < 9, \\ f_n - k - l - 4 + OD & \text{for } n \geqslant 9 \end{cases}$$

Notice that vertices 0 and 1 are adjacent; vertices from $L_{n,1}$ are adjacent to vertex 0; vertices from $L_{n,2}$ have two common neighbours with vertex 0. Therefore, we have $OD \geqslant 2 + 2k + 2l$. So

$$\gamma(\Gamma_n)(n-2) \geqslant \begin{cases} f_n + k + l - 3 & \text{for } 4 \leqslant n < 9, \\ f_n + k + l - 2 & \text{for } n \geqslant 9, \end{cases}$$

and $\gamma(\Gamma_n) \geqslant \left\lceil \frac{f_n - 3}{n - 3} \right\rceil$ for $4 \leqslant n < 9$, and $\gamma(\Gamma_n) \geqslant \left\lceil \frac{f_n - 2}{n - 3} \right\rceil$ for $n \geqslant 9$.

3) Vertex 0 is not in D, while both the vertices 1 and f_{n-1} are in. The number of dominated vertices is

$$2n + (k + l)(n - 1) + (\gamma(\Gamma_n) - 2 - k - l)(n - 2) - OD \geqslant f_n.$$

That is

$$\gamma(\Gamma_n)(n - 2) \geqslant f_n - k - l - 4 + OD.$$

Notice that vertices 1 and f_{n-1} have two common neighbours (vertices 0 and y); each vertex from $L_{n,1}$ is adjacent to vertex 0; vertices x, y and z from $L_{n,2}$ are adjacent to vertices 1 and/or f_{n-1}. Therefore, we have $OD \geqslant 2 + k + 2l$. So

$$\gamma(\Gamma_n)(n - 2) \geqslant f_n + l - 2,$$

and $\gamma(\Gamma_n) \geqslant \left\lceil \frac{f_n - 2}{n - 2} \right\rceil$.

4) Vertex 0 is in D, while none of the vertices 1 and f_{n-1} is in. The number of dominated vertices is

$$n + 1 + (k + l)(n - 1) + (\gamma(\Gamma_n) - 2 - k - l)(n - 2) - OD \geqslant f_n.$$

That is

$$\gamma(\Gamma_n)(n - 2) \geqslant f_n - k - l - 2 + OD.$$

Since vertices from $L_{n,1}$ are adjacent to vertex 0; vertices from $L_{n,2}$ have two common neighbours with vertex 0, we have $OD \geqslant 2k + 2l$. Hence

$$\gamma(\Gamma_n)(n - 2) \geqslant f_n + k + l - 2,$$

and $\gamma(\Gamma_n) \geqslant \left\lceil \frac{f_n - 3}{n - 2} \right\rceil$ for $n \geqslant 4$. Similar argument in case 2) yields $\gamma(\Gamma_n) \geqslant \left\lceil \frac{f_n - 2}{n - 2} \right\rceil$ for $n \geqslant 9$

5) Vertex 0 is not in D, and exactly one of the vertices 1 and f_{n-1} is in. Suppose that vertex 1 is in D. The number of dominated vertices is

$$n + (k + l)(n - 1) + (\gamma(\Gamma_n) - 1 - k - l)(n - 2) - OD \geqslant f_n.$$

That is

$$\gamma(\Gamma_n)(n - 2) \geqslant f_n - k - l - 2 + OD.$$

Notice that vertices from $L_{n,1}$ dominate vertex 0; vertices x, z are adjacent vertex 1. We have

$$OD \geqslant \begin{cases} k, & \text{if } l = 0; \\ k + l - 1, & \text{if } l = 1, 2, \text{ or } 3. \end{cases}$$

Hence $\gamma(\Gamma_n)(n-2) \geqslant f_n + k + l - 3$, and $\gamma(\Gamma_n) \geqslant \left\lceil \frac{f_n - 3}{n-2} \right\rceil$ for $n \geqslant 4$. Similarly, we have $\gamma(\Gamma_n) \geqslant \left\lceil \frac{f_n - 2}{n-2} \right\rceil$ for $n \geqslant 9$.

6) None of the vertices $0, 1, f_{n-1}$ is in D. Therefore D contains at least one degree $n-2$ vertex from $L_{n,1}$, and $k \geqslant 1$. The number of dominated vertices is

$$(k + l)(n - 1) + (\gamma(\Gamma_n) - k - l)(n - 2) - OD \geqslant f_n.$$

That is

$$\gamma(\Gamma_n)(n - 2) \geqslant f_n - k - l + OD.$$

Referring to Figure 3, vertex y has a common neighbour with each of vertices x and z. Vertices from $L_{n,1}$ dominate vertex 0. Therefore

$$OD \geqslant \begin{cases} k - 1, & \text{if } l \leqslant 2; \\ k - 1 + 2, & \text{if } l = 3. \end{cases}$$

So

$$\gamma(\Gamma_n)(n - 2) \geqslant \begin{cases} f_n - 3, & \text{if } l \leqslant 2; \\ f_n - 2, & \text{if } l = 3. \end{cases}$$

We have $\gamma(\Gamma_n) \geqslant \left\lceil \frac{f_n - 3}{n-2} \right\rceil$ for $n \geqslant 4$. Also, by similar argument, we have $\gamma(\Gamma_n) \geqslant \left\lceil \frac{f_n - 2}{n-2} \right\rceil$ for $n \geqslant 9$.

In all of above cases, we have $\gamma(\Gamma_n) \geqslant \left\lceil \frac{f_n - 3}{n-2} \right\rceil$ for $n \geqslant 4$ and, $\gamma(\Gamma_n) \geqslant \left\lceil \frac{f_n - 2}{n-2} \right\rceil$ for $n \geqslant 9$. □

4 Domination Numbers

Proposition 4.1 $\gamma(\Gamma_0) = \gamma(\Gamma_1) = \gamma(\Gamma_2) = 1$.

Proof. It is obvious after referring to Figure 1. □

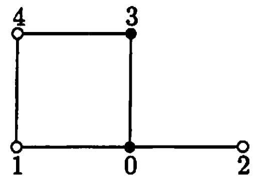


Figure 4. A minimum dominating set of Γ_3.

Proposition 4.2 $\gamma(\Gamma_3) = 2$

Proof. Referring to Figure 4 in which each vertex labeled by "•" is in D, the result is obvious. □

In Γ_3, there are 4 dominating sets of size 2, namely $\{0, 1\}$, $\{0, 3\}$, $\{0, 4\}$ and $\{2, 4\}$.

Proposition 4.3 $\gamma(\Gamma_4) = 3$.

Proof. By Theorem 3.2, $\gamma(\Gamma_4) \geqslant \left\lceil \frac{f_4 - 3}{4 - 2} \right\rceil = 3$. In total we found 12 dominating sets of size 3 in Γ_4. They are $\{0, 1, 2\}$, $\{0, 1, 5\}$, $\{0, 1, 7\}$, $\{0, 3, 5\}$, $\{0, 4, 5\}$, $\{1, 3, 7\}$, $\{1, 4, 7\}$, $\{2, 3, 6\}$, $\{2, 4, 5\}$, $\{2, 4, 6\}$, $\{3, 6, 7\}$ and $\{4, 5, 7\}$. Figure 5 shows a minimum dominating set of size 3 in Γ_4. □

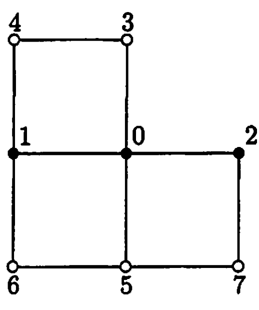


Figure 5. A minimum dominating set of Γ_4.

Proposition 4.4 $\gamma(\Gamma_5) = 4$.

Proof. By Theorem 3.2, $\gamma(\Gamma_5) \geqslant \left\lceil \frac{f_5 - 3}{5 - 2} \right\rceil = 4$, and there are 24 dominating sets of size 4 in Γ_5, all listed here: $\{0, 1, 2, 11\}$, $\{0, 1, 2, 12\}$, $\{0, 2, 5, 12\}$, $\{0, 2, 6, 12\}$, $\{0, 4, 5, 8\}$, $\{0, 5, 8, 12\}$, $\{0, 5, 10, 12\}$, $\{1, 2, 5, 11\}$, {1, 2, 6, 11}, {1, 2, 7, 11}, {1, 4, 7, 8}, {1, 5, 10, 11}, {1, 7, 8, 11}, {1, 7, 10, 11}, {2, 3, 5, 9}, {2, 3, 6, 9}, {2, 4, 5, 8}, {2, 4, 6, 8}, {2, 6, 11, 12}, {3, 5, 9, 10}, {4, 5, 7, 8}, {4, 5, 8, 10}, {4, 5, 10, 12} and {4, 6, 7, 8}. □

Proposition 4.5 $\gamma(\Gamma_6) = 5$.

Proof. Applying Theorem 3.2, $\gamma(\Gamma_6) \geqslant \left\lceil \frac{f_6 - 3}{6 - 2} \right\rceil = 5$. By an exhaustive computer search, we determined that there are 4 dominating sets of size 5 in Γ_6 (viz. {1, 2, 11, 16, 18}, {1, 2, 11, 17, 18}, {4, 6, 7, 8, 13}, {4, 7, 8, 13, 19}). □

By applying Theorem 3.2, we have $\gamma(\Gamma_7) \geqslant 7$. However, an exhaustive search revealed that there is no dominating set of size 7 in Γ_7; therefore $\gamma(\Gamma_7) \geqslant 8$. By computer search, there are 71 dominating sets of size 8 in Γ_7. Similarly, an exhaustive search produced 509 minimum dominating sets of size 12 in Γ_8. The results about the domination number of Γ_n, $1 \leqslant n \leqslant 8$, are summarized in Table 1, where $N_\gamma(\Gamma_n)$ denotes the number of minimum dominating sets of the Fibonacci cube of order n.

n	1	2	3	4	5	6	7	8
$\gamma(\Gamma_n)$	1	1	2	3	4	5	8	12
$\left\lceil \frac{f_n-3}{n-2} \right\rceil$				3	4	5	7	9

Table 1. Domination number of Γ_n.

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