Computation of Ramsey Numbers $R(C_m, W_n)$

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Abstract. For given finite simple graphs F and G, the Ramsey number $R(F, G)$ is the minimum positive integer n such that for every graph H of order n either H contains F or the complement of H contains G. In this note, with the help of computer, we get that $R(C_5, W_6) = 13$, $R(C_5, W_7) = 15$, $R(C_5, W_8) = 17$, $R(C_6, W_6) = 11$, $R(C_6, W_7) = 16$, $R(C_6, W_8) = 13$, $R(C_7, W_6) = 13$ and $R(C_7, W_8) = 17$.

1 Introduction

For a finite simple graph G, the complementary graph of G is denoted by $\overline{G}$. If G contains H, we write $G \supseteq H$. The vertex and edge number of G is denoted by $p(G)$ and $q(G)$ respectively. We denote by C_n a *cycle* of order n. A wheel W_n is a graph of order $n+1$ obtained by adding a vertex v to C_n with v adjacent to each vertex of C_n. For given finite simple graphs F and G, the Ramsey number $R(F, G)$ is the minimum positive integer n such that for every graph H of order n either $H \supseteq F$ or $\overline{H} \supseteq G$. H is an (F, G)-graph if neither $H \supseteq F$ nor $\overline{H} \supseteq G$; H is an $(F, G; n)$-graph if H is an (F, G)-graph of order n. The set of all nonisomorphic (F, G)-graphs and $(F, G; n)$-graphs are denoted by $\mathcal{R}(G_1, G_2)$ and $\mathcal{R}(G_1, G_2; n)$ respectively.

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Table 1: Known Ramsey numbers $R(C_m, W_n)$

n \ m	3	4	5	6	7	8	...
3	9 [5]	10 [3]	13 [3]	16 [3]	19 [3]	22 [3]	$3m-2$ $(m \geq 4)$ [3]
4	11 [8]	9 [7]	9 [6]	11 [6]	13 [6]	15 [6]	$2m-1$ $(m \geq 5)$ [6]
5	11 [2]	10 [7]	13	16	19	22	$3m-2$ $(m \geq 5)$ [6]
6	13 [2]	9 [7]	**13***	**11***	**13***	15 [13]	$2m-1$ $(m \geq 8)$ [13]
7	15 [2]	11 [7]	**15***	**16***	19[12]	22[12]	$3m-2$ $(m \geq 7)$ [12]
8	17 [2]	12 [7]	**17***	**13***	**17***		$2m-1$ $(m \geq 11)$ [13]
...	$2n+1$ [2]		$2n+1$ [14]		$2n+1$ [14]		$3m-2$ (odd $n \geq 3$ $m \geq n, m \neq 3$)[12]

In this note, with the help of computers, we get that $R(C_5, W_6) = 13$, $R(C_5, W_7) = 15$, $R(C_5, W_8) = 17$, $R(C_6, W_6) = 11$, $R(C_6, W_7) = 16$, $R(C_6, W_8) = 13$, $R(C_7, W_6) = 13$, $R(C_7, W_8) = 17$.

2 Some values of $R(C_m, W_n)$

Many researchers considered the Ramsey numbers $R(C_m, W_n)$ for various positive integers m and n.

In [1], it was proved that

Theorem 1 $R(C_m, W_n) = 2m - 1$ *for n even and $m \geq 5n/2$.*

It was also conjectured that

Conjecture 1 $R(C_m, W_n) = 3m - 2$ *for n odd and $m \geq n \geq 3$ and $(m, n) \neq (3, 3)$; $R(C_m, W_n) = 2m - 1$ for even $n \geq 4$ and $m \geq n \geq 3$ and $(m, n) \neq (4, 4)$.*

In [4], it was proved that Conjecture 1 is true for odd $n \geq 20$. There are also many other known values of $R(C_m, W_n)$. Table 1 gives known $R(C_m, W_n)$ for certain m and n. Here we present a table for known $R(C_m, W_n)$ for some m and n and corresponding references. These results can also be found in [11]. The results marked with * are from Theorem 2 in this note.

In [9], an efficient algorithm called one-vertex extension method with feasible intervals was introduced to extend $(4,5;n)$-graphs to $(4,5;n+1)$-graphs. In order to compute the Ramsey numbers $R(C_m, W_n)$, we applied this technique with slight modifications. The obtained results are shown in Table 1, which can be verified by naive one-vertex extension method. In addition, the powerful tool *nauty*, *shortg*[10] are used to reject isomorphic graphs. The corresponding statistics for nonisomorphic $(C_m, W_n; k)$-graphs are listed in Table 2.

Table 2: The number of nonisomorphic $(C_m, W_n; k)$-graphs

k	$\lvert\mathcal{R}(C_5, W_6; k)\rvert$	$\lvert\mathcal{R}(C_5, W_7; k)\rvert$	$\lvert\mathcal{R}(C_5, W_8; k)\rvert$	$\lvert\mathcal{R}(C_6, W_6; k)\rvert$
9	191	917	2907	437
10	155	496	3221	133
11	85	371	1174	0
12	61	342	1080	0
13	0	163	923	0
14	0	92	771	0
15	0	0	305	0
16	0	0	141	0
17	0	0	0	0
k	$\lvert\mathcal{R}(C_6, W_7; k)\rvert$	$\lvert\mathcal{R}(C_6, W_8; k)\rvert$	$\lvert\mathcal{R}(C_7, W_6; k)\rvert$	$\lvert\mathcal{R}(C_7, W_8; k)\rvert$
9	1989	5409	1624	13015
10	818	5289	1359	20444
11	284	589	1082	10066
12	138	28	252	3445
13	81	0	0	1043
14	22	0	0	785
15	0	5	0	305
16	0	0	0	141
17	0	0	0	0

From Table 2, we have

Theorem 2 $R(C_5, W_6) = 13$, $R(C_5, W_7) = 15$, $R(C_5, W_8) = 17$, $R(C_6, W_6) = 11$, $R(C_6, W_7) = 16$, $R(C_6, W_8) = 13$, $R(C_7, W_6) = 13$, $R(C_7, W_8) = 17$.

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