A degree condition for k-uniform graphs *

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Abstract

Let G be a graph of order $n \geq 4k+8$,where k is a positive integer with kn is even and $\delta(G) > k+1$. We show that if $max\{d_G(u), d_G(v)\} > n/2$ for each pair of nonadjacent vertices u, v , then G has a connected $[k, k+1]$-factor excluding any given edge e.

Key words: degree condition; connected $[k, k+1]$-factor; prescribed properties;
AMS(2000) subject classification: 05C70

1 Introduction

The graphs considered in this paper will be simple undirected graphs. Let G be a graph with vertex set $V(G)$ and edge set $E(G)$. Denote by $d_G(x)$ the degree of a vertex x in G. We use $\delta(G)$ for the minimum degree of G and use $G-S$ for the subgraph of G obtained from G by deleting the vertices in S together with the edges incident with them. Let S and T be disjoint subsets of $V(G)$, denote by $e_G(S,T)$ the number of edges that join a vertex in S and a vertex in T. If $S = \{x\}$, then $e_G(x,T)$ denotes the number of edges that join x and a vertex in T. Let $S, T \subseteq V(G)$ with $S \cap T = \emptyset$. For an integer $k \geq 1$, a component C of $G-(S \cup T)$ is called a k-odd component or k-even component according to $k \mid V(C) \mid +e_G(V(C),T)$ is odd or even. We denote by $h(S,T)$ the number of k-odd components of $G-(S \cup T)$. A k-factor of G is a spanning subgraph F of G such that $d_F(x) = k$ for each $x \in V(G)$. A graph G is called a k-uniform graph if for each edge of $E(G)$,

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there is a k-factor containing this edge and another k-factor excluding it. Notations and definitions not given here can be found in [1].

Many authors have investigated (g, f)-*factors* and f-*factors* in [2,3,6,7,9]. And The following theorems of k-factors in terms of degree conditions were known.

Theorem 1(Nishimura [7]). *Let k be an integer such that $k \geq 3$, and let G be a connected graph of order n with $n \geq 4k-3$, kn is even, and minimum degree at least k. Suppose that*

$$max\{d_G(u), d_G(v)\} \geq \frac{n}{2}$$

for each pair of nonadjacent vertices u, v of $V(G)$.Then G has a k-factor.

Theorem 2(Chen [3]). *Let G be a graph of order n and k be a positive integer where $k \geq 3$, $n \geq 4k-6$ and kn even. If $\delta > \frac{n}{2}$, then G has a k-factor including any given edge and a k-factor excluding any given edge.*

The following Theorem is essential to the proof of our main theorem.

Theorem 3[5]. *Let G be a graph, and g and f be two integer-valued functions defined on $V(G)$ such that $g(x) \leq f(x)$ for all $x \in V(G)$. If G has both a (g,f)-factor and a Hamilton path, then G contains a connected (g, f+1)-factor.*

Theorem 4[4]. *Let G be a 2-connected graph with $\nu \geq 3$¿ If for any two vertices x and y of G such that the distance between x and y is two,*

$$maxdeg_G(x), deg_G(y) \geq \frac{\nu}{2},$$

then G has Hamiltonian cycle.

Extending Theorem 1 and Theorem 2, we prove the following results.

Theorem 5 *Let $k \geq 2$ be a positive integer and G be a graph of order $n \geq 4k+8$ with $\delta(G) > k+1$ and kn even. Suppose that*

$$max\{d_G(x), d_G(y)\} > \frac{n}{2}$$

for any nonadjacent vertices x and y of $V(G)$. Then G is a k-uniform graph.

Corollary *Let $k \geq 2$ be a positive integer and G be a graph of order $n \geq 4k+8$ with $\delta(G) > k+1$ and kn even. Suppose that*

$$max\{d_G(x), d_G(y)\} > \frac{n}{2}$$

for any nonadjacent vertices x and y of $V(G)$. Then G has has a connected $[k, k+1]$-factor excluding any given edge e.

2 Lemmas

In order to prove Theorem 5, we need the following lemmas.

Lemma 1(Tutte [8]). *Let G be a graph and k be a positive integer. Then for all $S, T \subseteq V(G)$ with $S \cap T = \emptyset$,*

(i) G has a k-factor if and only if $\delta_G(S,T) \geq 0$;

(ii) $\delta_G(S,T) \equiv kn(mod\ \ 2)$. where $\delta_G(S,T) = k \mid S \mid + d_{G-S}(T) - k \mid T \mid - h(S,T)$.

Lemma 2 (Chen [3]). *Let G be a graph and $k \geq 1$ be an integer. Assume that there exists a real number θ and disjoint subsets S and T of $V(G)$ satisfying*

(i) $\delta_G(S,T) < \theta$;

(ii) $\delta_G(S^*, T^*) \geq \theta$ for all $S^*, T^* \subseteq V(G)$ such that $S \subseteq S^* \subseteq V(G) - T^* \subseteq V(G) - T$ and $\mid S \mid + \mid T \mid < \mid S^* \mid + \mid T^* \mid$. Then $d_{G-S}(u) \geq k+1$ and $e_G(u,T) \leq k-1$ for $u \in V(G) - (S \cup T)$. Moreover, the order of each component of $V(G) - (S \cup T)$ is at least 3.

Lemma 3 (Nishimura [7]) *Let $m,n,s,t,$ and w_0 be nonnegative integers. Suppose that $m \geq 3$, $w_0 \geq 4$ and $m(w_0 - 1) \leq n - s - t - 3$. Then it holds that*

$$m - 1 + s + t \leq \frac{1}{3}[n + 2(s + t + 1 - w_0)].$$

Lemma 4(Liu [6]) *Let G be a graph and k be a positive integer . Then G is a k-uniform graph if and only if G is 2-connected and for any $S, T \subseteq V(G)$ with $S \cap T = \emptyset$,*

$$\delta_G(S,T) \geq \theta(S,T),$$

where $\theta(S,T) = 2$ if S or T is not independent or $S \cup T \neq \emptyset$ and $G - (S \cup T)$ has a k-even component; $\theta(S,T) = 0$, otherwise.

3 Proof of Theorem 5

We prove the Theorem 5 by contradiction. Suppose that G is not a k-uniform graph. By lemma 4, there exist $S, T \subseteq V(G)$ such that $\delta_G(S,T) < \theta(S,T)$. Set $\theta = \theta(S,T)$. We choose disjoint subsets S and T of $V(G)$ such that θ, S and T satisfy the condition of Lemma 4. Since $\delta_G(S,T) < \theta$ and $\delta_G(S,T) = kn(mod\ \ 2)$, by (ii) of Lemma 1, then

$$k \mid S \mid + \sum_{x \in T} d_{G-S}(x) - k \mid T \mid - h(S,T) \leq \theta - 2 \leq 0. \qquad (1)$$

Let $U = G - (S \cup T)$ and let $C_1, C_2 \dots, C_\omega$ denote the components of $G - (S \cup T)$, where $|C_1| \le |C_2| \le \dots \le |C_\omega|$. For convenience, Set $s = |S|, t = |T|$ and $m_i = |V(C_i)|$. From Lemma 2 we have $d_{G-S}(u) \ge k+1$, $e_G(u, T) \le k-1$ for any $u \in V(U)$ and $m_i \ge 3$ for $i = 1, 2, \dots, \omega$.

If $S \cup T = \emptyset$, then $h(S,T) = h(\emptyset, \emptyset) = 0$. Since G is Hamiltonian, G is 2-connected and so $\theta = \theta(S,T) = 0$ by lemma 4. Therefore, we have $\delta_G(S,T) = 0 < \theta(S,T) = 0$, a contradiction.

So we may assume that $S \cup T \neq \emptyset$. If $T \neq \emptyset$, we write $h_1 = \min\{d_{G-S}(x) \mid x \in T\} = d_{G-S}(x_1)$. And if $T - N_T[x_1] \neq \emptyset$, let $h_2 = \min\{d_{G-S}(x) \mid x \in T - N_T[x_1]\} = d_{G-S}(x_2)$. By Lemma 2 we know

$$n - s - t \ge 3\omega. \tag{2}$$

If $\omega \ge 2$, we have

$$m_1 \le \frac{n-s-t}{\omega}, \quad m_2 \le \frac{n-s-t-3}{\omega - 1}. \tag{3}$$

Let $p = |N_T[x_1]|$, $s = |S|$ and $t = |T|$, then from (1) we get

$$ks + (h_1 - k)p + (h_2 - k)(t-p) - h(S,T) \le \theta - 2 \le 0. \tag{4}$$

To prove the Theorem, we distinguish five cases.

Case 1. $T = \emptyset$.

Because of $S \cup T \neq \emptyset$, $s \ge 1$. By (1), $\omega \ge 2s$, which contradicts the fact that G is hamiltonian.

Case 2. $T \neq \emptyset, h_1 \ge k+1$.

If $s = 0$ and $t = 1$, then from (1),

$$\sum_{x \in T} d_{G-S}(x) - k\,|T| - h(S,T) \le \theta - 2 \le 0.$$

So $h(S,T) \ge 1$ and $\theta = 2$. By Lemma 4, there exists a k-even component in $U = G - (S \cup T)$. Thus $\omega \ge 2$, contradicting the fact that G is Hamiltonian. Therefore, $s \ge 1$ or $t \ge 2$. Then $\omega \ge ks + t \ge 2$. Clearly,

$$n - s - t \ge m_1 + m_2(\omega - 1) \ge m_1 + m_2(2s + t - 1). \tag{5}$$

Obviously for any $x \in V(C_1)$, $d_G(x) \le m_1 - 1 + s + t$; for any $y \in V(C_2)$, $d_G(y) \le m_2 - 1 + s + t$. Since x and y are nonadjacent and the assumption of Theorem 3, so if $d_G(x) > \frac{n}{2}$, then $m_1 - 1 + s + t > \frac{n}{2}$. By the above inequality and (5), we get

$$2m_1 - 2 + 2s + 2t > m_1 + m_2(2s + t - 1) + s + t,$$

which yields

$$m_1 + m_2 > 2 + (2m_2 - 1)s + (m_2 - 1)t.$$

This is a contradiction since $s \geq 1$ or $t \geq 2$.

If $d_G(y) > \frac{n}{2}$, then $m_2 - 1 + s + t > \frac{n}{2}$. Then we may have

$$s + t > 2 + m_1 + m_2(2s + t - 3). \tag{6}$$

Since $T \neq \emptyset$, if $s \geq 1$, obvious we obtain a contradiction from (6). If $s = 0$ and $t = 2$, then by (1), $\sum_{x \in T} d_{G-S}(x) - k \mid T \mid -h(S,T) \leq 0$. Since G is Hamiltonian and $\mid S \cup T \mid = 2$, $h(S,T) \leq \omega \leq 2$. So there is a contradiction since $d_G(x) > k + 1$ and $t = 2$. If $s = 1$ and $t = 1$, then we get $2 > 2 + m_1$ by (6), contradicting the fact that $m_1 \geq 3$. If $s + t \geq 3$, from (6) we may obtain $0 > 2 + m_1 + (2m_2 - 1)s + (m_2 - 1)t - 3m_2$. This is a contradiction.

Case 3. $T \neq \emptyset$, $0 \leq h_1 \leq k$ *and* $T - N_T[x_1] = \emptyset$.

In this case, $t \leq h_1 + 1$, i.e. $t \leq k$ otherwise $h_1 = k$. By (4), $ks + (h_1 - k)t - h(S,T) \leq 0$. Since $s > k + 1 - h_1$, we get

$$\omega > k + (k - t)(k - h_1)$$

Since $(k - t)(k - h_1) \geq 0$, $\omega \geq 3$.

At first, we claim that there exists $y_1 \in V(C_1)$ such that x_1 and y_1 are nonadjacent. Otherwise, for any $y \in V(C_1)$, $x_1 y \in E(G)$. Then by Lemma 2,

$$k+1 \leq d_{G-S}(y) \leq m_1 - 1 + t = e_G(x_1, V(C_1)) - 1 + N_T[x_1] \leq d_{G-S}(x_1) \leq k.$$

This is a contradiction. From (2) and (4), we have

$$n - s > n - s - t \geq 3\omega \geq 3ks + 3(h_1 - k)t \geq 3ks + 3(h_1 - k)(h_1 + 1).$$

So

$$n > (3k + 1)s + 3(h_1 - k)(h_1 + 1). \tag{7}$$

Therefore, according to the assumption of the Theorem, $\max\{d_G(x_1), d_G(y_1)\} > \frac{n}{2}$. If $d_G(y_1) > \frac{n}{2}$, then $\frac{n}{2} < d_G(y_1) \leq m_1 - 1 + s + t - 1 \leq \frac{n-s-t}{3} + s + t - 2$. Then

$$4s > n - 4t + 12 \geq n - 4h_1 + 8. \tag{8}$$

Combining (7) and (8), we have

$$\frac{3(k-1)n}{4} < -3h_1^2 + (6k - 2)h_1 - 3k - 2$$

Let $f(h_1) = -3h_1^2 + (6k - 2)h_1 - 3k - 2$. We can obtain its maximum value is $3k^2 + k - 2$, which is a contradiction since $n \geq 4k + 8$ and $k \geq 2$.

If $d_G(x_1) > \frac{n}{2}$, then $s > \frac{n}{2} - h_1$. So from (7) we have

$$\frac{(3k-1)n}{2} < -3h_1^2 + 6kh_1 - 2h_1 + 3k. \tag{9}$$

Let $f(h_1) = -3h_1^2 + 6kh_1 - 2h_1 + 3k$. Using the same method as above, we can show that its maximum value is $3k^2 + k$. This leads to $6k^2 + 10k - 4 < 3k^2 + k$ from (9) since $n \geq 4k + 8$, which contradicts $k \geq 2$.

Case 4. $0 \leq h_1 \leq k - 1$ and $T - N_T[x_1] \neq \emptyset$.

Subcase 4.1. $0 \leq h_1 \leq h_2 \leq k - 1$.

Since $k - h_2 \geq 1$, $(k - h_2)(n - s - t) \geq ks + (h_1 - k)p + (h_2 - k)(t - p)$. Then $(k - h_2)(n - s) - ks > (h_1 - h_2)p \geq (h_1 - h_2)(h_1 + 1)$. So

$$(k - h_2)n \geq (2k - h_2)s + (h_1 - h_2)(h_1 + 1). \tag{10}$$

Since x_1 and x_2 are nonadjacent, if $d_G(x_1) > \frac{n}{2}$, then $s > \frac{n}{2} - h_1$. So $(k-h_2)n \geq (2k-h_2)(\frac{n}{2}-h_1)+(h_1-h_2)(h_1+1)$. From the above inequality we get

$$(\frac{n}{2} - 1)h_2 < (2k - 1)h_1 - h_1^2. \tag{11}$$

Clearly, when $h_1 = k - 1$, the right side of the above inequality attains its maximum value. Therefore $h_2 = k-1$. (9) implies that $((\frac{4k+8}{2}-1))(k-1) < (2k - 1)(k - 1) - (k - 1)^2$ since $n \geq 4k + 8$, which is a contradiction.

If $d_G(x_2) > \frac{n}{2}$, then $s > \frac{n}{2} - h_2$, from (10) we obtain $0 < (2k + 1 - \frac{n}{2})h_2 + h_1h_2 - h_2^2 - h_1^2 - h_1$, which contradicts $n \geq 4k + 8$ and $h_1 \leq h_2$.

Subcase 4.2. $0 \leq h_1 \leq k - 1$ and $h_2 = k$.

Clearly $t \geq p+1$, then by (2) and (4) $n-s-(p+1) \geq 3ks+3(h_1-k)p$, therefore

$$(3k + 1)s \leq n + (3k - 3h_1 - 1)p - 1. \tag{12}$$

According to the assumption, $\max\{d_G(x_1), d_G(x_2)\} > \frac{n}{2}$, which means $s > \frac{n}{2} - h_1$ or $s > \frac{n}{2} - h_2$.

If $s > \frac{n}{2}-h_1$, then from (12) and $p \leq h_1+1$. We obtain $(\frac{n}{2}-h_1)(3k+1) < n + (3k - 3h_1 - 1)(h_1 + 1) - 1$. Then $\frac{(3k-1)n}{2} < (6k - 3)h_1 - 3h_1^2 + 3k - 2$. Let $f(h_1) = (6k - 3)h_1 - 3h_1^2 + 3k - 2$. Obviously, the maximum value of $f(h_1)$ is $3k^2 + 1$ when $h_1 = k - 1$, which contradicts $n \geq 4k + 8$ and $k \geq 2$.

If $s > \frac{n}{2} - h_2 = \frac{n}{2} - k$, from (12) $(\frac{n}{2} - k)(3k + 1) < n + (3k - 3h_1 - 1)(h_1 + 1) - 1$. We can get the desired contradiction by employing the same argument as above.

Subcase 4.3. $h_2 \geq k+1$ and $0 \leq h_1 \leq k-1$. Obviously, $p \leq h_1+1 \leq k$.

Subcase 4.3.1. $p = k$. In this case $h_1 = k - 1$. From (4) we have $ks - k + (h_2 - k)(t - k) - h(S,T) \leq 0$. Since $s > k + 1 - h_1 \geq 2$, $\omega \geq h(S,T) \geq 3k + 1 \geq 7$. First we claim that there exists a $y_2 \in V(C_1)$ such that x_1 and y_2 are nonadjacent. Otherwise, x_1 is adjacent to every vertex in $V(C_1)$. Then by Lemma 2, for any $y \in V(C_1)$,

$$k+1 \leq d_{G-S}(y) \leq m_1-1+k-1 \leq e_G(x_1, V(C_1))-2+N_T[x_1] \leq d_{G-S}(x_1) \leq k-1.$$

This is a contradiction. Hence, if $d_G(x_1) > \frac{n}{2}$, then $s > \frac{n}{2} - h_1$. By (2) and (4), $n - s \geq 3ks + 3(h_1 - k)k + 3$. So $(3k+1)s \leq n - 3(h_1 - k)k - 3$. Since $s > \frac{n}{2} - h_1 = \frac{n}{2} - (k-1)$, we get $(3k+1)(\frac{n}{2} - (k-1)) < n + 3k - 3$, or $\frac{(3k-1)n}{2} < 3k^2 + k - 4$, which contradicts the fact that $n \geq 4k + 8$.

Therefore we may assume that $d_G(y_2) > \frac{n}{2}$ for some $y_2 \in V(C_1)$, then $m_1 - 1 + s + k - 1 > \frac{n}{2}$. Since $t \geq p + 1 = k + 1$, $\frac{n-s-(k+1)}{3} - 1 + s + k - 1 > \frac{n}{2}$. Then $s > \frac{n-4k+14}{4}$. In this case by (2) and (4) we obtain $n - s - (k+1) \geq 3ks + 3(h_1 - k)k + 3$. So $(3k+1)s \leq n - (k+1) - 3 - 3(h_1 - k)k$. From $s > \frac{n-4k+14}{4}$, we get $\frac{(3k-3)n}{4} < \frac{(4k-14)(3k+1)}{4} + 2k - 4$, or $0 < -\frac{21k}{4} - \frac{27}{4}$ since $n \geq 4k + 8$. This is a contradiction since $k \geq 2$.

Subcase 4.3.2. $p \leq k - 1$. From (4) and $s \geq d_G(x_1) - h_1 > \delta(G) - h_1 > k + 1 - h_1$, we have

$$\begin{aligned} \omega &\geq ks + (h_1 - k)p + (h_2 - k)(t - p) \\ &> k(k + 1 - h_1) + (h_1 - k)p + t - p \\ &\geq k + (k - h_1)(k - p) + 1 \geq 4. \end{aligned}$$

Since $n - s - t \geq 3ks + 3(h_1 - k)p + 3(h_2 - k)(t - p)$, we get

$$(3k+1)s \leq n + (3k - 3h_1 - 1)p - 3. \tag{13}$$

We may suppose that $m_2 - 1 + s + t > \frac{n}{2}$ since otherwise there is a contradiction. Let $\omega_0 = 2k - h_1 + t - p$. Then $\omega \geq \omega_0$ and $\omega_0 \geq 4$ in this case. By Lemma 3 we obtain

$$\begin{aligned} \frac{n}{2} &< m_2 - 1 + s + t \\ &\leq \frac{1}{3}[n + 2(s + t + 1 - 2k + h_1 - t + p)] \\ &= \frac{1}{3}[n + 2(s - 2k + h_1 + p + 1)]. \end{aligned}$$

This yields

$$4s > n + 4(2k - h_1 - p - 1). \tag{14}$$

Combining (12) with (13), we get

$$\begin{aligned} 3(k-1)n &< 4[(-2k + h_1 + p + 1)(3k+1) + (3k - 3h_1 - 1)p - 3] \\ &\leq [3(2k - h_1)(k-1) + (-2k + h_1 + 1)(3k+1) - 3] \\ &= 4[4h_1 - 5k - 2] < 0. \end{aligned}$$

This is a contradiction.

Case 5. $h_1 = k$ and $T - N_T[x_1] \neq \emptyset$.

Subcase 5.1. $h_1 = k$ and $h_2 \leq k+2$. By (4) $h(S,T) \geq ks$, then from (2) $n-s-t \geq 3ks$. So $s \leq \frac{n-t}{3k+1} \leq \frac{n-2}{3k+1}$. At first we may suppose $d_G(x_2) > \frac{n}{2}$. Then $\frac{n}{2} < d_G(x_2) \leq s + h_2 \leq \frac{n-2}{3k+1} + k + 2$. Since $n \geq 4k+8$, $6k^2+6k-8 < 0$, which contradicts $k \geq 2$.

Subcase 5.2. $h_1 = k$ and $h_2 \geq k+3$. Obviously, $t > p$. By (4) $ks+(h_2-k)(t-p)-h(S,T) \leq 0$. If $s=0$ and $t-p=1$, then $\omega \geq h(S,T) \geq 3$ and $t = p+1 \leq h_1+2 = k+2$. Using the same method as in case 3, we can show that there exists some $y \in V(C_1)$ such that y and x_1 are nonadjacent. Hence if $d_G(y) > \frac{n}{2}$, then $\frac{n}{2} < \frac{n-s-t}{3} + s + t - 1 = \frac{n+2t-3}{3} \leq \frac{n+2k+1}{3}$. This implies that $n < 4k+2$, which contradicts the assumption that $n \geq 4k+8$. If $d_G(x_1) > \frac{n}{2}$, then $s > \frac{n}{2} - h_1 = \frac{n}{2} - k$. Since $n-s > n-s-t \geq 3ks + 3(h_2-k)(t-p)$, or $(3k+1)s < n - 3(h_2-k)(t-p)$. Combining this inequality with $s > \frac{n}{2} - k$, we get $3k^2+9k+5 < 0$. This is a contradiction.

Therefore we can assume that $s \geq 1$ or $t-p \geq 2$. Since $\omega \geq h(S,T) \geq ks + (h_2-k)(t-p)$, we can know that in either case $\omega \geq 4$. First suppose $d_G(x_1) > \frac{n}{2}$, then $s > \frac{n}{2} - h_1 = \frac{n}{2} - k$. Since $n-s > n-s-t \geq 3ks + 3(h_2-k)(t-p)$, or $(3k+1)s < n - 3(h_2-k)(t-p)$. Combining this inequality with $s > \frac{n}{2} - k$, we get $3k^2+9k+5 < 0$. This is a contradiction.

If $d_G(y) > \frac{n}{2}$ for any $y \in V(C_1)$ such that x_1 and y are nonadjacent, then by Lemma 3 and $t \geq 2$ we have $\frac{n}{2} < m_1 - 1 + s + k - 1 \leq \frac{n-s-2}{4} - 1 + s + k - 1$. So $s > \frac{n-4k+10}{3}$. On the other hand , by (4) in this case, $(3k+1)s \leq n-2$. Combining the above two inequalities we have $(3k-2)n < (3k+1)(4k-10) - 6$, which yields $40k < 0$ since $n \geq 4k+8$. This is a contradiction . Finally, this contradiction complete the proof of the Theorem 5.

4 Proof of Corollary

Based on Theorem 5, we know that G has a k factor excluding any given edge e. Further, according to Theorem 4, $G - e$ contains a Hamiltonian path. From Theorem 3 the proof of Corollary is complete.

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References

[1] J. A. Bondy and U. S. R. Murty, *Graph Theory with Applications*, Macmillan, London (1976).

[2] J. Cai and G. Liu, Binding number and Hamiltonian (g, f)-factors in graphs, *J. Appl. Math. & Computing*, Vol. 25(2007), No. 1-2, pp, 383-388.

[3] C.P. Chen, Minimum degree and binding number for k-factors with prescribed properties. *Acta Math. Scientia* 12(1992), 374-380.

[4] G.Fan, New sufficient conditions for cycles in graphs, *J. Combin. Theory Ser. B* 37(1984) 221-227.

[5] G. Li, Y. Xu, C. Chen and Z. Liu, On connected $(g, f+1)$-factors in graphs, Combinatorica, 25(2005), 393-405.

[6] G.Liu and Y.Liu, On (g, f)-uniform graphs, *Acta Math. Appl. Sinica*, English Ser.21(2005),57-76.

[7] T. Nishmura, A degree condition for the existence of k-factors, *J. Graph Theory,* 16(1992), 141-151.

[8] W. T. Tutte, The factor of graphs, *Can. T. Math* 4(1952), 314-328.

[9] Y. Zhu and G. Liu, Some results on binding number and fractional perfect matching, *J. Appl. Math. & Computing*, Vol. 25(2007), No. 1-2, pp, 339-344.