Some Irreducible Codes Invariant under the Janko Group, J_1 or J_2.

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Abstract

A construction of graphs, codes and designs acted on by simple primitive groups described in [9, 10] is used to find some self-orthogonal, irreducible and indecomposable codes acted on by one of the simple Janko groups, J_1 or J_2. In particular, most of the irreducible modules over the fields $\mathbb{F}_p$ for $p \in \{2, 3, 5, 7, 11, 19\}$ for J_1, and $p \in \{2, 3, 5, 7\}$ for J_2, can be represented in this way as linear codes invariant under the groups.

1 Introduction

Moori and Rodrigues [13] found binary codes of small dimension and length 2300 that are invariant under the Conway group Co_2 and irreducible under this action. They used the construction from a proposition in [9, 10]. We use the same construction here to obtain some similar results for the Janko groups, J_1 and J_2, for codes over the prime fields of order p dividing the

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order of the group, i.e. $\mathbb{F}_p$, for $p = 2, 3, 5, 7, 11, 19$ for J_1 (of order 175560), and $p = 2, 3, 5, 7$ for J_2 (of order 604800). All of the irreducible modules of J_1 over these fields apart, possibly, from $p = 19$, and most of those of J_2, can be represented in this way as the code, the dual code or the hull of the code of a design. For $p = 19$ for J_1 and for the largest degree (10080) for J_2 we have incomplete results, due to the length of the computations. Our results are described explicitly in Section 5 in 20 tables, two for each of the six representations of J_1 and the four representations of J_2.

In summary, we obtained:

Proposition 1 *From the construction described in Result 2, with unions of orbits, the following constructions of the irreducible modules of the Janko groups J_1 and J_2 as the code, the dual code or the hull of the code of a design, over $\mathbb{F}_p$ where $p = 2, 3, 5, 7, 11, 19$ for J_1 and $2, 3, 5, 7$ for J_2, were found to be possible:*

1. *J_1: all of the irreducible modules for $p = 2, 3, 5, 7, 11$, and many of those for $p = 19$;*
2. *J_2: all for $p = 2$; all for $p = 3$ apart from dimensions $26, 378$; all for $p = 5$ apart from dimensions $70, 189, 300$; all for $p = 7$ apart from dimensions $140, 378, 448$. For these exclusions, none exist of degree strictly less than 10080.*

We give a summary of the tables in Section 5 in the Tables 1 and 2 below, showing only dimensions and degrees for each of the groups and fields $\mathbb{F}_p$. The row labelled "dim" denotes the dimensions of the possible distinct irreducible modules that can occur (see Section 5), and the row labelled "deg" denotes the degree of the permutation representation, i.e. the length of the code. An entry "–" indicates that none were found for that dimension; this only applies to $\mathbb{F}_{19}$ for J_1 and, in the case of J_2, it implies that if any do exist they must have degree 10080. These were generally beyond our computing capabilities.

Note: We do not claim that we have all the constructions of the irreducible modular representations as codes; we were seeking mainly existence.

We have also obtained computationally a 6-PD-set of size 404 in $\bar{J_2}$ for the $[100, 36, 16]_p$ codes, where $p = 2, 3$. Most likely full PD-sets can be found in $\bar{J_2}$ (or J_2) with the same information set but since our method was computational rather than theoretical, we decided the computation would take too long. Here $\bar{J_2} = \mathrm{Aut}(J_2) = J_2 : 2$.

$\mathbb{F}_2$	dim	20	76	76	112	112	360
	deg	1045	266	1463	266	1463	1045

$\mathbb{F}_3$	dim	76	76	112	133	154	360
	deg	266	1596	1045	1045	4180	1045

$\mathbb{F}_5$	dim	56	76	76	77	133	360
	deg	266	1045	4180	266	1596	1045

$\mathbb{F}_7$	dim	31	45	75	77	89	112	120
	deg	4180	1596	1045	1463	1045	266	1045

$\mathbb{F}_7$	dim	133	154	266
	deg	1045	1045	1596

$\mathbb{F}_{11}$	dim	27	49	56	64	69	77	77	77
	deg	1463	1045	266	4180	1596	1045	1463	1045

$\mathbb{F}_{11}$	dim	106	119	209
	deg	1045	1596	1045

$\mathbb{F}_{19}$	dim	22	34	43	55	76	76	77	133	133
	deg	266	1463	-	-	1045	-	-	1045	-

$\mathbb{F}_{19}$	dim	133	209
	deg	-	1045

Table 1: Codes of irreducible modules of J_1 for $p = 2, 3, 5, 7, 11, 19$

The work is mostly computational, using Magma [3, 4], and the Magma libraries *pergps* or *simgps*, or using references to the modular representations of these groups as given in [8, 5, 1], and the Magma database. We expect our results to be reproducible with Magma since we give sufficient information for the codes to be constructed using the Magma libraries, even though the labelling of orbits in the libraries does vary for different versions of Magma.

The paper is arranged as follows: in Section 2 the necessary background is given. In Section 3 we give some examples using the construction, with

$\mathbb{F}_2$	dim	28	36	84	160
	deg	315	100	840	315

$\mathbb{F}_3$	dim	26	36	63	90	133	225	378
	deg	-	100	100	280	525	1008	-

$\mathbb{F}_5$	dim	14	41	70	85	90	175	189	225	300
	deg	315	280	-	1008	315	525	-	840	-

$\mathbb{F}_7$	dim	28	36	63	89	101	124	126	140	175
	deg	315	100	100	1008	10080	840	840	-	525

$\mathbb{F}_7$	dim	199	336	378	448
	deg	10080	1800	-	-

Table 2: Codes of irreducible modules of J_2 for $p = 2, 3, 5, 7$

minimum weight and weight distributions where possible. In Section 4 a sample Magma run is given to illustrate the construction. In Section 5 we give the 20 tables giving the representations of the irreducible modules that we found for the relevant primes. These tables provide the proof of Proposition 1. In Section 6 we establish some decompositions of the full space using some of the codes from irreducible modules constructed by this method.

2 Background and terminology

Our notation for designs and codes will follow [2]. An incidence structure $\mathcal{D} = (\mathcal{P}, \mathcal{B}, \mathcal{I})$, with point set $\mathcal{P}$, block set $\mathcal{B}$ and incidence $\mathcal{I}$ is a t-(v, k, λ) design, if $|\mathcal{P}| = v$, every block $B \in \mathcal{B}$ is incident with precisely k points, and every t distinct points are together incident with precisely λ blocks. The design is **symmetric** if it has the same number of points and blocks.

The **code** $C_F(\mathcal{D})$ **of the design** $\mathcal{D}$ over the finite field F is the space spanned by the incidence vectors of the blocks over F. Codes here will be **linear codes**, i.e. subspaces of the ambient vector space. A code C over a field of order q, of length n, dimension k, and minimum weight d, is denoted by $[n, k, d]_q$. A **generator matrix** for the code is a $k \times n$ matrix made up of a basis for C. The **dual** code $C^{\perp}$ is the orthogonal under the standard inner product $(,)$, i.e. $C^\perp = \{v \in F^n | (v,c) = 0 \text{ for all } c \in C\}$. The **hull** of a code or a design, where $C = C(\mathcal{D})$, is $\text{Hull}(C) = C \cap C^\perp$. A code C is **self-orthogonal** if $C \subseteq C^\perp$. Thus C is self-orthogonal if and only if it is equal to its hull. A **check** matrix for C is a generator matrix for $C^\perp$. The all-one vector will be denoted by $\jmath$, and is the constant vector with all coordinate entries equal to 1. Two linear codes are **isomorphic** if they can be obtained from one another by permuting the coordinate positions. An **automorphism** of a code C is an isomorphism from C to C. The automorphism group will be denoted by $\text{Aut}(C)$. Note that here we are not including multiplication of the coordinate positions by non-zero field elements as automorphisms: see Huffman [7] for a detailed treatment of the various types of automorphism groups of codes.

Any code is isomorphic to a code with generator matrix in so-called **standard form**, i.e. the form $[I_k \,|\, A]$; a check matrix then is given by $[-A^T \,|\, I_{n-k}]$. The first k coordinates are the **information symbols** and the last $n-k$ coordinates are the **check symbols.**

Permutation decoding was first developed by MacWilliams [11] and involves finding a set of automorphisms of a code called a PD-set. The method is described fully in MacWilliams and Sloane [12, Chapter 15] and Huffman [7, Section 8]. We extend the definition of PD-sets to s-PD-sets for s-error-correction:

Definition 1 *If C is a t-error-correcting code with information set $\mathcal{I}$ and check set $\mathcal{C}$, then a* **PD-set** *for C is a set $\mathcal{S}$ of automorphisms of C which is such that every t-set of coordinate positions is moved by at least one member of $\mathcal{S}$ into the check positions $\mathcal{C}$.*

*For $s \le t$ an s-***PD-set** *is a set $\mathcal{S}$ of automorphisms of C which is such that every s-set of coordinate positions is moved by at least one member of $\mathcal{S}$ into $\mathcal{C}$.*

The algorithm for permutation decoding is given in [7] and requires that the generator matrix is in standard form, so an information set needs to be known. The property of having a PD-set will not, in general, be invariant under isomorphism of codes, i.e. it depends on the choice of information set. Furthermore, there is a bound on the minimum size of $\mathcal{S}$ (see [6],[14], [7]):

Result 1 *If $\mathcal{S}$ is a PD-set for a t-error-correcting $[n,k,d]_q$ code C, and $r = n-k$, then*

$$|\mathcal{S}| \ge \left\lceil \frac{n}{r} \left\lceil \frac{n-1}{r-1} \left\lceil \cdots \left\lceil \frac{n-t+1}{r-t+1} \right\rceil \cdots \right\rceil \right\rceil \right\rceil.$$

This result can be adapted to s-PD-sets for $s \leq t$ by replacing t by s in the formula.

If G is a group and M is a G-module, the **socle** of M, written Soc(M), is the largest semi-simple G-submodule of M. It is the direct sum of all the irreducible G-submodules of M. In Section 5 we determine Soc(V) for each of the relevant full-space G-modules V for $G = J_1, J_2$.

The construction from [9, Proposition 1] we will use is the following, stated here as the correction [10]:

Result 2 *Let G be a finite primitive permutation group acting on the set Ω of size n. Let $\alpha \in \Omega$, and let $\Delta \neq \{\alpha\}$ be an orbit of the stabilizer G_α of α. If $\mathcal{B} = \{\Delta^g \mid g \in G\}$ and, given $\delta \in \Delta$, $\mathcal{E} = \{\{\alpha, \delta\}^g \mid g \in G\}$, then $\mathcal{D} = (\Omega, \mathcal{B})$ forms a symmetric 1-$(n, |\Delta|, |\Delta|)$ design. Further, if Δ is a self-paired orbit of G_α then $\Gamma = (\Omega, \mathcal{E})$ is a regular connected graph of valency $|\Delta|$, $\mathcal{D}$ is self-dual, and G acts as an automorphism group on each of these structures, primitive on vertices of the graph, and on points and blocks of the design.*

In fact we can use any union of orbits of a point-stabilizer in this construction, and this is the approach we will follow here.

The indices (degrees) of the maximal subgroups of J_1 and J_2 are given in Table 3 and 4, respectively, taken from Magma data. Tables 5 and 6 are from [9] and show the orbit lengths for point stabilizers in J_1 and J_2, respectively, in the relevant primitive representations. In Tables 5 and 6 the first column gives the degree, the second the number of orbits, and the remaining columns give the length of the orbits of length greater than 1, with the number of that length in parenthesis behind the length in case there is more than one of that length.

No.	Order	Index	Structure
Max[1]	660	266	$PSL(2,11)$
Max[2]	168	1045	2^3:7:3
Max[3]	120	1463	$2 \times A_5$
Max[4]	114	1540	19:6
Max[5]	110	1596	11:10
Max[6]	60	2926	$D_6 \times D_{10}$
Max[7]	42	4180	7:6

Table 3: Maximal subgroups of J_1

No.	Order	Index	Structure
Max[1]	6048	100	$PSU(3,3)$
Max[2]	2160	280	$3{\cdot}PGL(2,9)$
Max[3]	1920	315	$2^{1+4}{:}A_5$
Max[4]	1152	525	$2^{2+4}{:}(3 \times S_3)$
Max[5]	720	840	$A_4 \times A_5$
Max[6]	600	1008	$A_5 \times D_{10}$
Max[7]	336	1800	$PSL(2,7){:}2$
Max[8]	300	2016	$5^2{:}D_{12}$
Max[9]	60	10080	A_5

Table 4: Maximal subgroups of J_2

Degree	#	length				
266	5	132	110	12	11	
1045	11	168(5)	56(3)	28	8	
1463	22	120(7)	60(9)	20(2)	15(2)	12
1540	21	114(9)	57(6)	38(4)	19	
1596	19	110(13)	55(2)	22(2)	11	
2926	67	60(34)	30(27)	15(5)		
4180	107	42(95)	21(6)	14(4)	7	

Table 5: Orbits of a point-stabilizer of J_1

3 Examples of codes from the construction

In this section we give some examples of the irreducible modules of the smaller dimensions and smaller primes that can be obtained as the code, the dual code, or the hull of a 1-design, using the construction. We obtain the minimum weight and weight distribution of the code, when possible. We also apply permutation decoding to some of these codes. Most of these examples are listed in Section 5. Generally here C_m will refer to a code of dimension m. Note that for any code C, Hull(C) is self-orthogonal, so if the code is irreducible, then it is self-orthogonal if and only if it is the hull. Otherwise the hull is $\{0\}$.

3.1 Self-orthogonal binary code from J_1

Degree 1045, dimension 20

$[1045, 20, 456]_2$ code; dual code: $[1045, 1025, 4]_2$

Degree	#	length						
100	3	63	36					
280	4	135	108	36				
315	6	160	80	32(2)	10			
525	6	192(2)	96	32	12			
840	7	360	240	180	24	20	15	
1008	11	300	150(2)	100(2)	60(2)	50	25	12
1800	18	336	168(6)	84(3)	42(3)	28	21	14(2)
2016	18	300(2)	150(6)	75(5)	50(2)	25	15	
10080	191	60(153)	30(24)	20(4)	15	12(4)	10(3)	6

Table 6: Orbits of a point-stabilizer of J_2

The smallest dimension for J_1 over $\mathbb{F}_2$ is 20, of degree 1045. The code c of Table 8 is the hull H of the code C of the 1-(1045, 421, 421) design $\mathcal{D}$ from the orbits $\{1, 3, 5, 10, 11\}$. $\dim(C) = 21$ and $\dim(H) = 20$. Further, $C = H \oplus \mathbb{F}_2\jmath$ and H is self-orthogonal. The automorphism group is J_1.

```
WeightDistribution(H);
[ <0, 1>, <456, 3080>, <488, 29260>, <496, 87780>, <504, 87780>,
 <512, 36575>, <520, 299706>, <528, 234080>, <536, 175560>,
 <544, 58520>, <552, 14630>, <560, 19019>, <608, 1540>,<624, 1045>]
Those of weight 456, 504, 544, 552, 624, 608 are single orbits;
 the others split.
```

3.2 Self-orthogonal binary codes from J_2

For the smallest representations for J_2 we give below three examples of J_2 acting on self-orthogonal binary codes of small degree that are irreducible or indecomposable codes over J_2. The full automorphism group of each of these codes is $\bar{J}_2$.

1. **Degree 100, dimension 36**

   $[100, 36, 16]_2$ code; dual code: $[100, 64, 8]_2$

   The code a of Table 20 is the code $C = C_{36} = \text{Hull}(C)$ of the 1-(100, 36, 36) design from orbit $\{2\}$.

   ```
   > WeightDistribution(C);
   [ <0, 1>,<16, 1575>,<24, 105000>,<28, 1213400>,<32, 29115450>,
    <36,429677200>,<40, 2994639480>,<44, 10672216200>,
    <48, 20240374350>,<52,20217640800>,<56, 10675819800>,
    <60, 3004193640>,<64, 422248725>,<68,30819600>,<72, 1398600>,
   ```

```
<76, 12600>,<80, 315> ]
```

$C = C_{36}$ is irreducible and $C^{\perp} = C_{64}$ has an invariant subcode C_{63} of dimension 63 that is spanned by the weight-8 vectors and that contains $\jmath$ and C_{36}. All these codes are indecomposable, by Magma. From Table 19 we see that, for J_2, $\mathrm{Soc}(\mathbb{F}_2^{100}) = C_{37} = C_{36} \oplus \mathbb{F}_2 \jmath$ where, from the weight-distribution for C_{36}, we see that C_{37} is a $[100, 37, 16]_2$ code.

We have found 6-PD-sets in $\bar{J}_2$ of size 404 for the code C_{36}, but note that the representation of the code given directly from Magma does not have standard form, so an equivalent code is taken. The code corrects seven errors. The Gordon bound is 47 for full correction, 30 for correcting six errors. The full automorphism group of this code is $\bar{J}_2$.

2. **Degree 280, dimension 13**
   $[280, 13, 128]_2$ code; dual code: $[280, 267, 4]_2$

   In degree 280, orbit $\{3\}$ gave a 1-$(280, 108, 108)$ design with code $C = C_{14} = \mathrm{Hull}(C)$ of dimension 14 and weight distribution

```
[ <0, 1>, <108, 280>, <128, 1575>, <136, 2520>, <140, 7632>,
<144, 2520>, <152,1575>, <172, 280>, <280, 1> ]
```

   with dual code $[280, 266, 4]_2$. The code spanned by the weight-128 words has dimension 13 and weight distribution

```
[ <0, 1>, <128, 1575>, <136, 2520>, <144, 2520>, <152, 1575>,
<280, 1> ]
```

   C_{14} has for its minimum words the incidence vectors of the blocks of the 1-$(280, 108, 108)$ design. The code C_{13} is the code of a 1-$(280, 128, 720)$ design and also has for its minimum words the incidence vectors of the blocks of the design. Both codes are reducible but indecomposable. The full automorphism group of these codes is $\bar{J}_2$.

3. **Degree 315, dimension 28**
   $[315, 28, 64]_2$ code; dual code: $[315, 287, 3]_2$

   The code b of Table 20 was constructed from a 1-$(315, 64, 64)$ design from orbits $\{3, 4\}$; it is equal to its hull, has dimension 28 and weight distribution

```
[ <0, 1>, <64, 315>, <96, 6300>, <104, 25200>, <112, 53280>,
<120, 242760>,<124, 201600>, <128, 875700>, <132, 1733760>,
```

```
<136, 4158000>,<140, 5973120>,<144, 12626880>, <148, 24232320>,
<152, 35151480>,<156, 44392320>,<160,53040582>,<164, 41731200>,
<168, 28065120>, <172, 13023360>, <176, 2129400>,
<180, 685440>, <184, 75600>, <192, 10710>, <200, 1008> ]
```

It is self-orthogonal and from the weight distribution we see that the minimum words of C are the incidence vectors of the blocks of the 1-design. The full automorphism group of this code is $\bar{J}_2$.

3.3 Ternary code from J_2

The code a of Table 10 is the $[100, 36, 16]_3$ code C_{36} of a 1-$(100, 36, 36)$ design from the orbit $\{3\}$. It has dual code $[100, 64, 8]_3$. The hull is the trivial space. The dual code decomposes into a 63-dimensional subcode C_{63} and the 1-dimensional $\mathbb{F}_3\jmath$. Thus

$$\mathbb{F}_3^{100} = C_{36} \oplus C_{63} \oplus \mathbb{F}_3\jmath = \mathrm{Soc}(\mathbb{F}_3^{100}),$$

and hence $\mathbb{F}_3^{100}$ is completely reducible. The code C_{63} is the code b of Table 10, and is irreducible, by Magma, or [8]. It is the ternary code of the 1-$(100, 36, 36)$ design obtained from the other non-trivial orbit $\{2\}$, and is a $[100, 63, 8]_3$ code. A weight-8 vector orbited under J_2 spans the code C_{63}.

The 6-PD-sets found for the binary code of these parameters will also apply to this code, with the same change of information set to get the code into standard form.

3.4 Self-orthogonal 5-ary code from J_2

J_2 on $[315, 14, 180]_5$, dual $[315, 301, 3]_5$

We obtained a 5-ary code of dimension 14 from J_2 of degree 315, not listed in Section 5 since it is not the code, the dual code or the hull of a design obtained from the construction of Result 2. In the representation of degree 315, the orbit $\{2\}$ gives a 1-$(315, 10, 10)$ design with 5-ary code $C = C_{265}$ of dimension 265. $H = \mathrm{Hull}(C)$ has dimension 15 and weight distribution

```
[ <0, 1>, <155, 1260>, <180, 25200>, <200, 120960>,<205, 40320>,
<210,985500>, <215, 1882944>, <220, 5443200>, <225, 22320480>,
<230, 112150080>,<235, 544079340>, <240, 1804274304>,
<245, 5141181600>, <250, 8341211844>,<255, 8215482240>,
<260, 4489949520>, <265, 1445449824>, <270, 308548800>,
<275, 71318520>, <280, 10376640>, <285, 2180640>,
```

```
<290, 209664>,<295, 302400>, <300, 40320>, <305, 1260>,<315, 1264> ]
```

So $H = C_{15} = [315, 14, 155]_3$. The subcode C_{14} of H spanned by the words of weight 180 has dimension 14, does not contain all-one vector, and has weight distribution

```
[ <0, 1>, <180, 25200>, <200, 120960>, <210, 711900>,<220, 3225600>,
 <225,4872000>, <230, 39513600>, <235, 112694400>,<240, 386994720>,
 <245,909993600>, <250, 1653873984>, <255, 1674086400>,
 <260, 1005051600>, <265,266303520>, <270, 41428800>,
 <275, 4243680>, <280, 172800>, <285, 201600>,<315, 1260> ]
```

Then $C = C_{265} = C_{14} \oplus \mathbb{F}_5 \jmath \oplus C_{250}$, where C_{14} is self-orthogonal, by the above, and irreducible, by [8], or Magma data, and C_{250} is indecomposable. The automorphism group of these codes is $\bar{J}_2$.

4 Sample Magma run

We give here a sample of the program and the Magma functions used to establish these codes and the properties. Here we construct the binary code of dimension 28 from J_2 of degree 315, having already established that this choice of orbits is what we are looking for (see [9]).

```
load simgps;
g:=SimGroup(''J2'');
re:=SimRecord(''J2'');
ma:=re'Max;
Dim:=Dimension;
h:=ma[3];
a1,a2,a3:=CosetAction(g,h);a2;
st:=Stabilizer(a2,1);
orbs:=Orbits(st); #orbs;
v:=Index(a2,st);v;
vs:=VectorSpace(GF(2),v);
lo:=[#orbs[j]: j in [1..#orbs]];lo;
y:={3,4}
py:=&join[{z: z in orbs[x]}:x in y];
blox:=Setseq(py^a2);
des:=Design<1,v|blox>;
des;
C:=LinearCode(des,GF(2));
d:=Dim(C);d,''=Dim(C)'';
dh:=Dim(C meet Dual(C));dh;
```

```
Loading ''MAG/PMB07/j2.m''
Loading ''/Users/keyj/Mag2.14/libs/simgps/simgps''
Permutation group a2 acting on a set of cardinality 315
Order = 604800 = 2^7 * 3^3 * 5^2 * 7
6
315
[ 1, 10, 32, 32, 80, 160 ]
28 =Dim(C)
28
pm:=PermutationModule(a2,GF(2));
cm:=[pm!(Basis(C)[i]):i in [1..#Basis(C)]];
mcm:=sub<pm|cm>;
IsIrreducible(mcm);
true
```

5 Irreducible modules

We now examine the lists of irreducible modules of J_1 over $\mathbb{F}_p$ for $p \in \{2,3,5,7,11,19\}$, and J_2 over $\mathbb{F}_p$ for $p \in \{2,3,5,7\}$, and show how most of the irreducible modules of J_1 and J_2 can be represented as linear codes over these fields using the described construction, and hence establishing the proof of Proposition 1. We have summarised the results in Tables 1 and 2 in Section 1.

Firstly we determined which representations were possible by finding the list of composition factors of the socle in each representation. We used the following program provided by a reviewer of an earlier version of this paper, and thank that reviewer for suggesting it.

```
load simgps;
G:=SimGroup("J1");
M:=MaximalSubgroups(G);
CS:=[];SS:=[];
for i in [1..#M] do
X:=M[i];
Y:=X`subgroup;
gp:=CosetImage(G,Y);
v:=GModule(gp,GF(p));
S:=Socle(v);SS:=Append(SS,S);
csi:=CompositionFactors(S);
CS:=Append(CS,csi);
i,M[i],CompositionFactors(S);
```

```
end for;
```

The distinct composition factors showed the irreducible modules that were possible, allowing us to find the feasible dimensions for a code in each representation, along with the possibility of non-isomorphic modules of the same dimension: see, for example, Table 7, dimension 76. From this information we were able to search for codes for each irreducible module that occurred. Our results are shown in ten pairs of tables, A and B, (Tables 7 to 26), below. Note that C always denotes the code of the design constructed as in Section 4 from the given union of orbits, using the labelling from the *pergps* library. The precise labelling of the orbits might differ when Magma is called up, but generally the codes can easily be constructed from the information we give.

In each of the ten pairs of tables, the first column of Table A shows the degrees of the representations, i.e. the indices of the maximal subgroups, seven for J_1 and nine for J_2. In Table A, $*^m$ indicates that a non-trivial irreducible module of the corresponding degree exists in the given representation and occurs m times in the socle S when $m \geq 2$; $*x$ or $*^m x$, where $x \in \{a, b, c, \ldots\}$, indicates that we have constructed the irreducible module as either the code C, the dual code $C^\perp$ or the hull H of a design described below in Table B from the orbits given there. They are self-orthogonal when they are the hull. The last column, headed "S", gives the dimension of the socle S, which is the sum of the dimensions in the row, with multiplicities as given, plus 1 for the irreducible module of dimension 1. An empty column means that an irreducible module of that dimension cannot occur in the representation. (We have omitted these dimensions from Tables 1 and 2.)

In Table B, the design is a 1-(v, k, k) design, C is the p-ary code of the design, H its hull, and $C^\perp$ the dual code of C. The entries in the column labelled "orbits" refer to the position in the sequence of orbit lengths as given by Magma (see the program described in Section 4). The column labelled "lengths" denotes the lengths of the orbits; "dims" shows the dimension of C and H; "code" denotes the irreducible one, and "dim" is the dimension of the irreducible one.

From the ten tables A below we see that for J_2, $\mathrm{Soc}(\mathbb{F}_3^{100}) = \mathbb{F}_3^{100}$ and $\mathrm{Soc}(\mathbb{F}_7^{100}) = \mathbb{F}_7^{100}$ and hence they are the only full spaces that are semisimple, i.e. completely reducible J_2-modules.

The twenty tables then establish the proof of Proposition 1.

5.1 Tables for J_1

J_1 $\mathbb{F}_2$	20	76	76	112	112	360	S
266		$*a$		$*b$			189
1045	$*c$	$*$		$*$		$*d$	569
1463	$*$	$*^2$	$*^2e$	$*$	$*f$	$*$	909
1540	$*$	$*^2$		$*$		$*$	645
1596		$*$	$*$	$*$		$*$	625
2926		$*^3$	$*^2$	$*$	$*^2$	$*^2$	1437
4180	$*$	$*^3$	$*$	$*^2$	$*$	$*^3$	1741

Table 7: A: Degrees of irreducible modules of J_1 over $\mathbb{F}_2$

J_1 $\mathbb{F}_2$	v	k	orbits	lengths	dims	code	dim
a	266	122	3,4	12,110	76,0	C	76
b	266	12	3	12	112,0	C	112
c	1045	421	1,3,5,10,11	1,28,56,168,168	21,20	H	20
d	1045	225	1,4,7	1.56,168	685,0	$C^{\perp}$	360
e	1463	20	5	20	264,76	H	76
f	1463	61	1,11	1,60	1351,0	$C^{\perp}$	112

Table 8: B: Designs and codes for the irreducible modules of J_1 over $\mathbb{F}_2$

J_1 $\mathbb{F}_3$	76	76	112	133	154	360	S
266	$*a$		$*$				189
1045	$*$		$*b$	$*c$	$*$	$*d$	836
1463	$*^2$		$*$	$*^2$		$*$	891
1540	$*^2$		$*$	$*^2$	$*$	$*$	1045
1596	$*$	$*e$	$*$	$*$		$*$	758
2926	$*^3$		$*$	$*^4$		$*^2$	1593
4180	$*^3$	$*$	$*^2$	$*^4$	$*^2f$	$*^3$	2449

Table 9: A: Degrees of irreducible modules of J_1 over $\mathbb{F}_3$

J_1 $\mathbb{F}_3$	v	k	orbits	lengths	dims	code	dim
a	266	242	4,5	110,132	189,76	H	76
b	1045	449	1,4,5,9,10	1,56,56,168,168	933,0	$C^{\perp}$	112
c	1045	168	9	168	493,133	H	133
d	1045	504	9,10,11	168,168,168	360,0	C	360
e	1596	110	10	110	1387,76	H	76
f	4180	42	30	42	3725,154	H	154

Table 10: B: Designs and codes for the irreducible modules of J_1 over $\mathbb{F}_3$

J_1 $\mathbb{F}_5$	56	76	76	77	133	360	S
266	$*a$	$*$		$*b$			210
1045	$*$	$*c$				$*d$	493
1463	$*$	$*^2$		$*^2$		$*$	723
1540	$*$	$*^2$		$*$		$*$	646
1596	$*$	$*$	$*$	$*^2$	$*e$	$*$	856
2926	$*$	$*^2$		$*$	$*$	$*$	1369
4180	$*^2$	$*^3$	$*f$	$*^2$	$*^2$	$*^3$	1917

Table 11: A: Degrees of irreducible modules of J_1 over $\mathbb{F}_5$

J_1 $\mathbb{F}_5$	v	k	orbits	lengths	dims	code	dim
a	266	110	4	110	56,56	H	56
b	266	13	1,3	1,12	189,0	$C^{\perp}$	77
c	1045	336	9,11	168,168	836,76	H	76
d	1045	421	1,3,5,9,11	1,28,56,168,168	685,0	$C^{\perp}$	360
e	1596	22	4	22	1387,133	H	133
f	4180	126	24,48,54	42,42,42	4026,76	H	76

Table 12: B: Designs and codes for the irreducible modules of J_1 over $\mathbb{F}_5$

J_1 $\mathbb{F}_7$	31	45	75	77	89	112	120	133	154	266	S
266				$*$		$*a$					190
1045			$*b$		$*c$	$*$	$*d$	$*e$	$*f$		684
1463			$*$	$*^2g$	$*$	$*$	$*$	$*^2$			817
1540			$*$	$*$	$*$	$*$	$*$	$*^2$	$*$		894
1596		$*h$		$*^2$	$*$	$*$	$*$	$*$		$*i$	920
2926			$*^2$	$*^3$	$*^2$	$*$	$*^2$	$*^4$		$*$	1710
4180	$*j$		$*^3$	$*$	$*^2$	$*^2$	$*^3$	$*^4$	$*^2$	$*^2$	2545

Table 13: A: Degrees of irreducible modules of J_1 over $\mathbb{F}_7$

J_1 $\mathbb{F}_7$	v	k	orbits	lengths	dims	code	dim
a	266	111	1,4	1,110	154,0	$C^\perp$	112
b	1045	225	1,5,7	1,56,168	924,75	H	75
c	1045	232	2,6,11	8,56,168	805,89	H	89
d	1045	9	1,2	1,8	925,0	$C^\perp$	120
e	1045	64	2,5	8,56	912,0	$C^\perp$	133
f	1045	400	2,5,9,10	8,56,168,168	891,0	$C^\perp$	154
g	1463	60	10	60	1386,0	$C^\perp$	77
h	1596	55	6	55	1445,45	H	45
i	1596	176	2,6,19	11,55,110	1330,0	$C^\perp$	266
j	4180	42	35	42	4014,31	H	31

Table 14: B: Designs and codes for the irreducible modules of J_1 over $\mathbb{F}_7$

J_1 $\mathbb{F}_{11}$	7	14	27	49	56	64	69	77	77	77	106
266				$*$	$*a$				$*$		
1045				$*b$	$*$			$*c$		$*d$	$*e$
1463			$*g$	$*$	$*$				$*^2h$		$*$
1540			$*$	$*$	$*$			$*$	$*$	$*$	$*$
1596				$*$	$*$		$*i$		$*^2$		$*$
2926			$*$	$*$	$*$	$*k$			$*^3$		$*$
4180			$*$	$*$	$*^2$	$*$	$*$	$*^2$	$*^2$	$*^2$	$*^3$

J_1 $\mathbb{F}_{11}$	119	209	S
266			183
1045		$*f$	575
1463		$*^2$	811
1540		$*^2$	888
1596	$*j$	$*^2$	972
2926	$*$	$*^4$	1622
4180	$*^2$	$*^5$	2434

Table 15: A: Degrees of irreducible modules of J_1 over $\mathbb{F}_{11}$

J_1 $\mathbb{F}_{11}$	v	k	orbits	lengths	dims	code	dim
a	266	123	1,3,4	1,12,110	210,0	$C^\perp$	56
b	1045	28	3	28	962,49	H	49
c	1045	168	11	168	968,0	$C^\perp$	77
d	1045	56	6	56	968,0	$C^\perp$	77
e	1045	224	4,8	56, 168	898,106	H	106
f	1045	232	2,4,9	8,56,168	836,0	$C^\perp$	209
g	1463	180	7,22	60,120	1281,27	H	27
h	1463	120	17	120	1386,0	$C^\perp$	77
i	1596	110	9	110	1455,69	H	69
j	1596	111	1,11	1,110	1462,119	H	119
k	2926	60	53	60	2737,64	H	64

Table 16: B: Designs and codes for the irreducible modules of J_1 over $\mathbb{F}_{11}$

J_1 $\mathbb{F}_{19}$	22	34	43	55	76	76	77	133	133	133	209	S
266		$*$			$*$							111
1045	$*a$				$*b$		$*$	$*c$			$*d$	518
1463		$*e$	$*$		$*$			$*^2$			$*^2$	914
1540		$*$		$*$	$*$		$*$	$*^2$			$*^2$	1003
1596		$*$	$*$		$*$	$*$		$*$	$*$	$*$	$*^2$	1047
2926		$*$	$*^2$		$*^3$			$*$	$*$	$*$	$*^4$	1983
4180	$*$	$*$	$*$	$*$	$*^3$	$*$	$*^2$	$*^4$	$*^2$	$*^2$	$*^5$	2722

Table 17: A: Degrees of irreducible modules of J_1 over $\mathbb{F}_{19}$

J_1 $\mathbb{F}_{19}$	v	k	orbits	lengths	dims	code	dim
a	1045	337	1,8,11	1,168,168	934,22	H	22
b	1045	225	1,5,9	1,56,168	969,0	$C^\perp$	76
c	1045	224	4,8	56,168	912,0	$C^\perp$	133
d	1045	344	2,8,9	8,168,168	836,0	$C^\perp$	209
e	1463	120	17	120	1364,34	H	34

Table 18: B: Designs and codes for the irreducible modules of J_1 over $\mathbb{F}_{19}$

5.2 Tables for J_2

J_2 $\mathbb{F}_2$	12	28	36	84	128	160	S
100			$*a$				37
280							1
315		$*b$	$*$			$*c$	225
525			$*$			$*$	197
840			$*$	$*d$		$*$	281
1008		$*$				$*$	189
1800			$*$	$*$		$*$	281
2016						$*$	161
10080		$*$	$*$	$*$		$*^4$	789

Table 19: A: Degrees of irreducible modules of J_2 over $\mathbb{F}_2$

J_2 $\mathbb{F}_2$	v	k	orbits	lengths	dims	code	dim
a	100	36	2	36	36,36	C	36
b	315	64	3,4	32,32	28,28	C	28
c	315	33	1,4	1,32	155,0	$C^{\perp}$	160
d	840	384	4,7	24,360	84,84	C	84

Table 20: B: Designs and codes for the irreducible modules of J_2 over $\mathbb{F}_2$

J_2 $\mathbb{F}_3$	26	36	42	63	90	114	133	225	378	S
100		$*a$		$*b$						100
280				$*$	$*c$					154
315		$*$			$*$					127
525		$*$		$*$	$*$		$*d$			323
840				$*$	$*$		$*$	$*$		512
1008					$*^2$			$*e$		406
1800		$*$		$*^2$	$*$		$*$			386
2016				$*$	$*^2$		$*$	$*$		602
10080	$*$	$*$		$*$	$*^3$		$*^2$	$*^6$	$*^3$	3146

Table 21: A: Degrees of irreducible modules of J_2 over $\mathbb{F}_3$

J_2 $\mathbb{F}_3$	v	k	orbits	lengths	dims	code	dim
a	100	63	3	63	36,0	C	36
b	100	36	2	36	63,0	C	63
c	280	37	1,2	1,36	91,90	H	90
d	525	45	1,2,3	1,12,32	286,133	H	133
e	1008	50	4	50	630,225	H	225

Table 22: B: Designs and codes for the irreducible modules of J_2 over $\mathbb{F}_3$

J_2 $\mathbb{F}_5$	14	21	41	70	85	90	175	189	225	300	S
100											1
280			$*$			$*$					132
315	$*$					$*a$					105
525						$*$	$*b$				266
840			$*c$			$*$	$*$		$*d$		532
1008	$*e$				$*f$	$*^2$			$*$		505
1800			$*$		$*$	$*$	$*$				392
2016			$*$		$*$	$*^2$	$*$		$*$		707
10080	$*$		$*^2$	$*$	$*^2$	$*^3$	$*^2$	$*^3$	$*^6$	$*^5$	4374

Table 23: A: Degrees of irreducible modules of J_2 over $\mathbb{F}_5$

J_2 $\mathbb{F}_5$	v	k	orbits	lengths	dims	code	dim
a	315	112	3,5	32,80	155,90	H	90
b	525	224	3,6	32,192	350,0	$C^{\perp}$	175
c	840	265	1,4,6	1,24,240	691,41	H	41
d	840	421	1,5,6	1,180,240	615,0	$C^{\perp}$	225
e	1008	160	6,7	60,100	368,14	H	14
f	1008	25	3	25	560,85	H	85

Table 24: B: Designs and codes for the irreducible modules of J_2 over $\mathbb{F}_5$

J_2 $\mathbb{F}_7$	28	36	42	63	89	101	124	126	140	175	199
100		$*a$		$*b$							
280				$*$				$*$			
315	$*c$	$*$									
525		$*$		$*$						$*d$	
840				$*$			$*e$	$*f$		$*$	
1008	$*$				$*g$		$*$	$*$			
1800		$*$		$*^2$	$*$			$*^2$		$*$	
2016				$*$	$*$		$*$	$*^2$		$*$	
10080	$*$	$*$		$*$	$*^2$	$*^3 i$	$*^3$	$*^4$	$*$	$*^2$	$*^2 j$

J_2 $\mathbb{F}_7$	336	378	448	S
100				100
280				190
315				65
525				275
840				489
1008				368
1800	$*^2 h$			1351
2016	$*^2$			1376
10080	$*^6$	$*^3$	$*^2$	6419

Table 25: A: Degrees of irreducible modules of J_2 over $\mathbb{F}_7$

J_2 $\mathbb{F}_7$	v	k	orbits	lengths	dims	code	dim
a	100	37	1,2	1,36	64,0	$C^{\perp}$	36
b	100	63	3	63	63,0	C	63
c	315	81	1,5	1,80	287,0	$C^{\perp}$	28
d	525	44	2,3	12,32	350,0	$C^{\perp}$	175
e	840	16	1,2	1,15	579,124	H	124
f	840	20	3	20	714,0	$C^{\perp}$	126
g	1008	150	9	150	719,89	H	89
h	1800	211	1,7,12	1,42,168	1464,0	$C^{\perp}$	336
i	10080	120	59,183	60,60	9628,101	H	101
j	10080	120	62,102	60,60	9306,199	H	199

Table 26: B: Designs and codes for the irreducible modules of J_2 over $\mathbb{F}_7$

6 Decompositions

We examine some decompositions of the full space F^n, where n is the degree of the permutation representation, into G-modules, where $G = J_1, J_2$, using codes obtained by our construction, where F is one of the prime fields F_p for $p \in \{2,3,5,7,11,19\}$ for J_1, $p \in \{2,3,5,7\}$ for J_2. Note that if C is the code of one of the designs, and if $\mathrm{Hull}(C) = \{0\}$, then $F^n = C \oplus C^{\perp}$.

In all cases C_m denotes an indecomposable linear code of dimension m over the relevant field and group. If the codes are irreducible they have been obtained according to our method and are either described or listed in the Section 5. Some of the codes are of codimension 1 in the code obtained by our method of obtaining the code, the dual, or the hull.

1. J_1 of degree 266 over $\mathbb{F}_2$:

$$\mathbb{F}_2^{266} = C_{76} \oplus C_{112} \oplus C_{78},$$

   where C_{76} and C_{112} are irreducible and as listed in Section 5, Table 8, a,b, and C_{78} has the submodule $\mathbb{F}_2\jmath$.

2. J_1 of degree 1045 over $\mathbb{F}_2$:

$$\mathbb{F}_2^{1045} = C_{76} \oplus C_{112} \oplus C_{360} \oplus C_{496} \oplus \mathbb{F}_2\jmath,$$

   where all but C_{496} are irreducible. C_{76} is of index 1 in the dual of the code from a 1-$(1045, 56, 56)$ design from orbit $\{6\}$; C_{112} is of index 1 in the dual of the code from a 1-$(1045, 84, 84)$ design from orbits $\{3, 5\}$; C_{360} is from Table 8, d; C_{496} has composition factors:

```
GModule of dimension 20 over GF(2),
GModule of dimension 112 over GF(2),
GModule of dimension 1 over GF(2),
GModule of dimension 76 over GF(2),
GModule of dimension 20 over GF(2),
GModule of dimension 1 over GF(2),
GModule of dimension 112 over GF(2),
GModule of dimension 20 over GF(2),
GModule of dimension 1 over GF(2),
GModule of dimension 1 over GF(2),
GModule of dimension 112 over GF(2),
GModule of dimension 20 over GF(2)
```

3. J_1 of degree 266 over $\mathbb{F}_3$:

where C_{112} is of index 1 in the code from a 1-$(266, 121, 121)$ design from orbits $\{2, 4\}$; C_{153} is the code of the 1-$(266, 132, 132)$ design obtained from the orbit $\{5\}$.

4. J_1 of degree 1045 over $\mathbb{F}_3$:

$$\mathbb{F}_3^{1045} = C_{76} \oplus C_{112} \oplus C_{360} \oplus C_{154} \oplus C_{342} \oplus \mathbb{F}_3\jmath,$$

where all but C_{342} are irreducible. C_{76} is the dual of the code of the 1-$(1045, 112, 112)$ design obtained from the orbits $\{4, 5\}$; C_{112} and C_{360} are b and d of Table 10. C_{154} is a subcode of the ternary code C_{703} of the 1-$(1045, 308, 308)$ design obtained from the orbits $\{3, 4, 6, 10\}$. If $E = C_{76} \oplus C_{112} \oplus C_{360} \oplus \mathbb{F}_3\jmath$, then $C_{154} = E^{\perp} \cap C_{703}$. The code C_{342} is reducible with composition factors:

```
GModule of dimension 133 over GF(3),
GModule of dimension 76 over GF(3),
GModule of dimension 133 over GF(3)
```

5. J_2 of degree 315 over $\mathbb{F}_2$:

$$\mathbb{F}_2^{315} = C_{160} \oplus C_{154} \oplus \mathbb{F}_2\jmath,$$

where C_{160} is c of Table 20, and $C_{154} \oplus \mathbb{F}_2\jmath = C_{160}^{\perp}$ is the binary code of the 1-$(315, 33, 33)$ design from orbits $\{1, 4\}$. Note that $\mathbb{F}_2^{100}$ and $\mathbb{F}_2^{280}$ are indecomposable as J_2 modules.

6. J_2 of degree 100 over $\mathbb{F}_3$:

$$\mathbb{F}_3^{100} = C_{36} \oplus C_{63} \oplus \mathbb{F}_3\jmath = \mathrm{Soc}(\mathbb{F}_3^{100}) = C_{36} \oplus C_{36}^{\perp},$$

where C_{36} and C_{63} are a and b of Table 22. Note that $\mathbb{F}_3^{100}$ is completely reducible as a J_2-module.

7. J_2 of degree 280 over $\mathbb{F}_3$:

$$\mathbb{F}_3^{280} = C_{63} \oplus C_{216} \oplus \mathbb{F}_3\jmath,$$

where C_{63} is the dual of the code of the 1-$(280, 145, 145)$ design obtained from the orbits $\{1, 2, 3\}$; C_{216} is the code of the 1-$(280, 135, 135)$ design obtained from the orbit $\{4\}$.

8. J_2 of degree 525 over $\mathbb{F}_5$:

$$\mathbb{F}_5^{525} = C_{175} \oplus C_{100} \oplus C_{250},$$

where C_{175} is b of Table 24, i.e. the dual of the code C_{350} of a 1-$(525, 224, 224)$ design; C_{100} is the dual of the code C_{425} of the 1-$(525, 140, 140)$ design obtained from the orbits $\{2, 3, 4\}$; $C_{250} = C_{425} \cap$

9. J_2 of degree 100 over $\mathbb{F}_7$:

$$\mathbb{F}_7^{100} = C_{36} \oplus C_{63} \oplus \mathbb{F}_7\jmath = \text{Soc}(\mathbb{F}_7^{100}) = C_{36} \oplus C_{36}^\perp,$$

where C_{36} and C_{63} are a and b of Table 26. Note that $\mathbb{F}_7^{100}$ is completely reducible as a J_2-module.

7 Conclusion

We have constructed many of the irreducible modules as codes invariant under the group, but we have not located all of them for any particular degree. For example, for J_1 of degree 266 over $\mathbb{F}_5$, we know there is an irreducible module of dimension 76. We have C_{56} and C_{77} and hence $C_{133} = C_{56} \oplus C_{77}$. Then $\text{Soc}(\mathbb{F}_5^{266}) = C_{210} = C_{133} \oplus C_{133}^\perp = C_{56}^\perp$. However, we know that $C_{210} = C_{56} \oplus C_{77} \oplus \mathbb{F}_5\jmath \oplus C_{76}$, but we were not able to explicitly describe the J_1-invariant code C_{76}, even though we have all the others. Note that C_{210} contains two invariant subcodes of dimension 77: the irreducible C_{77} and the decomposable $\mathbb{F}_5\jmath \oplus C_{76}$, where C_{76} is irreducible.

There are many cases where non-isomorphic irreducible modules occur in the same degree, but we have not concentrated on examining these. We have also generally not constructed the same irreducible module for different degrees, since we were concentrating on existence. Some examples are:

- J_1 over $\mathbb{F}_2$ in degree 1463 has both types of irreducible modules of dimension 76, but only e of the second type is listed in Table 7. The first type can be obtained from the orbits $\{1, 6\}$, giving a 1-$(1463, 21, 21)$ design with $C^\perp$ the irreducible code of dimension 76, and $\text{Hull}(C) = \{0\}$. The listed one (e of the table) is a hull, and thus self-orthogonal.

- J_1 over $\mathbb{F}_3$ also has two irreducible modules of dimension 76, constructed as a (degree 266) and e (degree 1596) of Table 9 giving the codes. The first type also occurs in degree 1045, taking orbits $\{4, 5\}$, giving a 1-$(1045, 112, 112)$ design with $\text{Hull}(C) = \{0\}$ and $C^\perp = C_{76}$. Thus this code is not self-orthogonal, whereas the code for degree 266, from the same irreducible module, is a hull, and thus self-orthogonal. This module occurs along with the other one (from e in the table) in degree 1596: take the orbit $\{15\}$ which gives a 1-$(1596, 110, 110)$ design with code C of dimension 1519, and $\text{Hull}(C)$ of dimension 76.

We have shown that the construction from Result 2 leads to many interesting, and possibly usable, codes acted on by the groups. We have looked particularly at those from the irreducible modules, and thus of small dimension. These are good candidates for permutation decoding, due to the size of the group and the large size of the check set.

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